Guidelines for Review and Evaluation of Backcalculation Results

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FOREWORD

This report documents a new approach, called forwardcalculation, used for determining layered elastic moduli from in situ load-deflection data. Guidelines are provided for carrying out forwardcalculation procedures as well as screening backcalculation results using forwardcalculated moduli.

This report will be of interest to highway agency engineers involved in pavement analysis, design, construction, and deflection data collection, as well as researchers who will use the Long-Term Pavement Performance (LTPP) load-deflection data to improve design procedures and standards for constructing and rehabilitating pavements.

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Notice

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16. Abstract

This document presets a new approach to determining layered elastic moduli from in situ load-deflection data, which was developed under the Federal Highway Administration’s project for reviewing Long-Term Pavement Performance (LTPP) backcalculation data. This approach is called forwardcalculation, and it differs from backcalculation in that modulus estimates are calculated directly from the load and deflection data using closed-form formulae rather than through iteration. The closed-form forwardcalculation equations are used for the subgrade and the bound surface course, respectively, for both flexible and rigid pavement falling weight deflectometer (FWD) data. Intermediate layer moduli are estimated through commonly used modular ratios between adjacent layers.

The audience for this document includes highway agency engineers, researchers, and consultants who are involved in pavement analysis, design, construction, and deflection data analysis.

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
^{o}F	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	^{o}C
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
^{o}C	Celsius	1.8C+32	Fahrenheit	^{o}F
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.
(Revised March 2003)

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CHAPTER 1. INTRODUCTION

BACKGROUND

Backcalculation has long created exciting opportunities along with puzzling obstacles to the analyst assigned to the task of deriving layered elastic parameters based on falling weight deflectometer (FWD) load-deflection data. Each available program has its own set strengths and weaknesses, and no two programs give exactly the same set of results. With luck, the results of two different backcalculation programs may be close to one another.

An approach called forwardcalculation has been developed through the Federal Highway Administration (FHWA) Long-Term Pavement Performance (LTPP) program in a report titled, *Review of the Long-Term Pavement Performance Backcalculation Results—Final Report*.(1) This spreadsheet-based procedure may be used to screen other methods of determining layered elastic properties or as a stand-alone method of determining layered elastic properties for routine pavement rehabilitation design.

This document presents a screening approach for review and evaluation of backcalculated moduli. This approach, which allows users to choose any backcalculation program they wish, offers forwardcalculated values that may be used to compare the results of the two or more methods of evaluation. Ideally, if these approaches give similar (but still not identical) results, one can be reasonably confident that the results obtained through either method will be reasonable and tenable for further use in pavement evaluation and rehabilitation design.

BACKCALCULATION IN A NUTSHELL

Most backcalculation programs, including those used to generate the backcalculated modulus data in the LTPP-computed parameter tables, involve the use of numerical integration subroutines that are capable of calculating FWD pavement deflections and other parameters, given the stiffnesses (or moduli) of the various pavement layers and their thicknesses, etc. If all assumptions are correct, meaning each layer is an elastic layer, is isotropic and homogeneous, and all other boundary conditions are correct, then it is possible to iterate various combinations of moduli until there is a virtually perfect match between measured and theoretical FWD deflections. In this manner, a solution to the problem of deriving moduli from deflections, instead of the other way around, is obtained.

A serious drawback to this approach is the fact that one or more of the many input assumptions mentioned above may be incorrect and therefore do not apply to the actual in situ pavement system where FWD was used to measure deflections. In spite of this potential drawback, many of the moduli derived through backcalculation will appear both reasonable and rational, based on common engineering sense and a working knowledge of pavement materials. This conclusion appears to be especially true when relatively intact, well-defined, and undistressed pavement sections are tested with FWD.

For any pavement system, the engineer using a backcalculation program of choice should be very well versed in its proper use and inherent limitations. Such an expert is able to fine-tune the input data to better model the layered elastic system in a manner that is both rational and suits the particular backcalculation program and the structure of the input data in an advantageous manner. Accordingly, backcalculation is arguably more of an art than a science.

Through forwardcalculation, presented below, it is now possible to screen backcalculated results to see if these results are in the ballpark. For routine testing that is not research related, forwardcalculation may also be used as a stand-alone method of pavement analysis and rehabilitation design.

INTRODUCTION TO FORWARDCALCULATION

Through the forwardcalculation method presented below, it was possible to screen the entire LTPP backcalculation database, which consists of a series of backcalculated computed parameter tables based on the pre-1998 FWD data in the database. Forwardcalculation was used to generate a set of layered elastic moduli that is independent of the backcalculated values, for comparison purposes, to screen the backcalculated moduli in the database. This approach is based on the premise that two substantially different approaches to calculated layered elastic parameters from the same deflection data should produce at least somewhat similar moduli if one is to believe that either approach is credible.

In its current form, forwardcalculation only involves the use of certain portions of the FWD deflection basin to derive an apparent or effective modulus (stiffness) of the subgrade and/or the bound surface course, using closed-form as opposed to iterative solutions. In other words, there is only one directly calculated solution for each of these values, given the deflection data and the layer thicknesses.

The forwardcalculation formulae used to deduce the subgrade modulus mainly use deflections measured at larger distances from the load as well as the center deflection, while the surface course modulus is mainly a function of the near-load deflections and/or the radius of curvature of the deflection basin.

The advantages of forwardcalculation are as follows:

1. Since the subgrade and bound surface course stiffnesses obtained are not dependent on the other moduli within the pavement system, as is the case with backcalculation, there is a unique solution to each problem.
2. Forwardcalculation is easy to understand and use, whereas backcalculation is more of an art than a science. Forwardcalculation can be performed by anyone, while backcalculation requires expert engineering judgment along with the art of running the iterative program of choice. The art is in the evaluation of the reasonableness of the results and proper selection of the model and other input parameters used for backcalculation.
3. Using an elastic layered system and the MODCOMP program, forwardcalculation techniques developed for the LTPP database produce considerably less scatter in the flexible pavement system results (for the same layer and test section) than do backcalculation programs run in batch mode.

Nothing in the way of pavement analysis comes without its own unique drawbacks. As such, these drawbacks are not limited to backcalculation alone, as for example:

1. Since the subgrade and surface course stiffnesses are calculated independently of one another through different forwardcalculation formulae, in combination the values obtained may or may not be reasonable with respect to the total center deflection.
2. The forwardcalculated bound surface course modulus has to be a single value, with all bound layers combined into a single, effective surface course layer.
3. To obtain a third, intermediate layer stiffness (if present), such as a granular layer or crushed rock base, one can assume that the surface and subgrade stiffnesses are correct and then fit the center deflection to the remaining unknown stiffness of, perhaps, a base course layer. This base layer determination approach suffers from the same drawback as backcalculation—one layer's modulus is dependent on the other layers' analysis results.
4. It is also possible to use a ratio between the subgrade modulus calculated through forwardcalculation and apply the modular ratio relationship for unbound base materials developed by Dorman and Metcalf (2) or else apply any other suitable ratio between an intermediate layer and the subgrade or surface course, as deemed appropriate. However, there is no assurance that such a solution is correct, so ultimately one must apply the test of reasonableness to the results.
5. Since forwardcalculation produces approximate values (particularly for the base or intermediate layer/s), these should only be used as modulus estimates, as for example for screening backcalculated moduli, quality assurance/quality control (QA/QC) applications, or routine pavement evaluation purposes.

ORGANIZATION OF REPORT

This report is organized into three chapters. This chapter presents a background of the development of the guidelines, followed by an introduction to backcalculation and an introduction to forwardcalculation. In chapter 2, development of the forwardcalculation technique is described and discussed. Chapter 3 documents the use of the accompanying forwardcalculation spreadsheets, as well as of the forwardcalculation results in reviewing and screening backcalculation or any other modulus determination results.

The Microsoft® Excel spreadsheets containing all formulae used in phase I of this study have been provided to FHWA, so all forwardcalculation input quantities are totally transparent to those who wish to use the methodology, whether for screening or in rehabilitation design. To this end, four spreadsheets are available—two for asphalt-bound surfaces (using SI and U.S. Customary units) and two for cement-bound surfaces (SI and U.S. Customary). These spreadsheets can be obtained by contacting LTPP Customer Support Services by phone at 202–493–3035 or by e-mail at ltppinfo@fhwa.dot.gov.

CHAPTER 2. DEVELOPMENT OF FORWARDCALCULATION METHODOLOGY

BACKGROUND AND PREVIOUS DEVELOPMENTS

Closed-form solutions for determining select layered elastic properties of pavement systems have been used extensively in the past.

In 1884, Boussinesq developed a set of closed-form equations for a semi-infinite, linear elastic half-space (a semi-infinite layer), including the modulus of elasticity of the median, based on a point load. Subsequently, it has been shown that the apparent or composite subgrade modulus derived from any FWD sensor at offset "r" can be calculated from the equation in: (3)

$$E_{o,r} = (0.84 \cdot a^2 \cdot \sigma_o) / (d_r \cdot r)$$

Figure 1. Equation. Composite subgrade modulus at an offset.

where: $E_{o,r}$ = Surface or composite modulus of the subgrade beneath the sensor used
a = Radius of FWD loading plate
σ_o = (Peak) pressure of FWD impact load under loading plate
d_r = (Peak) FWD deflection reading at offset distance r
and r = Distance of deflection reading d_r from center of loading plate

The suggested constant of 0.84 assumes that Poisson's ratio is 0.4 (from the calculation $1-\mu^2$). If d_r is a reasonably large distance from the edge of the loading plate, the load can be assumed to be a point load, so the plate pressure distribution does not matter. Furthermore, small changes in Poisson's ratio have only a minimal impact on this equation.

Subsequent developments have allowed the use of the shape of the deflection basin to estimate various layered elastic (or slab-on-dense-liquid) moduli from FWD deflection readings.

UPPER SUBGRADE MODULUS BASED ON THE HOGG MODEL

One method to ascertain the approximate subgrade stiffness, or elastic modulus, directly under an imposed surface load and in the upper portion of the subgrade is the Hogg model. The Hogg model is based on a hypothetical two-layer system consisting of a relatively thin plate on an elastic foundation. The method in effect simplifies the typical multilayered elastic system with an equivalent two-layer stiff layer on elastic foundation model.

Depending on the choice of values along the deflection basin used to calculate subgrade stiffness, there can be a tendency to either over- or underestimate the subgrade modulus. The Hogg model uses the deflection at the center of the load and one of the offset deflections. The offset distance where the deflection is approximately one-half of that under the center of the load plate was shown to be effective at removing estimation bias. Variations in pavement thickness and the ratio of pavement stiffness to subgrade stiffness are taken into account, since the distance to where the deflection is one-half of the deflection under the load plate is controlled by these factors.

The underlying model development for a finite subgrade was first published in 1944 by Hogg.(4) The implementation of the model used in these guidelines was subsequently published in 1983 by Wiseman and Greenstein.(5)

The equations used are as follows:

$$E_0 = I \frac{(1+\mu_0)(3-4\mu_0)}{2(1-\mu_0)} \left(\frac{S_0}{S} \right) \left(\frac{p}{\Delta_0 l} \right)$$

Figure 2. Equation. Hogg subgrade modulus.

$$r_{50} = r\frac{(1/\alpha)^{1/\beta} - B}{\left[\frac{1}{\alpha}\left(\frac{\Delta_0}{\Delta_r} - 1\right)\right]^{1/\beta} - B}$$

Figure 3. Equation. Offset distance where deflection is half of center deflection.

$$l = y_0\frac{r_{50}}{2} + \left[(y_0 r_{50})^2 - 4mar_{50}\right]^{\frac{1}{2}}$$

if $\frac{a}{l} < 0.2$**, then** $l = (y_0 - 0.2m)r_{50}$

Figure 4. Equation. Characteristic length of deflection basin.

$$\left(\frac{S_0}{S}\right) = 1 - \overline{m}\left(\frac{a}{l} - 0.2\right)$$

if $\frac{a}{l} < 0.2$**, then** $\left(\frac{S_0}{S}\right) = 1.0$

Figure 5. Equation. Theoretical point load stiffness/pavement stiffness ratio.

where:

E_0 = Subgrade modulus
μ_0 = Poisson's ratio for subgrade
S_0 = Theoretical point load stiffness
S = Pavement stiffness = p/Δ_0 (area loading)
p = Applied load
Δ_0 = Deflection at center of load plate
Δ_r = Deflection at offset distance r
r = Distance from center of load plate
r_{50} = Offset distance where $\Delta_r/\Delta_0 = 0.5$
l = Characteristic length
h = Thickness of subgrade
I = Influence factor—see Table 1 below
α = Curve fitting coefficient—see Table 1
β = Curve fitting coefficient—see Table 1
B = Curve fitting coefficient—see Table 1
y_0 = Characteristic length coefficient—see Table 1
m = Characteristic length coefficient—see Table 1
$\overline{m}$ = Stiffness ratio coefficient—see Table 1

The implementation of the Hogg model described by Wiseman and Greenstein included three cases. Case I is for an infinite elastic foundation, while cases II and III are for a finite elastic layer with an effective thickness that is assumed to be approximately 10 times the characteristic length, *l*, of the deflection basin. These two finite thickness cases are for subgrades with Poisson's ratios of 0.4 and 0.5, respectively. The various constants used for the three versions of the Hogg model are shown in Table 1. Use of Case II is recommended to obtain realistic design values, and it has been used extensively to calculate subgrade moduli for purposes of pavement evaluation through forwardcalculation.

Table 1. Hogg model coefficients.

	Hogg model case		Case I	Case II	Case III
	Depth to hard bottom	H/l	10	10	Infinite
Eqn.	Poisson's ratio	M_0	0.50	0.40	All Values
2	Influence factor	I	0.1614	0.1689	0.1925
3	Range Δ_r/Δ_0		> 0.70	> 0.43	All Values
	$r_{50}=f(\Delta_r/\Delta_0)$	A	0.4065	0.3804	0.3210
		B	1.6890	1.8246	1.7117
		B	0	0	0
	Range Δ_r/Δ_0		< 0.70	< 0.43	
	$r_{50}=f(\Delta_r/\Delta_0)$	A	2.6947E-3	4.3795E-4	
		B	4.5663	4.9903	
		B	2	3	
4	$l=f(r_{50},a)$	Y_0	0.642	0.603	0.527
		M	0.125	0.108	0.098
5	$(S/S_0)=f(a/l)$	$\overline{m}$	0.219	0.208	0.185

Case II of the Hogg model has been used extensively over the past 15 years or more, and it has been found to be reasonably stable on a wide variety of pavement types and locations, tending to have a high correlation with backcalculated subgrade moduli but with significantly lower (and therefore more conservative) results than the corresponding backcalculated values. This difference is generally due to the presence of apparent or actual subgrade nonlinearity (effectively, stress softening) as well as the calculation of a finite depth of subgrade = 10 x l (as defined by Case II) to an effective hard bottom layer of either deeper lying subgrade material or actual bedrock.

In addition, less variation is indicated between FWD test points along the same test section when the Hogg model is used in forwardcalculation when compared to virtually any backcalculation approach.

Both as a screening tool and to derive relatively realistic, in situ subgrade stiffnesses, the Hogg model is effective and very easy to use compared to other methods.

BOUND SURFACE COURSE MODULUS BASED ON THE AREA METHOD

A viable method to determine the apparent surface course stiffness of the uppermost bound layer(s), under an imposed surface load is called the AREA method.

This approach was first introduced in the National Cooperative Highway Research Program (NCHRP) Study 20-50(09), *LTPP Data Analysis: Feasibility of Using FWD Deflection Data to Characterize Pavement Construction Quality*.(6) More recently, the equations originally suggested have been updated and calibrated for both asphalt concrete (AC) and portland cement concrete (PCC) pavement surfaces.

For both pavement types, the radius of curvature method is based on the AREA concept (a deflection basin curvature index) and the overall composite modulus of the entire pavement structure, E_o, as defined in Figure 6.

$$E_o = (1.5 \bullet a \bullet \sigma_o) / d_o$$

Figure 6. Equation. Composite modulus under FWD load plate.

where: E_o = Composite modulus of the entire pavement system beneath the load plate
a = Radius of FWD load plate

σ_o = (Peak) pressure of FWD impact load under the load plate
and d_o = (Peak) center FWD deflection reading

Figure 6 has been extensively used over the past three to four decades. An excellent introduction to this approach is presented by Ullidtz in *Pavement Analysis*. (3)

Figure 6 is the most commonly used version of this equation. In theory, it is based on an evenly distributed and uniform FWD test load and a Poisson's ratio of 0.5. Generally, Poisson's ratio is less than 0.5 (between 0.15 and 0.20 for PCC layers and between 0.3 and 0.5 for most other pavement materials). On the other hand, the distribution of the load under the FWD plate will not be exactly uniform, but rather somewhat nonuniform because of the rigidity of the loading plate. These two offsetting factors have resulted in the widely used and straightforward 1.5-times composite modulus formula, which was therefore chosen for the development of the forwardcalculation spreadsheets.

AREA, as used for rigid pavements in this study and reported by the American Association of State Highway and Transportation Officials (AASHTO) in the 1993 *AASHTO Guide for Design of Pavement Structures,* (7) has been historically calculated as:

$$A_{36} = 6 * [1 + 2(d_{12}/d_0) + 2(d_{24}/d_0) + (d_{36}/d_0)]$$

Figure 7. Equation. 914-millimeter (mm) (36-inch) AREA equation for rigid pavements.

where: A_{36} = AREA beneath the first 914 mm (36 inches) of the deflection basin
d_0 = FWD deflection measured at the center of the FWD load plate
d_{12} = FWD deflection measured 305 mm (12 inches) from the center of the plate
d_{24} = FWD deflection measured 610 mm (24 inches) from the center of the plate
and d_{36} = FWD deflection measured 914 mm (36 inches) from the center of the plate

When calculating AREA_{36}, the diameter of the loading plate must be between 300 mm (11.8 in) and 305 mm (12 in). An AREA_{36} calculation of 36 is achieved if all 4 deflection readings, at the 0-, 305-, 610-, and 914-mm (0-, 12-, 24-, and 36-inch) offsets, are identical, which is tantamount to an infinitely stiff upper layer.

While the equation in Figure 7 is well suited for rigid pavements with a large radius of curvature, flexible pavements generally have a much smaller radius of curvature (i.e., a steeper deflection basin). Accordingly, for AC pavements a new version of the AREA concept based on FWD sensors placed at 0-, 203-, and 305-mm (0-, 8-, and 12-inch) offsets was derived:

$$A_{12} = 2 * [2 + 3(d_8/d_0) + (d_{12}/d_0)]$$

Figure 8. Equation. 305-mm (12-inch) AREA equation for flexible pavements.

where: A_{12} = AREA beneath the first 305 mm (12 inches) of the deflection basin
d_0 = FWD deflection measured at the center of the FWD load plate
d_8 = FWD deflection measured 203 mm (8 inches) from the center of the plate
and d_{12} = FWD deflection measured 305 mm (12 inches) from the center of the plate

An AREA_{12} calculation of 12 is achieved if all three deflection readings, at the 0-, 203-, and 305-mm (0-, 8-, and 12-inch) offsets, are identical, which is tantamount to an infinitely stiff upper layer.

For AC and PCC pavement types, respectively, a series of calculations were made to see what the relevant AREA terms become if all layers in a multilayered elastic system have identical stiffnesses or moduli (and Poisson's ratios). This can be carried out using, for example, the CHEVRON, CHEVLAY2, ELSYM5, or BISAR multilayered elastic programs (CHEVLAY2 was used in this case). It turns out that, no matter which modulus value is selected, as long as all of the layers are assigned the same identical modulus of elasticity and there is continuity between layers (generally assumed in backcalculation as well), the $AREA_{36}$ term is always equal to 11.04 for rigid pavements (assuming no bedrock) and $AREA_{12}$ is always equal to 6.85 if bedrock is assumed for flexible pavements. (Note: The $AREA_{12}$ calculation for identical moduli with no bedrock = 6.91—nearly identical.) The reason that bedrock was assumed for AC and not PCC pavements is that FWD deflection readings generally reflect the presence of an apparent underlying stiff layer for flexible pavements but not for rigid pavements. (Using either approach, however, the resulting calculations for upper layer pavement stiffness will be nearly the same, whether or not bedrock is assumed.)

The minimum AREA values of 11.04 and 6.85 for the 914-mm (36-inch) and 305-mm (12-inch) areas, respectively, are important in the following equations because they can now be used to ascertain whether the upper layer has a significantly higher stiffness than the underlying layer(s), and to what extent this increase affects the stiffness of the upper, bound pavement layers. For example, if the $AREA_{36}$ term is much larger than 11.04, then the concrete layer is appreciably stiffer than the underlying (unbound) layer(s). The value 11.04 is therefore used in Figure 9, below, while Figure 10 can be thought of as a radius of curvature stiffness index, based on the stiffness of the bound upper layer(s) compared to the composite stiffness of the underlying unbound layers.

The calculation of E_o was previously explained in connection with the presentation of Figure 6. To reiterate, E_o is a composite, effective stiffness of all the layers under the FWD loading plate. If these two terms are combined such that the boundary conditions are correct and the logic of the two AREA concepts, for PCC and AC pavements respectively, are adhered to, the following equations result:

$$AF_{PCC} = [(k_2 - 1) / \{k_2 - (AREA_{36} / k_1)\}]^{1.79}$$

Figure 9. Equation. AREA factor for rigid pavements.

where: AF_{PCC} = AREA factor, i.e., the improvement in AREA from 11.04 to the 1.79 power
k_1 = 11.04 (the AREA when the stiffness of the concrete layer is the same as the lower layers)
k_2 = 3.262 (maximum possible improvement in AREA = 36 / 11.037)

$$AF_{AC} = [(k_2 - 1) / \{k_2 - (AREA_{12} / k_1)\}]^{1.35}$$

Figure 10. Equation. AREA factor for flexible pavements.

where: AF_{AC} = AREA factor, i.e., the improvement in AREA to the 1.35 power
k_1 = 6.85 (the AREA when the stiffness of the asphalt layer is the same as the lower layers)
k_2 = 1.752 (maximum possible improvement in AREA = 12 / 6.85)

$$E_{PCC} = [E_o * AF_{PCC} * k_3^{(1/AFpcc)}] / k_3^{2.38}$$

Figure 11. Equation. Stiffness or modulus of the upper PCC layer.

$$E_{AC} = [E_o * AF_{AC} * k_3^{(1/AFac)}] / k_3^{2}$$

Figure 12. Equation. Stiffness or modulus of the upper AC layer.

where: E_{PCC} = Stiffness or modulus of the upper PCC (bound) layer(s)
E_{AC} = Stiffness or modulus of the upper AC (bound) layer(s)
E_o = As defined in Figure 6
AF = As defined in Figure 9 for PCC or Figure 10 for AC
k_3 = Thickness ratio of upper layer thickness / load plate diameter = $h_1 / (2*a)$
and a = Radius of the FWD load plate.

Both Figure 11 and Figure 12 have been calibrated using a large number of CHEVLAY2 runs, and they work very well for typical pavement materials and modular ratios when the underlying materials are unbound. It should be noted that this approach is not totally rigorous but rather is empirical in nature. The approach can therefore be used effectively to approximate the stiffness of the upper (bound) layer(s) in a pavement cross section, for QC, comparative, or routine testing and analysis purposes.

The advantage of using the equations in Figure 9 through Figure 12, or similar equations developed elsewhere, is that forwardcalculation techniques, together with commonly used deflection-based quantities (such as AREA), can be employed. Only the composite modulus or stiffness of the pavement system, the AREA, and the pavement thickness normalized to the diameter of the loading plate, are needed to calculate the relative stiffness of the bound upper layer(s) of pavement.

INTERMEDIATE LAYER MODULUS CALCULATIONS

Forwardcalculation techniques, as shown in the two previous sections concerning the subgrade and bound surface courses, can in turn be used to estimate the modulus of any intermediate layer through the use of modular ratios. For example, the modulus relationship developed by Dorman and Metcalf between two adjacent layers of materials can be used if the base and subgrade layers are unbound.(\2) The Dorman and Metcalf method computes the base modulus, as shown by the equations in Figure 13 (U.S. Customary units) and Figure 14 (SI units).

$$E_{Base} = 0.86 \bullet h_2^{0.45} \bullet E_{Sub}$$

Figure 13. Equation. Modulus (psi) of the unbound base using the Dorman and Metcalf relationship.

where: E_{Base} = Dorman and Metcalf base modulus, psi
h_2 = Thickness of the intermediate base layer, inches
and E_{Sub} = Subgrade modulus, psi

$$E_{Base} = 0.2 \bullet h_2^{0.45} \bullet E_{Sub}$$

Figure 14. Equation. Modulus (MPa) of the unbound base using the Dorman and Metcalf relationship.

where: E_{Base} = Dorman and Metcalf base modulus, MPa
h_2 = Thickness of the intermediate base layer, mm
and E_{Sub} = Subgrade modulus, MPa

Another technique, sometimes used for rigid pavement sections with bound base courses, is to relate the apparent modulus of the PCC layer, $E_{pcc,app.}$, to a calculated modulus of the PCC layer (E_1) and the base course layer (E_2) expressed as a ratio between these two layers. This calculation is a function of the thickness of each layer and whether these layers are bonded or unbonded (i.e., whether there is slip between the two uppermost layers, under load).

As previously shown, only the PCC surface course and the subgrade moduli are forwardcalculated, essentially ignoring the effect of the base layer. Therefore, the forwardcalculated E_{PCC} actually reflects the effect of both the upper PCC layer and the underlying base layer. In other words, E_{PCC} is really an apparent modulus of the PCC layer, and needs to be divided into two parts, especially when a bound base layer is involved: the actual modulus of the PCC and the calculated modulus of the base. In these cases, E_{PCC} is called $E_{pcc,app.}$, which is the apparent modulus of the PCC layer alone when it is significantly influenced by the base.

The method to divide the calculated $E_{pcc,app.}$-value into two parts is adopted from Khazanovich, et al.(8) The upper PCC surface layer and the base layer may be bonded or unbonded, as appropriate, and are assumed to act as plates. Thus, no through-the-thickness compression is assumed. The details of this method are given below for an unbonded and a bonded condition between the PCC slab and the base, respectively.

For the unbonded case, the PCC slab modulus is computed from Figure 15.

$$E_1 = \frac{h_1^3}{h_1^3 + \beta h_2^3} E_{pcc}$$

Figure 15. Equation. PCC slab modulus—100 percent unbonded case.

For the bonded case, the PCC slab modulus is computed from Figure 16.

$$E_1 = \frac{h_1^3}{h_1^3 + \beta h_2^3 + 12h_1\left(x - \frac{h_1}{2}\right)^2 + 12\beta h_2\left(h_1 - x + \frac{h_2}{2}\right)^2} E_e$$

Figure 16. Equation. PCC slab modulus—100 percent bonded case.

where:

$$x = \frac{\frac{h_1^2}{2} + \beta h_2\left(h_1 + \frac{h_2}{2}\right)}{h_1 + \beta h_2}$$

Figure 17. Equation. Layer thickness relationship—both cases.

and:

$$\beta = \frac{E_2}{E_1}$$

Figure 18. Equation. Modular ratio β—both cases.

and:

$E_{pcc,app}$ = Apparent modulus of the PCC layer assuming no base course effect
E_1 = Modulus of upper plate, i.e., the PCC layer
E_2 = Modulus of lower plate, i.e., the base layer
h_1 = Thickness of upper plate, i.e., the PCC slab
h_2 = Thickness of lower plate, i.e., the base layer

The procedures presented above require the modular ratio as an input parameter. This ratio should be assigned based on engineering judgment. It is assumed that if the ratio is assigned within reasonable limits, the PCC modulus (= E_1) results are insensitive to the chosen ratio. Table 2 presents a set of recommended modular ratios (β) of the PCC (E_1) and base (E_2) moduli for each type of base layer. It should be noted that β from Figure 18 is defined as a ratio of base to PCC moduli. This was done to make it stable for the case of a weak base (i.e., when β approaches 0). Therefore, if the modular ratios from Table 2 are used, these should be inverted before applying them in the procedures described above.

Given the values for β and for the actual plate thicknesses, h_1 and h_2, the equations in Figure 15 and Figure 16 may used with the forwardcalculated $E_{pcc,app.}$-value to yield E_1 and E_2 for the two upper layers. Alternatively, any other modular ratio may be used, as appropriate, depending on the actual materials present in any given project.

Table 2. Back- and forwardcalculated modular ratios for E_{PCC} /E_{Base}.

LTPP Code	Base Type	Ratio $\beta^*=1/\beta$
1	Hot-mixed, hot-laid asphalt concrete (AC), dense graded	10
2	Hot-mixed, hot-laid AC, open graded	15
3	Sand asphalt	50
4	Jointed plain concrete pavement (JPCP)	1
5	Jointed reinforced concrete pavement (JRCP)	1
6	Continuously reinforced concrete pavement (CRCP)	1
7	PCC (prestressed)	1
8	PCC (fiber reinforced)	1
9	Plant mix (emulsified asphalt) material, cold-laid	20
10	Plant mix (cutback asphalt) material, cold-laid	20
13	Recycled AC, hot-laid, central plant mix	10
14	Recycled AC, cold-laid, central plant mix	15
15	Recycled AC, cold-laid, mixed-in-place	15
16	Recycled AC, heater scarification/recompaction	15
17	Recycled JPCP	100
18	Recycled JRCP	100
19	Recycled CRCP	100
181	Fine-grained soils: lime-treated soil	100
182	Fine-grained soils: cement-treated soil	50
183	Bituminous treated subgrade soil	100
292	Crushed rock	150
302	Gravel, uncrushed	200
303	Crushed stone	150
304	Crushed gravel	175
305	Crushed slag	175
306	Sand	250
307	Soil-aggregate mixture (predominantly fine-grained)	400
308	Soil-aggregate mixture (predominantly coarse-grained)	250
319	Hot-mixed AC	15

Table 2. Back- and forwardcalculated modular ratios for E_{PCC} /E_{Base}—*Continued*

LTPP Code	Base Type	Ratio $\beta^* = 1/\beta$
320	Sand asphalt	50
321	Asphalt-treated mixture	50
322	Dense-graded, hot-laid, central plant mix AC	10
323	Dense-graded, cold-laid, central plant mix AC	15
324	Dense-graded, cold-laid, mixed-in-place AC	15
325	Open-graded, hot-laid, central plant mix AC	15
326	Open-graded, cold-laid, central plant mix AC	15
327	Open-graded, cold-laid, mixed-in-place AC	15
328	Recycled AC, plant mix, hot-laid	10
329	Recycled AC, plant mix, cold-laid	15
330	Recycled AC, mixed-in-place	15
331	Cement aggregate mixture	5
332	Econocrete	4
333	Cement-treated soil	50
334	Lean concrete	2
335	Recycled portland cement concrete	100
338	Lime-treated soil	100
339	Soil cement	10
340	Pozzolanic-aggregate mixture	100
341	Cracked and seated PCC layer	25
351	Treatment: lime, all classes of quick lime and hydrated lime	100
352	Treatment: lime, fly ash	150
353	Treatment: lime and cement fly ash	150
354	Treated: portland cement	50
355	Treatment: bitumen (includes all classes of bitumen and asphalt treatments)	100
700	AC	15
730	PCC	1

FORWARDCALCULATION IN PRACTICE

In summary, it should be emphasized that forwardcalculated modulus data are not intended to replace backcalculation or any other form of modulus of elasticity measurements. Forwardcalculation, like all other methods of determining in situ stiffnesses or moduli, merely provide the analyst with approximations or estimates of such values. The only question is: how realistic are such estimates for pavement evaluation or design purposes?

Accordingly, forwardcalculation is designed for routine FWD-based project use, and for screening purposes for data derived using backcalculation techniques. It is ultimately intended to ascertain whether backcalculated modulus values—which are also estimates—are reasonable, since two distinctly different methods of deriving stiffnesses or moduli from the same FWD load-deflection data should not produce vastly dissimilar results.

In the following section, the use of the provided forwardcalculation spreadsheets is presented.

CHAPTER 3. FORWARDCALCULATION SPREADSHEETS

OVERVIEW

Using the forwardcalculation principles outlined in the preceding sections, four generic forwardcalculation spreadsheets are provided, as follows:

- Forwardcalculation for flexible pavement sections using U.S. Customary units.
- Forwardcalculation for flexible pavement sections using SI units.
- Forwardcalculation for rigid pavement sections using U.S. Customary units.
- Forwardcalculation for rigid pavement sections using SI units.

Please note, however, that there are several constraints to using these spreadsheets. One constraint is that they are presently formatted for seven deflection readings. Accordingly, if your FWD generates more than seven deflection readings, the analyst should only paste in data for the seven most appropriate sensor positions obtained, depending on the units and type of pavement, etc.

Before using the provided forwardcalculation spreadsheets, the following constraints should be noted:

Notes on the FWD Deflection Data

For flexible pavements, three of the seven chosen deflection readings must be positioned either at 0, 8, and 12 inches or 0, 200, and 300 mm. Further, these three must be ordered as the first three of the seven deflection positions selected. For rigid pavements, four of the chosen seven deflection readings must be positioned either at 0, 12, 24, and 36 inches or at 0, 300, 600, and 900 mm. Furthermore, these four must be the first, third, fifth, and sixth of the seven deflection positions selected. The use of these critical positions makes it possible to calculate either the $AREA_{12}$ term or the $AREA_{36}$ term for AC or PCC pavements, respectively. The appropriate AREA term is, in turn, used in the calculation of the bound surface course stiffness. The remaining sensor positions should be chosen such that they span the region in the deflection basin where one of these is approximately one-half of the center deflection reading, for all lines of FWD input data. This enables proper use of the Hogg model for forwardcalculating the effective subgrade modulus and the depth to the effective hard layer.

Although any drop-by-drop FWD data may be used as input, to improve the random accuracy of the deflection readings, it is recommended that averages of multiple drops are used, especially in the case of rigid pavements or flexible pavements where the asphalt temperature is very low. This is important because the basin will be very flat, and the method is highly sensitive to very small errors in any of the AREA terms used for forwardcalculation of the bound surface course.

Determine the Pavement Layer Structures for Forwardcalculation

When using the forwardcalculation techniques, the operator must forwardcalculate elastic moduli for up to three layers for each deflection basin. Given that requirement, the pavement system being analyzed must be divided or combined into a two- or three-layer structure, as follows:

1. Surface (bound) layer (AC or PCC).
2. Base layer (unbound or granular for flexible pavements, unbound or treated for rigid pavements).
3. Subgrade layer; depth to apparent stiff layer calculated from the deflection basin.

For rigid pavements, the uppermost base layer below the PCC slab was considered the base layer.

USING THE FORWARDCALCULATION SPREADSHEETS

The four forwardcalculation spreadsheets provided are self-explanatory in almost all circumstances. For first-time users, the following steps should be carried out when using any of these spreadsheets:

1. Load the Microsoft® EXCEL spreadsheet of choice (flexible or rigid pavement type, U.S. Customary or SI units). If the necessary options and/or add-ins are installed in your version of EXCEL, the spreadsheet will ask whether you want to load the macros or not. To this you should click the icon reading, "Enable Macros." If they are not installed, please install any necessary options or add-ins.
2. Once the spreadsheet is loaded, save it under a new name so the template is not lost—forever!
3. All gray shaded areas in the spreadsheet must be filled in with correct input data. If these data are pasted in from another worksheet, to retain the gray shading the input data should also be shaded gray.
4. In cell I-2, type in a name or identifier for the project.
5. In cells C-8 through R-8, and down to a maximum of about 1,000 lines of load-deflection data, paste in the FWD data you wish to process through forwardcalculation. Use the format and (especially) the units shown in the example provided. (Note: Columns C through J of input data are for identification purposes only, and are not used for any forwardcalculation purposes—you may change the headings or choose to not use, as appropriate.)
6. In cell U-5, fill in the plate radius. Please note, as indicated in cell U-6, that there are only two possible radii: 300 mm or 12 inches (or equivalent in opposite units).
7. In cells W-5 through AB-5, fill in the sensor positions used, in the appropriate units. Note that, in cell V-5 (= 0), the center deflection is required and that some of the cells between W-5 and AA-5 are also required, as previously discussed in connection with the determination of the AREA term. The notes shown in cells W-6 through AA-6 indicate which of these are required, at which positions, and the required units.
8. The constant in cell AO-5 is needed only if you wish to run a calculation for stresses, strains, and deflections after the layer moduli are obtained. This allows for the use of a stiff or hard layer at depth in such calculations, together with the thickness of the upper subgrade and the modulus of this hard layer. In each example provided, the hard layer is assumed to have an effective modulus of three times the subgrade modulus. Change this factor if you have evidence that there is bedrock near the surface and this factor should be higher than three.
9. In cells AH-8 through AH-xxx (as far down in the worksheet as the deflection data are entered), enter the thickness of the bound surface course in the specified units. This can either be the same for the entire worksheet or different for every station, as desired. Please recall that the bound surface course thickness is the sum of all the bound layers in the pavement, not only the AC or PCC surfacing. An exception to this rule occurs if you intend to apply the formulas presented in the equations shown in Figure 11, Figure 12, Figure 15, and Figure 16, in which case you should enter only the thickness of the PCC slab.
10. If the surface course thickness is the same for the entire worksheet, it is possible to enter a constant value in cell AI-5 and create a formula for the entire column AH from cell AH-8 to the bottom of the data, to copy in this constant value. Otherwise, cell AI-5 is not used. (Note: It is highly recommended that the minimum surface course thickness used be 3 inches or 75 mm.)
11. For the flexible spreadsheets only, cells AV-8 through AV-xxx (as far down as the deflection data are entered) are available to employ the Dorman and Metcalf relationship discussed above, provided the base and subgrade consist of unbound materials. Enter the thickness of the unbound base material in the specified units. This can either be the same for the entire worksheet or different for every station, as desired. Please note that only one intermediate layer is allowed, so this must be the sum of all improved intermediate layers. These intermediate base course layers should be in the 2–24 inch or 50–600 mm range.
12. If the base course thickness is the same for the entire worksheet, it is possible to enter this constant value in cell AZ-5 and create a formula for the entire row AV, from cell AV-8 to the bottom of the file, to copy in this constant value. Otherwise, cell AZ-5 is not used. (Note: It is recommended that the use of the Dorman and Metcalf relationship is limited to granular-type bases, not improved subgrade materials, which should form part of the subgrade, not the base.)

13. The formulae and columns that are not shaded gray will need to be copied from the first few lines of data after all input data has been entered into the gray cells.
14. The statistical results displayed in the examples provided at the top of the worksheet are for the entire file, down to around 1000 lines of FWD load-deflection data. Of course, any portion of the data may be manipulated, as desired, for example, by eliminating spurious input data (e.g., nondecreasing deflections) or dividing the file into uniform subsections, with a separate worksheet for each, etc.

WHAT TO DO WITH THE RESULTS OF BACK- AND FORWARDCALCULATION

After either back- or forwardcalculation has been carried out to the satisfaction of the analyst, all values (or averages and coefficients of variance thereof) should be checked for reasonableness before using these data for pavement design purposes. Table 3 provides a broad range of modulus values for various pavement materials that may be considered reasonable.

When using the broad ranges shown in Table 3, common sense should be exercised as well. For example, the range for asphalt-bound surface courses covers a very broad temperature range, with the higher parts of the range shown covering colder pavement temperatures (down to freezing) and the lower part covering temperatures as high as 45° C (~115° F).

If the analyst feels that either back- or forwardcalculation resulted in any modulus values outside of the ranges shown in Table 3, or any other appropriate ranges, these should be rejected as either unrealistic or unreasonable. If both methods produced values within the ranges shown in Table 3, the pairs of values may be compared (or corresponded) and designated as recommended in Table 4.

As can be seen in Table 4 and with the variability of in situ materials in mind, it is felt that an acceptable correspondence between back- and forwardcalculated moduli can be considered to be within a factor of 1.5 (times or divided by) the screening value calculated through forwardcalculation. (Again, this is as long as both values are still within the reasonable ranges shown in Table 3.) In such a case, either value may be selected for pavement design purposes.

Consider as well that the subgrade modulus backcalculated through the Hogg model as presented herein is usually lower than that obtained through classical backcalculation. This is mainly because the Hogg model only calculates the effective subgrade modulus under the load plate and to a finite depth, as indicated in the forwardcalculation spreadsheet output. Backcalculation, in most cases, assumes that the subgrade extends to an infinite (or even a fixed finite) depth, and is the same at all deflection basin offset distances (sensors #2 through #7 or more), including under the load plate (sensor #1).

As a result, in the LTPP database, for example, the forwardcalculated subgrade modulus was around half of the backcalculated subgrade modulus, on average. Therefore, if one uses the traditional AASHTO design formula, where the design subgrade modulus is assumed to be one-third that of the backcalculated modulus, then the forwardcalculated subgrade moduli should not be divided by three, or the design will be far too conservative.

It is not recommended that some of the modulus values from backcalculation and some of the values from forwardcalculation be used for the design of a single uniform section, but rather the entire set of either backcalculated or forwardcalculated values should be used, depending on the reasonableness of the values obtained.

Microsoft® Excel spreadsheets have been prepared containing all formulae used in phase I of this study. All forwardcalculation input quantities are totally transparent to those who wish to use the methodology, whether for screening or in rehabilitation design. To this end, four spreadsheets are available—two for asphalt-bound surfaces (using SI and U.S. Customary units) and two for cement-bound surfaces (SI and U.S. Customary). These spreadsheets can be obtained by contacting LTPP Customer Support Services: by phone at 202–493–3035 or by e-mail at ltppinfo@fhwa.dot.gov.

Table 3. Reasonable ranges for various pavement layers from the LTPP database.

LTPP Code	Base Materials	Min. Range (MPa)	Max. Range (MPa)	Min. Range (psi)	Max. Range (psi)
321	Asphalt-treated mixture, not permeable asphalt-treated base (PATB)	700	25,000	101,500	3,625,000
302	Gravel, uncrushed	50	750	7,250	108,750
303	Crushed stone	100	1,500	14,500	217,500
304	Crushed gravel	75	1,000	10,875	145,000
306	Sand	40	500	5,800	72,500
307	Soil-aggregate mixture (predominantly fine-grained)	50	700	7,250	101,500
308	Soil-aggregate mixture (predominantly coarse-grained)	60	800	8,700	116,000
309	Fine grained soil or base	35	450	5,100	65,000
319	Hot-mixed AC	700	25,000	101,500	3,625,000
320	Sand asphalt	700	25,000	101,500	3,625,000
323	Dense-graded, cold-laid, central plant mix AC	700	25,000	101,500	3,625,000
325	Open-graded, hot-laid, central plant mix AC (PATB)	350	3,500	50,750	507,500
331	Cement aggregate mixture	2,000	20,000	290,000	2,900,000
332	Econocrete	3,500	35,000	507,500	5,075,000
334	Lean concrete	4,500	45,000	652,500	6,525,000
339	Soil cement	1,000	7,000	145,000	1,015,000
327	Open-graded, cold-laid, in-place mix AC	200	3,000	29,000	435,000
337	Limerock; caliche	150	1,500	21,750	217,500
350	Other—treated base	400	8,000	58,000	1,160,000
	Bound surface courses:				
	Concrete surface (uncracked)	10,000	70,000	1,450,000	10,150,000
	AC surface (>0^{O} C–<45^{O} C, not alligatored)	700	25,000	101,500	3,625,000
	Unbound subgrades:				
	Any unbound type	15	650	2,175	94,250

Table 4. Flagging codes used to screen the backcalculated LTPP database.

Description of the Correspondence Between the Forwardcalculated and the Backcalculated Modulus Values	Correspondence Codes	Ratio Between the Forwardcalculated and Backcalculated Modulus Values
Acceptable	0	2/3 < Ratio ≤ 1.5
Marginal	1	1/2 < Ratio ≤ 2 (& not code 0)
Questionable	2	1/3 < Ratio ≤ 3 (& not codes 0 or 1)
Unacceptable	3	Ratio ≤ 1/3 or Ratio > 3

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