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Development of Performance-Related Specifications for Portland Cement Concrete Pavement Construction

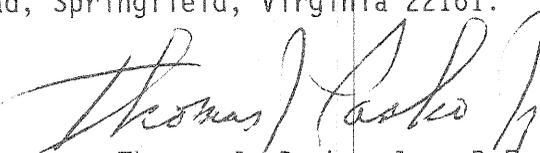
Research, Development, and Technology
Turner-Fairbank Highway Research Center
6300 Georgetown Pike
McLean, Virginia 22101-2296

FOREWORD

This report presents the results of a study to further the development of performance-related specifications for portland cement concrete pavement construction. Laboratory testing was conducted to investigate the relationships between materials variables and primary predictors of pavement distresses. A demonstration performance-related specifications system was developed, allowing users to determine the appropriate percentage of bid price that a contractor should receive for concrete of a given quality. Recommendations pertaining to future laboratory studies, field studies, and further development of performance-related specification systems are also summarized.

This work was conducted as part of Nationally Coordinated Program E8, "Construction Control and Management," and is intended for engineers concerned with quality assurance, specifications, and construction of concrete pavements.

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Thomas J. Pasko, Jr., P.E.
Director, Office of Engineering
and Highway Operations
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16. Abstract <p>The primary product of this study was a demonstration performance-related specification (PRS) system for PCC pavement construction. The system is designed to consider three key factors (i.e., PCC strength, slab thickness and initial serviceability) in assessing an as-constructed pavement delivered by a contractor and calculating an appropriate reward (bonus/incentive) or penalty (disincentive).</p> <p>Many pavement performance prediction relationships and PCC property prediction equations were evaluated to develop the performance-related aspects of the new system. In addition, a rather intensive experimental laboratory study of PCC material properties was conducted to develop better multi-factor prediction relationships.</p> <p>The demonstration PRS was developed using a computerized spreadsheet program. It was designed to be parallel to the demonstration PRS system developed under NCHRP Project 10-26A for asphalt concrete pavements. As a result of the study, recommendations were made for further research in several key areas related to PRS systems and field and laboratory studies.</p>					
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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol When You Know Multiply By To Find Symbol

LENGTH

in	inches	25.4	millimetres	mm
ft	feet	0.305	metres	m
yd	yards	0.914	metres	m
mi	miles	1.61	kilometres	km

AREA

in ²	square inches	645.2	millimetres squared	mm ²
ft ²	square feet	0.093	metres squared	m ²
yd ²	square yards	0.836	metres squared	m ²
ac	acres	0.405	hectares	ha
mi ²	square miles	2.59	kilometres squared	km ²

VOLUME

fl oz	fluid ounces	29.57	millilitres	mL
gal	gallons	3.785	litres	L
ft ³	cubic feet	0.028	metres cubed	m ³
yd ³	cubic yards	0.765	metres cubed	m ³

NOTE: Volumes greater than 1000 L shall be shown in m³.

MASS

oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams	Mg

TEMPERATURE (exact)

°F	Fahrenheit temperature	5(F-32)/9	Celsius temperature	°C
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APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol When You Know Multiply By To Find Symbol

LENGTH

mm	millimetres	0.039	inches	in
m	metres	3.28	feet	ft
m	metres	1.09	yards	yd
km	kilometres	0.621	miles	mi

AREA

mm ²	millimetres squared	0.0016	square inches	in ²
m ²	metres squared	10.764	square feet	ft ²
ha	hectares	2.47	acres	ac
km ²	kilometres squared	0.386	square miles	mi ²

VOLUME

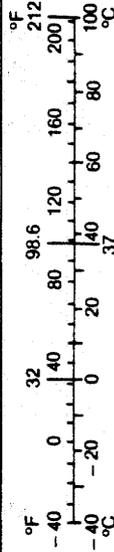
mL	millilitres	0.034	fluid ounces	fl oz
L	litres	0.264	gallons	gal
m ³	metres cubed	35.315	cubic feet	ft ³
m ³	metres cubed	1.308	cubic yards	yd ³

MASS

g	grams	0.035	ounces	oz
kg	kilograms	2.205	pounds	lb
Mg	megagrams	1.102	short tons (2000 lb)	T

TEMPERATURE (exact)

°C	Celsius temperature	1.8C + 32	Fahrenheit temperature	°F
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* SI is the symbol for the International System of Measurement

(Revised April 1989)

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CHAPTER 1 BACKGROUND, OBJECTIVES, AND SCOPE

Over the past 10 or 12 years considerable research has been directed towards the development of performance-related specifications (PRS) for measures of materials and construction (M&C) quality. In 1976, a National Cooperative Highway Research Program (NCHRP) synthesis was published on statistically oriented end-result specifications.^{(76.1)*} Fundamental concepts for performance-based acceptance plans and associated price-adjustment systems were reported in the late 1970's and many further developments were reported in the early 1980's as reflected in references 77.1, 78.1, 80.1, 82.1, 82.2, 82.3, and 84.1.

A state-of-the-art specification for flexible pavement was published by the Federal Highway Administration (FHWA) in 1984.^(84.5) At about the same time a research program for development of PRS was instituted by the NCHRP, beginning with NCHRP Project 10-26. The main objective for that study was to identify variables and existing data bases from which appropriate relationships between M&C factors and performance indicators might be derived as inputs for specifications development. It was concluded that existing data bases were probably inadequate for direct derivation of essential relationships.^(85.4)

As a consequence of the Project 10-26 study, the NCHRP decided that further research on PRS should be within a general framework that provides for multistage derivation of the needed relationships. In this framework primary relationships would be between performance indicators (e.g., distress levels or applications to "failure") and known performance predictors (e.g., surfacing thickness and mechanistic properties). Secondary relationships would show the nature and extent of associations among the performance predictors and other M&C factors that are amenable to M&C control (e.g., asphalt concrete (AC) or portland cement concrete (PCC) mix factors).

Under this new approach, NCHRP Project 10-26A was initiated in 1986 and is expected to be completed in 1990. Based on the scope of work, the research report should cover virtually all aspects of PRS development for AC materials and construction, including experimental results from laboratory studies and algorithmic demonstrations of particular M&C acceptance plans and payment schedules.^(88.1)

Much development of PRS for rigid pavements has been done by the New Jersey Department of Transportation (NJDOT). Comprehensive procedures for deriving acceptance plans and payment schedules are set forth in reference 82.4. For the most part, performance-relatedness of these specifications is based on the rigid pavement performance equation given in the American Association of State Highway and Transportation Officials (AASHTO) Interim Guide for Design of Pavement Structures.^(81.3) In turn, this equation is

* References in this report are identified by a superscript which includes the date of the report (first two digits) followed by a numerical designation.

based on a present serviceability index (PSI) prediction equation that was reported from the American Association of State Highway Officials (AASHO) Road Test, and that contains four performance predictors relating to M&C factors, namely, PCC thickness, PCC flexural strength, PCC elastic modulus, and modulus of subgrade reaction.^(62.1) The NJDOT specifications include acceptance and payment plans that are based on as-constructed PCC thickness and strength.

To provide a research program parallel to the NCHRP 10-26A study, the FHWA in 1987 requested proposals for development of PRS for PCC pavement construction. In response to the FHWA request, the research described in this report was initiated in mid-1987 and had the following objectives:

1. To identify relationships, between measures of material and construction quality and pavement performance, that are necessary for the development of performance-related rigid pavement specifications.
2. To develop a laboratory/field testing program designed to quantify the necessary relationships.
3. To conduct laboratory/field testing to quantify all necessary relationships between one materials and construction specification variable and rigid pavement performance.
4. To demonstrate the development of a performance-related specification (including incentive/disincentive provisions) for the one selected materials and construction specification variable.

The scope of work used to accomplish the foregoing objectives is implied by the following resumé of the report contents.

A general framework for specifications development is given in chapter 2. Existing primary relationships for the prediction of rigid pavement stress, distress, and performance are assessed in chapter 3, wherein specific variables and relationships are proposed for use in the demonstration specification development.

Existing secondary relationships and data bases for rigid pavement M&C variables are evaluated in chapter 4, relative to an "optimum" data base that would suffice for the development of demonstration specifications. The chapter includes discussion of M&C specifications in current use by a number of selected States.

Chapter 5 gives a description of the laboratory study of needed secondary relationships. Initial experiment design, implementation, data acquisition, variable classifications, and statistical analysis techniques are all discussed in detail in this chapter.

Statistical procedures and computer algorithms for development of performance-related M&C specifications are set forth in chapter 6, and provide a basis for the development of demonstration specifications.

Chapter 7 gives details of the development of a PRS demonstration program of performance-related M&C specifications for concrete pavement construction. This PRS program was developed around the methodology discussed in NCHRP Project 10-26A. This PRS demonstration system uses secondary relationships derived from the laboratory study data. Chapter 7 also includes a sensitivity study and comment section on the PRS demonstration system.

Finally, chapter 8 provides a summary and recommendations for further research on the development of PRS systems for PCC pavements. The recommendations include further work on (1) laboratory experiments for the development of better so-called secondary prediction relationships, (2) field studies for the development/verification of better primary performance prediction relationships and (3) analyses to improve the fundamental mechanics (i.e., cost components, acceptance and payment plans, operating characteristics, etc.) of future PRS systems.

Throughout the scope and work of this study, special efforts have been made to draw upon and ensure compatibility with relevant results from all cited developments of performance-related M&C specifications.

CHAPTER 2

FRAMEWORK FOR DEVELOPMENT OF PERFORMANCE-RELATED M&C SPECIFICATIONS

A general framework for the development of performance-related M&C specifications is shown schematically in figure 1. The framework is based on concepts that were presented in the Irick paper and is consistent with the framework that has been developed in NCHRP Project 10-26A for asphaltic-concrete specifications.^(87.4, 88.1)

Shown on the left of figure 1 are four sets of relationships (R1 through R4) and two boxes (B and C) that represent variables contained in the relationships. Box A represents data bases for all variables that are used to derive the relationships, including variables in box B and box C. The right side of the figure shows four types of additional inputs (boxes D through G) to algorithms (R5) that are used to produce the performance related M&C specifications represented by box H.

In this chapter, an overview is given for all 13 framework elements. More extensive discussion and examples for the elements are given in chapters 3, 4 and 6.

Primary relationships for this study are defined to be those for predicting pavement stress (R1), pavement distress (R2), and pavement performance (R3) from particular combinations of predictors (box B) that represent traffic, environmental, roadbed, and structural conditions. It is assumed that any relationship among R1 through R3 is an equation (or algorithm) that predicts values for an output variable that is a specific indicator of stress, distress, or performance. One stress indicator, for example, might be a particular strain in the PCC surfacing layer, one distress indicator might be inches of wheelpath faulting per mile, and one performance indicator might be the number of ESALs at which the pavement's present serviceability index has reached PSI=2.0.

Predictor variables represented by box B are well-defined independent variables that appear explicitly in one or another of the primary relationships. Examples are surfacing thickness (box B4), roadbed soil modulus (box B3), annual precipitation (box B2), and annual rate of equivalent single axle load (ESAL) accumulation (box B1).

A number of specific primary relationships for PCC pavements will be identified in chapter 3. In general, each has been derived either from mechanistic considerations (M in figure 1), from empirical models (E), or from some combination of the two methods (ME). A fourth method for deriving a particular relationship is through algebraic manipulation (A) of one or more relationships that were derived via methods M, E, or ME.

As indicated in box A, data bases used to derive primary relationships may be either observational, experimental, or some combination thereof. An observational data base, for example, might represent observations from a set of selected highway construction projects. An experimental data base arises from a designed study in which control is planned and exercised over the independent variables of the study. Thus, experimental data bases can result from sets of specially constructed test sections as in the AASHO Road Test, or from the test specimens of a designed laboratory experiment.

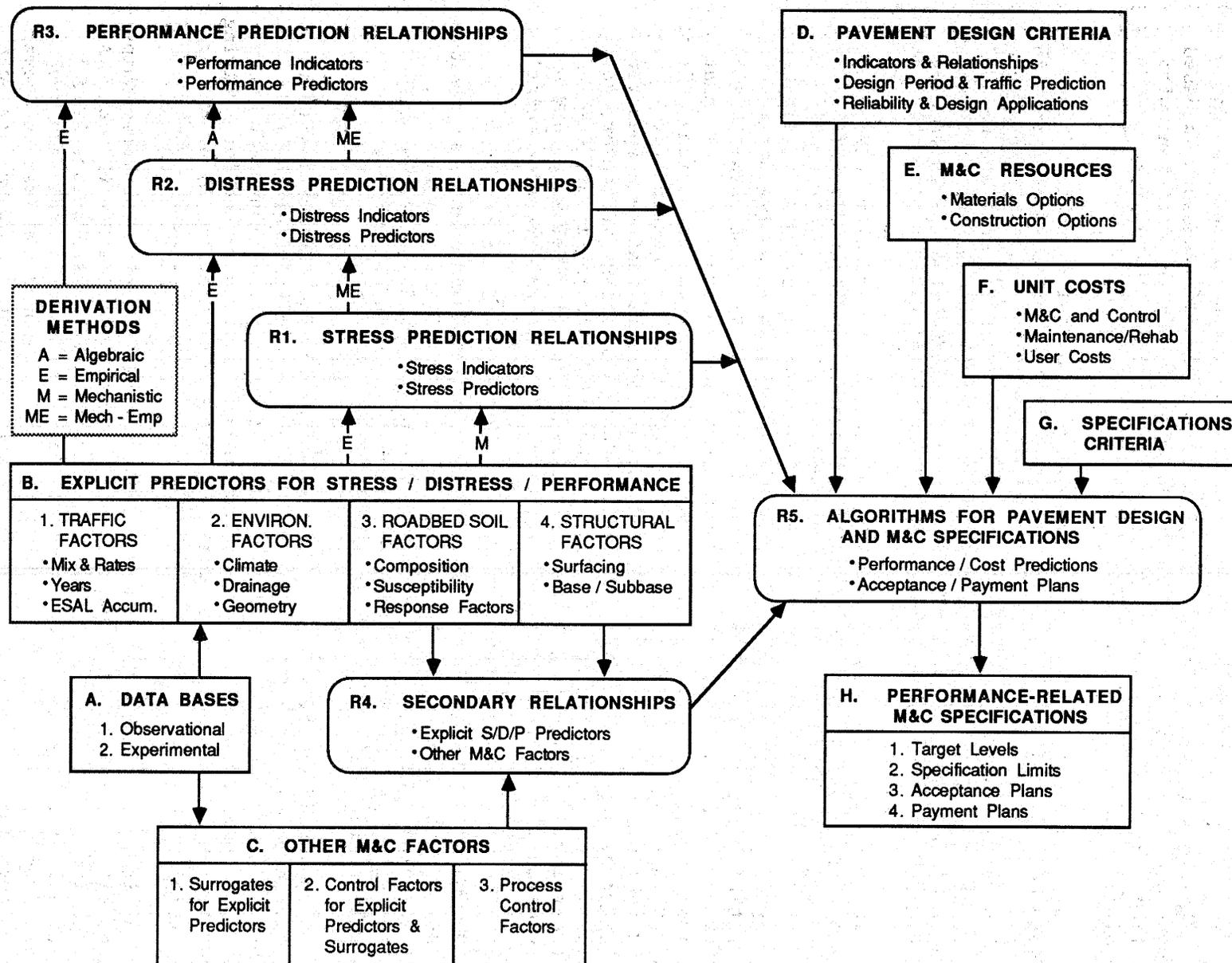


Figure 1. Framework for development of performance-related materials and construction (M&C) specifications.

Associated with every prediction relationship is at least one statistical distribution of prediction errors, i.e., differences between predicted values for a given indicator and corresponding observed values of the indicator. Characteristics of the error distribution (e.g., shape, mean value, standard deviation) are needed for the development of performance-based M&C specifications, as will be discussed in chapter 6. To the fullest extent possible, discussion of specific relationships in chapters 3 and 4 includes what is known about their methods of derivation and their error distributions.

Certain explicit predictors in boxes B3 and B4 may be materials and/or construction factors whose levels are controlled directly during the M&C process (e.g., layer thicknesses). In other cases, explicit predictors may be controlled indirectly through other M&C factors that are represented by box C. All M&C factors that are not explicit predictors for a particular set of relationships fall in one or another of three classes of "other" M&C factors.

Class C1 contains factors that are not explicit predictors but that may be used as surrogates for factors that appear in one or another of R1 through R3. For example, the relationships in use may contain PCC flexural strength as an explicit predictor, whereas PCC compressive strength might be controlled through M&C specifications. In this case, compressive strength is a surrogate for flexural strength.

Class C2 contains M&C factors that are not explicit predictors but have specifications to provide indirect control for explicit predictors or their surrogates. If, for example, a prediction relationship contains modulus of subgrade reaction as an explicit predictor of stress/distress/performance, then class C2 may contain factors, such as density and compaction, whose specifications provide at least partial control over soil modulus. Other examples of factors in class C2 are those which specify certain PCC mix properties (e.g., cement content) that are known to affect explicit predictors such as PCC flexural strength or modulus of elasticity.

The remaining M&C factors in box C are called process control factors (C3) whose specifications enhance the control of other M&C factors. Examples include moisture control during roadbed preparation so that specified levels of soil density and compaction can be attained. Other examples include control of subsurface profiles to enable attainment of specifications for surfacing profile. Some M&C factors may belong in two or more subclasses of box C. Slump of the PCC mix, for example, may be controlled to enhance both workability of the PCC and its ultimate strength.

Secondary relationships (R4) include all equations or algorithms that show interrelations among M&C factors that are represented by box C and boxes B3 and B4. By definition, secondary relationships do not contain indicators for stress/distress/performance, but should account for all M&C factors that are explicit predictors in the primary relationships. Chapter 4 discusses (1) existing secondary relationships that have been identified by the research team, and/or (2) available data bases that might be used or augmented to produce new secondary relationships among M&C factors. Chapter 5 describes new laboratory studies whose experimental design and implementation can lead to secondary relationships that are needed for the

development of performance-related M&C specifications, but are neither available from past research nor attainable from available data bases.

As for the derivation of primary relationships, existing data bases for secondary relationships may be either observational or experimental. In chapter 5, however, it is assumed that new data bases for R4 relationships should be developed from planned experiments.

As shown in figure 1, both primary relationships (R1 through R3) and secondary relationships (R4) are inputs to the algorithms (R5) that produce performance-related M&C specifications. The specific nature of these algorithms will be discussed in chapter 6. The algorithms depend upon criteria (box G) that are used to derive performance-related M&C specifications.

As shown in the figure, certain algorithms in R5 are needed for predictions of performance and operational costs associated with pavement deterioration and rehabilitation. Other algorithms are needed for the derivations of acceptance plans and payment schedules that are associated with the M&C specifications. The specifications criteria in box G include, for example, acceptance risks and performance-based economic criteria.

Boxes D through F in figure 1 represent conditions and constraints that must be taken into account by the specifications algorithms. Included are pavement design criteria (box D) that specify particular stress/distress/performance indicators, limiting values for the indicators, and particular primary relationships (R1 through R3) that are to be used as pavement design equations.

It is assumed that the design criteria will also include a design period (e.g., 15 years) during which the selected distress/performance indicators do not reach their limiting values, and associated predictions of expected traffic during the design period, perhaps in terms of ESAL accumulation. A third design criterion is design reliability, which is basically a factor incorporated in the AASHTO design guide to treat the variability of pavement performance from a design standpoint.

Another class of constraints for the specifications algorithms is represented by available M&C resources (box E) and their associated costs (box F). As indicated, the M&C resources will generally represent various options for materials (e.g., aggregate sources) and construction methods (e.g., paving equipment and procedures).

Unit costs in box F must cover not only options for materials and pavement construction, but should also include data for estimating routine maintenance costs and user costs for various levels of pavement condition. If the optimization criteria relate to performance periods beyond the initial period, the cost data must provide inputs for estimation of rehabilitation costs.

The final element of the framework (box H) represents performance-related M&C specifications that are derived via the algorithms in R5. It is assumed that the specifications include target levels (H1) and/or specification limits (H2) for all M&C factors that relate to the pavement's

structural design. Specifications for some factors might include target levels and lower limits only (e.g., PCC thickness), other specifications might have both upper and lower limits but no target level (e.g., aggregate gradation). Other specifications might have only a lower limit (e.g., PCC flexural strength) or an upper limit (e.g., surface profile deviation).

In general, it may be assumed that target levels are based on specific relationships among R1 through R4, subject to criteria, conditions and constraints imposed by items in boxes D through G. It can be expected that levels will be assumed for some factors and that the algorithms will indicate alternative combinations of levels for remaining M&C factors, at least whenever the necessary relationships (R1 through R4) are available. Levels for some factors will, of course, be specified through State requirements and/or through M&C standards that have been set by AASHTO or American Society for Testing and Materials (ASTM).

Although some specification limits may also be determined by requirements and standards, the algorithms should make appropriate use of (1) error distributions for the relationships that determine target levels and (2) normal variability in M&C factors. It will be assumed that item (2) is an essential aspect of all secondary relationships in R4.

After target levels and/or specification limits are produced by the algorithms, acceptance plans (H3) are developed for those factors whose levels can effect the acceptance or rejection of materials and/or pavement layers. In the simplest case an acceptance plan would define the "lots" to be sampled, time/space sampling points, measurement procedures for the samples, and measurement statistics (e.g., percent within tolerance limits) that will lead either to acceptance or rejection of a given lot. An essential aspect of any acceptance plan is its operating characteristic, i.e., the probability that lots of given quality (with respect to the M&C factor that has been evaluated) will be accepted. It is assumed that the unit costs in box F include M&C inspection and quality control expenditures.

The fourth facet of PRS includes payment plans (H4) that determine the extent to which the contractor's bid price will be adjusted as a consequence of specific (or multiple) characteristics of the as-built pavement lots. In general, payment plans may be expressed as pay factors (e.g., ranging from 0.5 to 1.2) that correspond to the differences between the expected performance of the design pavement and the as-constructed pavement.

The foregoing overview of the framework represented by figure 1 implies that the algorithms in R5 are necessarily extensive and complex. Although considerable research effort will be required to finalize other framework elements, particularly the secondary relationships (R4), it appears that the algorithm development will be even more demanding. To the fullest possible extent, the eventual algorithms will draw upon and be consistent with counterpart algorithms that have been developed in other studies, such as NCHRP Project 10-26A, and in those represented by references 78.1 and 82.4.

CHAPTER 3 PRIMARY RELATIONSHIPS

This chapter covers and provides specific examples of the three types of primary relationships that were shown in figure 1, namely:

- R1 - Stress prediction relationships for various indicators of pavement response to single loading applications.
- R2 - Distress prediction relationships for various indicators of pavement distress, including singular distress modes and composite indicators of overall distress.
- R3 - Performance prediction relationships for the time periods and/or traffic accumulations for which pavement distress remains at acceptable levels.

Table 1 is a general classification scheme for the variables that are contained in the primary relationships. The left-hand column lists the indicators whose values are functions of the predictors listed in the right-hand column. Thus, the dependent variable for any particular relationship is in the first column, the corresponding independent variables are among those listed in the second column.

Stress indicators are dependent variables in R1 relationships but can be predictor variables in R2 relationships (see class 226). Moreover, certain distress indicators can be dependent variables in some of R2, and auxiliary independent variables in other R2 relationships (see class 227).

Each type of relationship is discussed, respectively, in the sections that follow. Within each section, specific primary relationships are identified and the relevant portions of table 1 are expanded to include more specific indicators and predictors. Objectives for each section are to:

1. Identify all predictors that are related to rigid pavement materials and construction, particularly for the PCC surfacing component.
2. Select a small number of relationships that are candidate elements of the algorithms that will be used to derive performance-related M&C specifications.
3. Discuss for each selected relationship, the sensitivity of the predicted variable to changes in predictor variables.
4. Estimate the nature and extent of prediction errors that are not explained by the predictors.

STRESS PREDICTION RELATIONSHIPS

This section describes many of the available analytical (and empirical) response models that can be used to predict stresses, strains and/or deformations in portland cement concrete (PCC) pavements. This section is

Table 1. General classification of variables in primary relationships for rigid pavements.

R1. STRESS PREDICTION RELATIONSHIPS

11. STRESS INDICATORS	12. STRESS PREDICTORS
111. Deflections	121. Loading Factors
	122. Moisture/Temperature Conditions
112. Strain Components	123. Surfacing Factors
	124. Base/Subbase Factors
113. Stress Components	125. Roadbed Factors

R2. DISTRESS PREDICTION RELATIONSHIPS

21. DISTRESS INDICATORS	22. DISTRESS PREDICTORS
211. Singular Distress Indicators	221. Traffic Factors & Age
2111. Pumping	
2112. Cracking	222. Environmental Factors
2113. Faulting	
2114. Joint Deterioration	223. Surfacing Factors
2115. Other Slab Distresses	
2116. Swells and Depressions	224. Base/Subbase Factors
2117. Skid Resistance Loss	225. Roadbed Factors
212. Composite Distress Indicators	226. Stress Indicators
2121. Roughness	
2122. Serviceability Loss	227. Auxiliary Distress Indicators
2123. Condition Rating Loss	

R3. PERFORMANCE PREDICTION RELATIONSHIPS

31. PERFORMANCE INDICATORS	32. PERFORMANCE PREDICTORS
Number of Equivalent Single Axle Load Applications (ESAL) at Acceptable Levels of Distress Indicators	Distress Predictors in Classes 221-227.

mostly a condensation of reference 86.4 with some enhancements for the models that were not covered.

The models can each basically be classified under one of the following four categories:

- Empirical.
- Multi-layered elastic solid.
- Elastic plate on dense liquid.
- Finite element idealizations.

The first category refers to models that have been derived through mathematical or statistical analysis of field data. The last three categories are all mechanistic models that rely on theory and the fundamentals of engineering mechanics in solving for a particular response.

Empirical Models

Results of studies on the original AASHO Road Test data provide excellent examples of empirical pavement response models that have been derived through statistical analysis of field data.^(62.1) Following is a derived equation that relates dynamic edge strain (STRN, 10^{-6} in/in) to design slab thickness (D2, inches), single axle load (L1, kips) and temperature (T, °F):

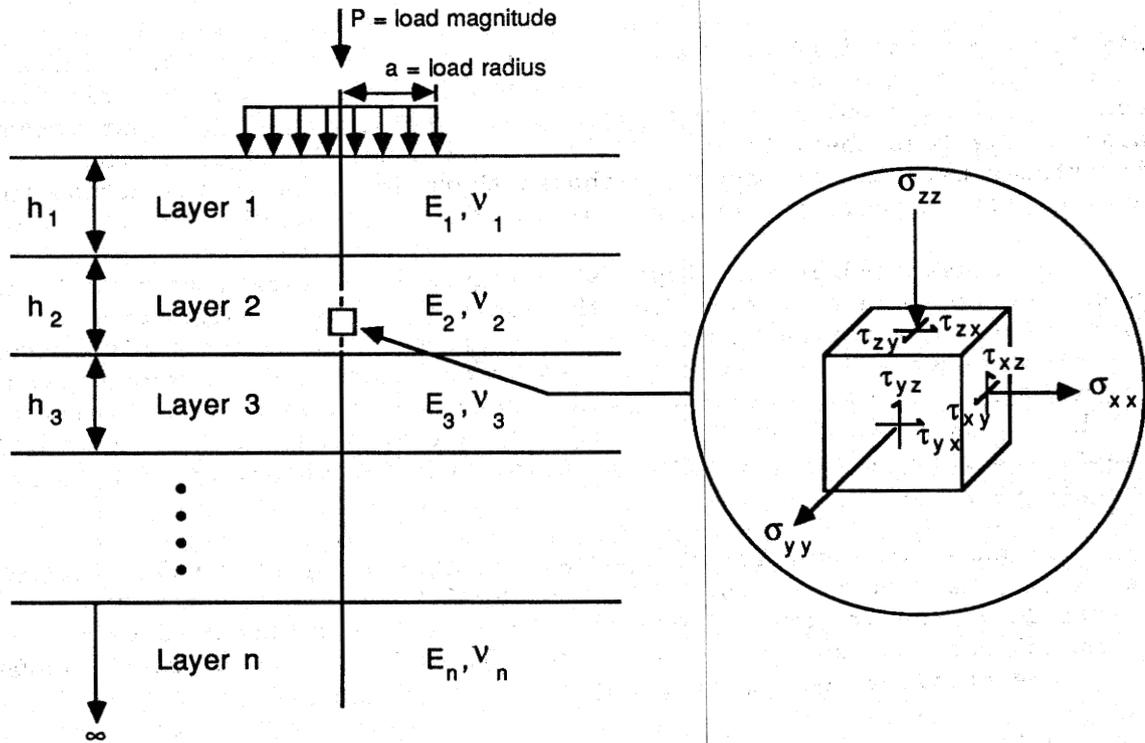
$$\text{STRN}/L1 = 20.54/[10^{0.0031(T)} D2^{1.278}] \quad (1)$$

Similar equations were derived for both static and dynamic deflections measured at the slab edge and corner. These equations are all very useful in evaluating pavement behavior and predicting performance at the Road Test. However, they lose their applicability once environmental and loading conditions outside those experienced at the Road Test are encountered. This explains why the analytical or mechanistic models described next are so much more attractive than any empirical models. They are capable of predicting pavement behavior and response for a much wider range of conditions.

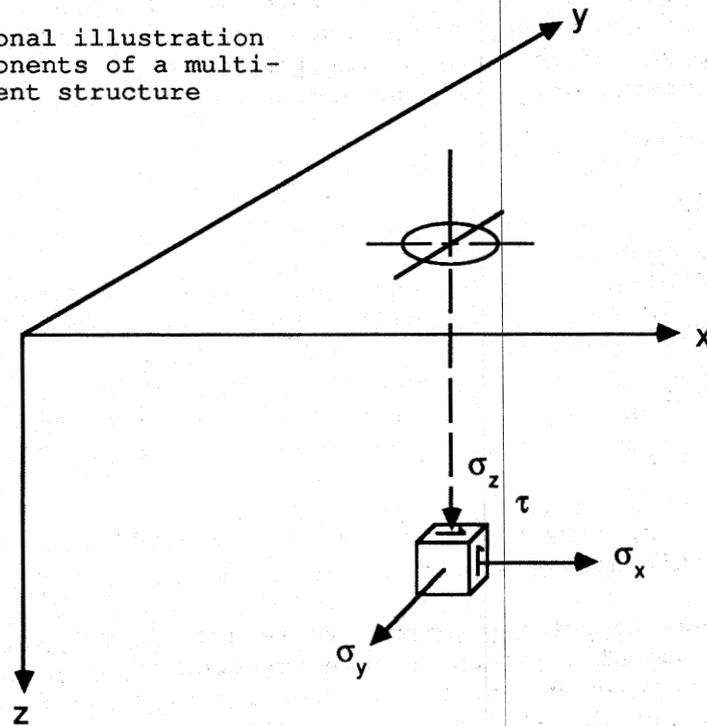
Multi-Layered Elastic Analysis Models

In this analytical methodology, the pavement is modeled as a series or "stack" of individual layers having unique characteristics (see figure 2). Each layer is assumed to be infinite in all horizontal directions, and the materials that compose the layers are considered to be homogeneous, isotropic and linear elastic in response. (Note: There are some models that incorporate ad-hoc procedures to treat the nonlinear response of materials to stress.) The materials in each layer are characterized by their thickness (h_i), elastic or Young's modulus (E_i), and Poisson's ratio (ν_i). Some methods also consider the unit weight of the layer materials, however, most assume the layers are weightless.

Loads applied to the pavement surface are assumed to have circular contact areas with uniform contact pressures. Most methods can only



a) Two-dimensional illustration of the components of a multi-layer pavement structure under load.



b) Illustration of coordinate system.

Figure 2. Schematic representation of multilayer elastic pavement structure.

simulate vertical loading; however, there is at least one that permits tangential surface loads. Many of the available methods also permit the consideration of multiple surface loads (usually up to 10). Most methods also assume that there is full friction (i.e., no slippage) at the interfaces between the layers, although there is at least one method that does permit variable friction at the layer interfaces.

As illustrated by the diagrams in figure 2, a variety of normal and shear stresses can be computed on the faces of a three-dimensional differential element anywhere within the structure. Corresponding strains and displacements due to load can also be determined. Some models even provide for the computation of maximum principal stresses and strains using a Mohr's circle-based procedure. For those that permit the use of multiple loads, the principle of superposition is used to combine the effects at any designated point.

For one-, two- and three-layer structures, hand/graphical solution techniques have been developed through an evolutionary process by a multitude of researchers. These equations and nomographs have been assembled and published under one textbook.^(75.1) These methods do, however, have some problems (see appendix A).

By far, the quickest and most accurate way to develop solutions is through the use of the computer programs that are currently available. These computer programs make use of integral transform procedures and are based on the solutions originally developed by Burmister:^(45.1)

- BISAR.^(73.3)
- CHEV.^(63.2)
- ELSYM.^(72.1)
- PDMAP.^(77.5)
- VESYS.^(78.2)
- CHEVIT.^(76.2)

Table 2 provides a summary comparison of the capabilities of each of these multilayered elastic analysis programs.

Although these computer programs are relatively fast compared to some of the other more complex methods, there are occasions (particularly on microcomputers) where even faster operational speeds are desirable. This and the need to study the statistical significance of many of the independent variables has led to the development of regression equations that simulate the output of the analytical programs. Appendix A provides some examples of these kinds of approximation functions.

Multilayered elastic solid based modeling procedures have been used for the analysis of both rigid (PCC) and flexible pavements. However, they do have their weaknesses for both pavement types:

(86.4)

Table 2. Comparison of multilayered elastic analysis computer programs.

Program (Ref)	Number of Layers (max.)	Number of Loads (max.)	Continuity Conditions at Interface	Probabilistic Considerations	Program Source	Remarks
BISAR ^(73.3)	10	10	Full continuity to no friction	No	Shell International Petroleum Co., Ltd., London, England	1. Relatively long running time since complete set of stresses and strains provided for each point. 2. Considers horizontal as well as vertical loads.
CHEV ^(63.2)	5	2	Full continuity	No	Chevron Research Company	1. Nonlinear response of granular materials accounted for in DAMA program of the Asphalt Institute which makes use of CHEV program.
ELSYM ^(72.1)	10	100	Full continuity to no friction	No	University of California, Berkeley	1. Short running time for particular point.
PDWAP ^(77.5)	5	2	Full continuity	Yes	National Cooperative Highway Research Program (Project 1-10B)	1. Running time is long for degrees of reliability other than 50 percent (the deterministic mode). 2. Iterative process used to arrive at moduli for untreated granular materials.
VESYS ^(78.2)	5	2	Full continuity	Yes	FHWA-US DOT	1. Running time is long in probabilistic mode. 2. Program considers materials both as time independent (elastic) and time dependent (viscoelastic).
CHEVIT ^(76.2)	5	12	Full continuity	Yes	U.S. Army CE Waterways Experiment Station	1. Modification of CHEV program. 2. Includes provision for stress sensitivity of granular layers.

- For PCC pavements, the procedures are unable to treat the effects of discontinuities that may exist in the structure (i.e., cracks, joints, non-uniform support, etc.). Direct computation of stresses, strains and displacements is only possible for interior load and full support conditions. Edge and corner loads, voids and variable load transfer at joints/cracks must all be treated by applying an adjustment factor derived by some other analytical means, such as finite element idealizations.
- For flexible pavements, there is a limitation when analyzing layered systems consisting of unbound granular layers. Because of their lack of cohesion, these materials have little capability to withstand the levels of tensile stress that might be generated by one of the theoretical elastic layer models. (This problem is less profound in rigid pavements since the PCC slab carries most of the stress.) The likelihood of prediction of this unrealistic condition is greatest when the ratio of elastic moduli between adjacent layers exceeds a practical value (generally between 1.5 and 4.0). To treat this phenomenon, some "ad hoc" procedures have been developed that essentially adjust layer moduli to ensure that significant tensile stresses are not developed in the unbound layers.

Elastic Plate on Dense Liquid Subgrade

H.M. Westergaard developed the original plate theory for pavements in 1925. He presented prediction models for stresses in slabs of uniform thickness resulting from loads and the effects of slab curling subjected to temperature gradients.^(26.1) Modifications to his models based on experimental findings have been suggested by various investigators but his basic considerations remain unchanged. Westergaard's original work provided equations (shown below) for the computation of deflections and stresses for interior, edge and corner loading of rigid slabs. No equations were presented for strain.^(70.1, 85.6)

Interior Deflection:

$$DEFI = 0.125 * P / (k * L^2) \quad (2)$$

Edge Deflection:

$$DEFE = 0.433 * P / (k * L^2) \quad (3)$$

Corner Deflection:

$$DEFC = [1.1 - 0.88 * (2/L)] * P / (k * L^2) \quad (4)$$

Interior Stress ($\nu = 0.15$):

$$SIGI = 0.316 * P * [4 * \log_{10}(L/b) + 1.069] / h^2 \quad (5)$$

Edge Stress* ($\nu = 0.15$):

$$SIGE = 0.572 * P * [4 * \log_{10}(L/b) + 0.359] / h^2 \quad (6)$$

* This equation has been corrected by Westergaard.^(48.1)

Corner Stress:

$$SIGC = 3 * P * [1 - (a_1/L)^{0.6}] / h^2 \quad (7)$$

Following are definitions for each of the variables in these equations:

- P = load magnitude (pounds).
- k = modulus of subgrade reaction (pci).
- L = radius of relative stiffness (inches).
= $[(E \cdot h^3)/(12 \cdot (1 - \nu^2) \cdot k)]^{0.25}$.
- E = PCC modulus of elasticity (psi).
- h = slab thickness (inches).
- ν = Poisson's ratio.
- k = modulus of subgrade reaction (pci).
- a = load radius (inches).
- a_1 = distance from center of load to corner of slab; normally
 $a_1 = (2 \cdot a^2)^{0.5}$.
- b = a; for $a \geq 1.742 \cdot h$ (ordinary theory) or
= $(1.6 \cdot a^2 + h^2)^{0.5} - (0.675 \cdot h)$; for $a < 1.742 \cdot h$ (special theory).

More recent studies by Ioannides based on the use of finite element analyses have further enhanced these equations.^(84.9, 85.6) Appendix A summarizes these equations which have also been incorporated into a computer program called WESTER.^(84.9)

The Westergaard idealization is presented in figure 3. The pavement structure is represented as a "medium-thick" elastic plate resting on a dense liquid (Winkler) foundation. Assumptions of this theory include the following:

- Surface forces act normal to the surfaces (i.e., there is no shear).
- The PCC slab is of uniform thickness, stiffness, and elasticity.
- There are no (axial) forces in the plane of the slab (such as compressive stresses due to thermal expansion).
- There are no vertical deformations in the slab.
- Perpendicular planes remain perpendicular after bending.
- All materials have linear stress-strain curves.
- There is no strain in the x-y plane.
- The interface between the PCC slab and the underlying material is assumed to be frictionless.
- The slab is uniformly supported.

Representing the PCC slab as a medium-thick elastic plate produces essentially the same responses as if it were modeled as an elastic layer. Thus, the major differences between plate theory and elastic layer theory for rigid pavements is in the representation of the remainder of the structure.

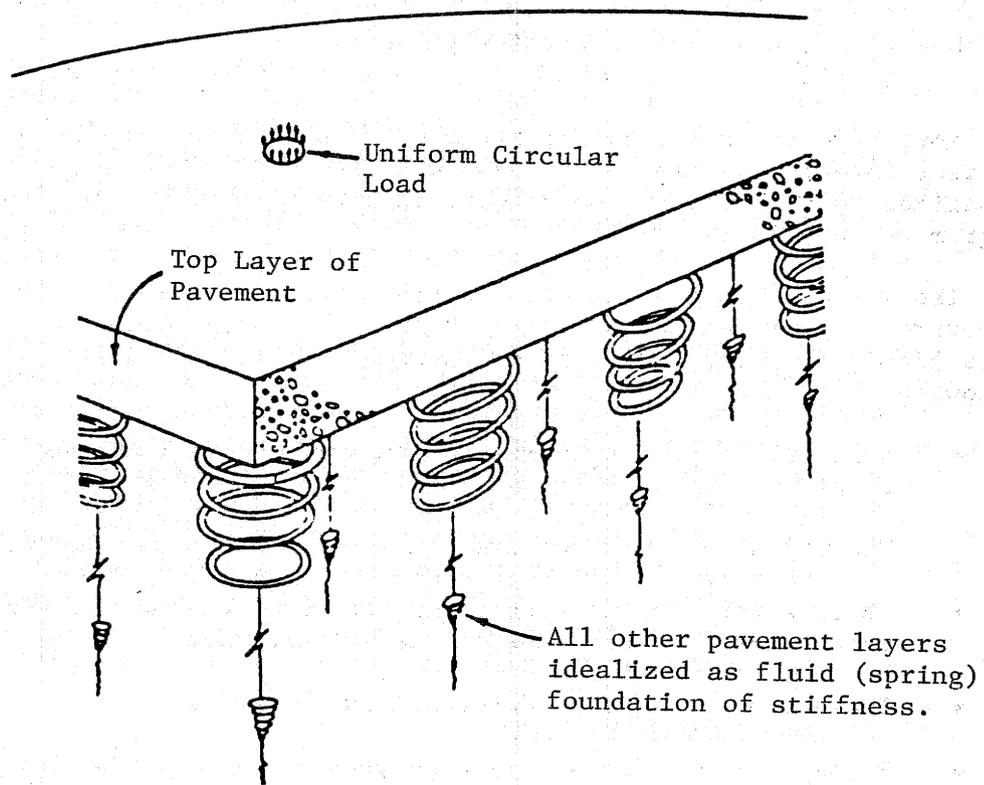


Figure 3. Illustration of plate theory idealization.

The Winkler foundation is described by a bed of closely-spaced, independent linear springs, each representing the effects of support provided over a unit area. Each spring deforms in response to the stress applied directly to it while neighboring springs remain unaffected. Thus, it is assumed that the deflection at any point is directly proportional to the contact stress at that point on the surface. Support is quantified by a spring constant, k , called the modulus of subgrade reaction, which is the ratio of pressure on the unit area divided by the resulting deflection. Representing support conditions with a single layer of material does present drawbacks in the analysis of multilayered support systems, particularly when one or more support layers are especially rigid, as happens when a stabilized base or concrete slab is present. Some researchers have attempted to address these conditions with a very high k value, but the validity of this approach is questionable.

In the basic Westergaard idealization, only uniform, circular-shaped load distributions can be handled. In his 1948 work, however, Westergaard did present an equation for an elliptically loaded area.^(48.1) Procedures have also been developed by Pickett, et al., and Pickett and Ray to handle uniform pressure distributions of any shape. These graphical procedures permit analysis with multiple wheel loads as well as single wheel loads.^(51.1,51.2)

The Westergaard model does provide a means of analyzing free edges and corners. The slab bending stress, vertical deflection and foundation reaction pressure can be computed at or near the free corner or edge of a slab that is semi-infinite horizontally. It cannot, however, handle more than one slab at a time. This prevents the direct analysis of stress reductions that result from various levels of load transfer across transverse joints and cracks or support provided by PCC shoulders. Empirical adjustments to the free edge responses can be made to account for the reduced deflections due to support provided by adjacent slabs. Equations have also been developed that provide a means of taking into account stresses caused by thermal gradients in the slab.^(38.1)

Finite Element Idealizations

The development of the finite element method has produced analysis capabilities that far exceed those of plate theory. There are some trade-offs, however, in that increased attention is required in data preparation and output interpretation.

For analysis by this method, the body to be analyzed is divided into a set of elements connected at their joints or nodal points. The cylinder shown in figure 4 is an example. The continuous variation of stresses and strains in the real system is replaced by an assumed linear variation of displacements, and hence constant stresses and strains within each element. This assumption satisfies the requirements of compatibility of displacements between elements. For a given element geometry and constitutive equation, the stiffness matrix relating displacements and loads at the corners of each of the basic triangular elements is established. The four triangular elements forming one rectangular element are generally combined, eliminating the common nodal point. Combinations of the element stiffness matrices yields the symmetric banded matrix for the entire structural assembly, which

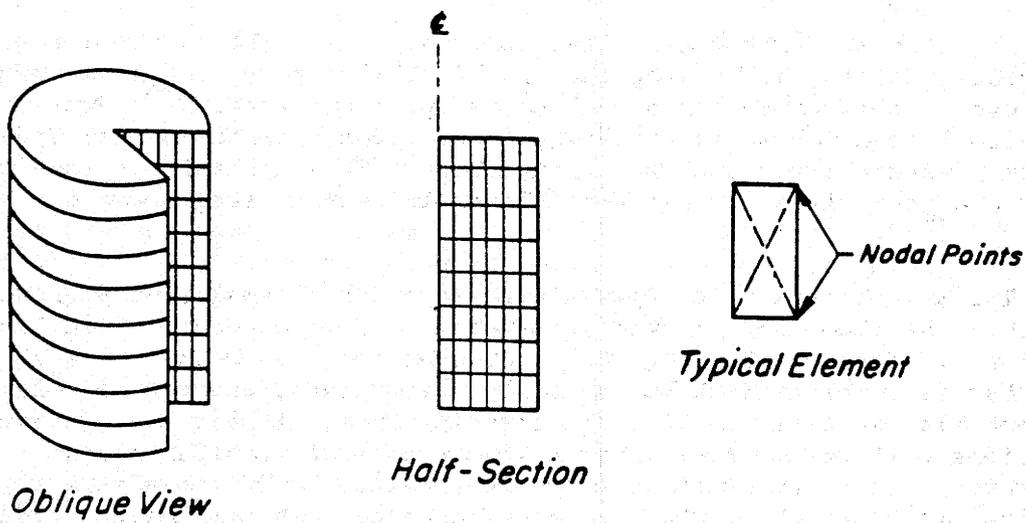


Figure 4. Finite element idealization of a cylinder.

is modified using known displacements at bound axes. Solution of this system of linear equations yields all nodal point displacements, from which the element strains and stresses are computed. The average of the stresses in the four triangular elements gives the best estimate of the stresses at the centroid of the rectangular element.

The element configuration must be carefully selected to optimize the results. (See figure 5 for example.) Generally, the accuracy is improved by the use of a finer mesh, particularly in areas of rapidly varying stresses. However, the greater number of elements increases the computational time and therefore the costs. Dehlen has suggested that an optimum rectangular mesh has finer vertical subdivisions near the surface and in both materials near layer interfaces; and finer radial subdivisions both near the axis of symmetry and near the edge of the loaded area (see figure 5).^(69.1)

For PCC pavements, many types of finite element idealizations exist, including plane strain, axisymmetric and prismatic solid elements. All of these idealizations introduce certain constraints to the model. Special types of elements are used to model discontinuities (i.e., cracks and joints), special interface conditions, reinforcing steel and dowel bars. Special computational techniques permit consideration of temperature and moisture gradients and voids within the pavement structure. Variable layer properties (thickness and deformation properties), nonlinear and nonelastic material responses can also be modeled.

Two- and three-dimensional finite element models are available. Ideally, it is desirable to use three-dimensional models to determine the response of the pavement to changes in temperature, moisture, etc. Unfortunately, the cost difference between two- and three-dimensional models can be several orders of magnitude, particularly when very small elements (fine meshes) are being used to increase the observed accuracy of small-scale responses. However, with the advent of increasingly advanced personal and micro-computers these problems are becoming less critical.

Finite element programs are available that are capable of modeling the following rigid pavement considerations:

- Various types of joint load transfer systems, including dowel bars, aggregate interlock, keyways, or any combination thereof.
- The effects of a stabilized base course.
- Placement of asphalt or concrete overlays with either perfect bonding or no bond.
- The effects of reinforcing steel on the behavior of cracks.
- The effects of concrete shoulders with or without tie bars.
- Concrete slabs of varying thickness and moduli of elasticity.
- Subgrades with varying moduli of support, including voids.

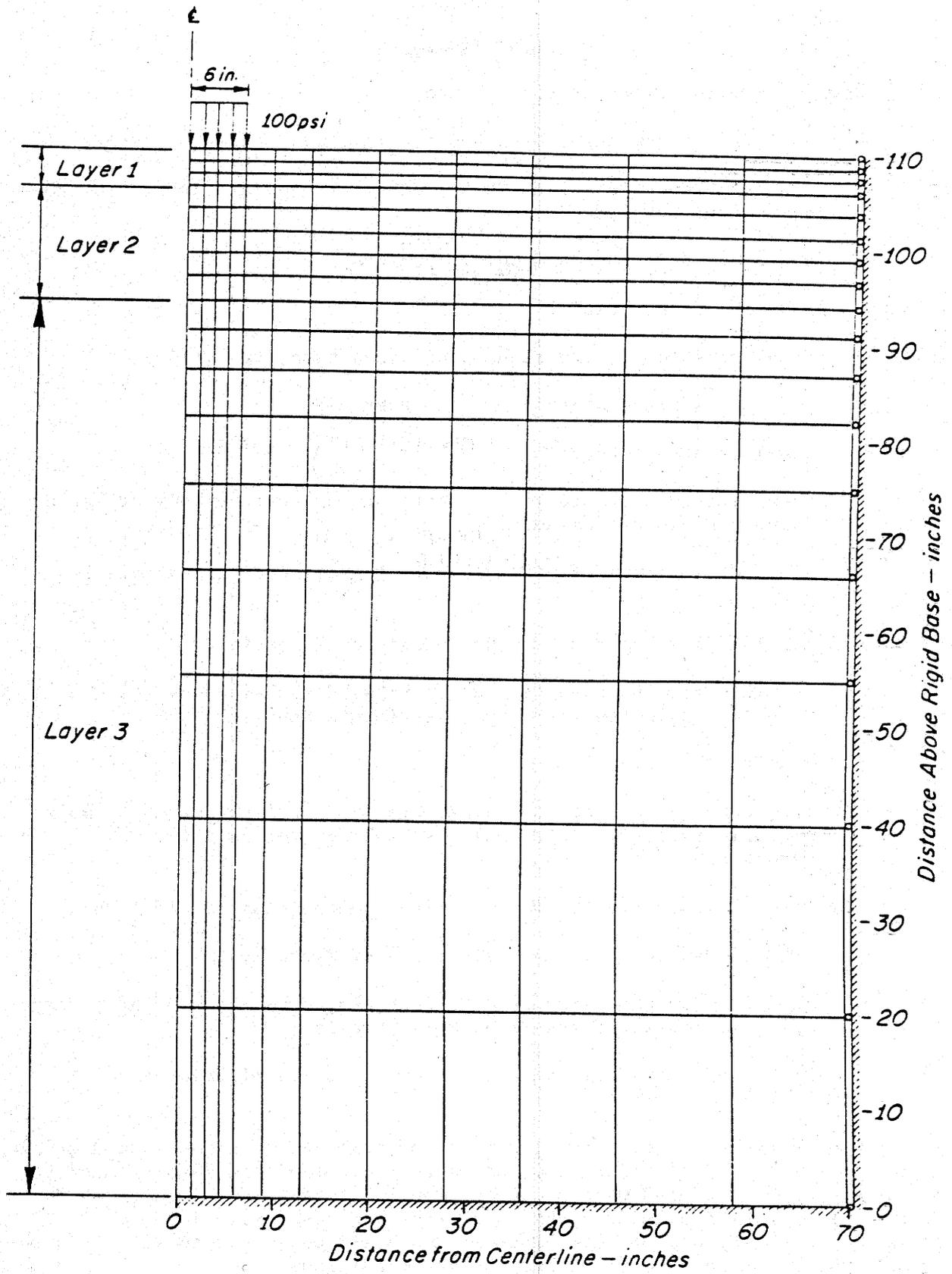


Figure 5. Finite element configurations used for analysis of homogeneous and layered systems.

Required program inputs generally include:

- Geometry of the slab, base and overlay, load transfer system.
- Elastic properties of the slab, base, or overlay, load transfer system and subgrade.
- Boundary conditions and wheel loadings.

Outputs produced often include:

- Nodal stresses in the slab, stabilized base, or overlay.
- Vertical surface stresses of the subgrade.
- Dowel bar reactions.
- Shear stresses at the joints where aggregate interlock or keyed joint systems are assumed.

Specific dense liquid finite element programs that are available include:

- KENWINK (developed at the University of Kentucky).
- WESLIQID (an enhanced version of KENWINK developed by the U.S. Army Corps of Engineers Waterways Experiments Station).^(81.7)
- FINITE.^(77.6)
- ILLI-SLAB (developed at the University of Illinois and the most flexible finite element analysis package available for pavements).^(78.3)

Elastic solid and elastic layer finite element programs include:

- KENELS (developed at the University of Kentucky).^(74.1, 74.2)
- WESLAYER (an extension of KENELS developed by the U.S. Army Corps of Engineers Waterways Experiments Station).^(81.7)
- SOLID SAP (developed at the University of California at Berkeley).^(71.2)

Good estimations of stress, strain and deflection can be obtained using the finite element technique provided a sufficiently fine mesh of mostly square elements is used with proper element properties and boundary locations. Finite element techniques offer the most valid approach to modeling the response of both flexible and rigid pavements to all types of loadings, climatic conditions and support conditions.

DISTRESS PREDICTION RELATIONSHIPS

This section is concerned with relationships (R2 in figure 1) for the prediction of specific distress indicators from predictors that include traffic factors, environmental factors, roadbed soil factors, and structural factors. A high percentage of all existing rigid pavement distress prediction relationships are identified in the 1984 FHWA cost allocation study and/or in the 1986 AASHTO Guide for Design of Pavement Structures as reflected in references 84.3, 86.3, and 86.4. Other sources of R2 relationships include an FHWA study on COPES and a 1977 FHWA study on zero-maintenance design. See references 85.2, 84.2, and 77.3.

Although additional relationships have been reported elsewhere in the pavement research literature, it is assumed that relationships in the foregoing references will provide a substantial and adequate basis for determining the degree to which various types of rigid pavement distresses depend on factors that are associated with the materials and construction of rigid pavements.

A logical structure for the identification of distress relationships and predictors is given in table 3 which is an extension of the R2 portion of table 1. Distress indicators (class 21) are again listed in two categories, one (class 211) for seven types of singular distress and one (class 212) for three types of composite distress. Thirteen different relationships have been selected from the references shown at bottom left of the table, and provide at least one case for each distress type, except for roughness (class 2121). This omission is not regarded as serious since serviceability loss (class 2122) is almost entirely associated with roughness. It is acknowledged that several distress types, most notably cracking, could be further classified into still more specific subclasses.

Appendix B contains details for a number of primary relationships that have been selected from the research literature. Some of the appendix B relationships are not included in table 3 because each is more or less redundant with one or another of the other selected relationships. To the fullest possible extent, appendix B details include specific prediction equations, the size of the data bases from which the equations were derived, and measures of the closeness of fit between the equations and the data base observations. Two such measures are the multiple R square and the standard deviation of prediction errors, i.e., the standard error of estimate (SEE).

Distress predictors are listed in the right-hand column of table 3 in seven major classes; 221 through 227. The first two classes are for traffic, age, and environmental factors that affect pavement distress, performance, and therefore pavement design, but do not relate specifically to M&C variables. They must, however, be included in the present study so that assessments can be made of the relative effects of traffic, environment, roadbed soil, and structure on any particular type of distress.

Primary structural variables are listed in some detail under surfacing (class 223), base/subbase (class 224), and roadbed (class 225) factors. More specificity for these factors, especially those that involve PCC, will be given in a later section.

Table 3. Distress prediction variables and selected relationships.

21. DISTRESS INDICATORS			22. DISTRESS PREDICTORS		
Dependent Variables	Rel. ¹		Independent Variables	Rel. ¹	
211. SINGULAR DISTRESS			221. TRAFFIC FACTORS AND AGE		
2111. Pumping	A		2211. Loading Characteristics	M	
2112. Cracking	B,C,G		2212. No. of Loadings	All but G,J,L	
2113. Faulting	H,I		2213. Age	B,G,L,O,R	
2114. Joint Deterior.	J		-----		
2115. Other Slab Distress	K		222. ENVIRONMENTAL FACTORS		
2116. Swells & Depressions	L		2221. Moisture/Pre-cipitation	A,L,O,R	
2117. Skid Resistance Loss	M		2222. Temperature/Freezing	H,J O,R	
-----			-----		
212. COMPOSITE DISTRESS			223. SURFACING FACTORS		
2121. Roughness			2231. PCC Thickness	All but J-M	
2122. Serviceability Loss	N,O		2232. PCC Strength	B,C,G,I,N,O,R	
2123. Condition Rating Loss	R		2233. PCC Stiffness	b,c,g,i,N,o	
-----			2234. PCC Durability	J,O	
			2235. Other PCC Factors	G,R	
			2236. Reinforcement Factors	G	
			2237. Joint/Dowel Factors	B,H,I,J,N,O,R	
			2238. Shoulder Factors	H	
			2239. Initial Profile	N	
-----			-----		
			224. BASE/SUBBASE FACTORS		
			2241. Type Material	B,H,L,O	
			2242. Thickness	b,c,g,i,o	
			2243. Stiffness	b,c,g,i,o	
			2244. Drainage	A,J,N	
-----			-----		
			225. ROADBED SOIL FACTORS		
			2251. Type/Gradation	A,B,H	
			2252. Strength	L,R	
			2253. Stiffness	b,c,g,i,N,o	
-----			-----		
			226. STRESS INDICATORS		
			2261. Deflections		
			2262. Strains		
			2263. Stresses	B,C,G,I,O	
-----			-----		
			227. AUXILIARY DISTRESS INDICATORS		
			2271. Pumping	B,H,O	
			2272. D-Cracking	J,O	
-----			-----		

Ref. No.	Abbreviated Citation	Rel. ¹
77.3	Zero-Maint., FHWA	C
84.2	Qual. Control, FHWA	R
84.3	Cost Alloc., FHWA	L,M
85.3	COPEs, NCHRP	A,B,H,J,O
86.3	AASHTO Guide, Vol. 1	N
86.4	AASHTO Guide, Vol. 3	G,I
81.6	CRCP Distress	K

¹Relationship References, and Appendix B Codes

The last two predictor classes are for stress indicators (class 226) and auxiliary distress indicators (class 227) that are used as predictor variables in certain distress relationships.

The final column of table 3 shows which relationships are associated with each predictor class. Uppercase letters are used for relationship codes in which the predictor appears explicitly in some term of the prediction equation.

Any distress relationship that includes a stress indicator as an explicit predictor implicitly includes the predictors that are required to predict stress. This situation is represented in the final column of table 3 by the use of lower case letters for relationships that include stress indicators and therefore implicit structural predictors. For example, the cracking prediction relationship B includes edge stress (as part of the RATIO term) which, in turn, depends upon PCC thickness (class 2231), PCC modulus (class 2233), and the stiffness of subsurface layers (classes 2243 and 2253). Thus, the code letter b appears in table 3 opposite each of the relevant implicit predictors associated with cracking relationship B.

The final class of distress predictors (class 227) includes forms of distress other than the distress indicator that is predicted by a given relationship. Thus, for example, cracking relationship B, faulting relationship H, and serviceability relationship O, all include pumping (class 2271) as an explicit predictor of cracking.

All but one of the selected relationships are truly distress prediction equations in that their distress indicators can cover a range of values. The remaining relationship (C) is anomalous in that it does not predict any particular amount of cracking, but rather predicts the number of stress applications at which fatigue cracking will occur. This special type of relationship will be discussed in the section that follows.

PERFORMANCE PREDICTION RELATIONSHIPS

For the purposes of this study, pavement performance will be defined as the amount of acceptable service that the pavement provides before major rehabilitation is required.

It is assumed that one or more distress indicators, D, are used as criteria for the level of service that is provided at any point in time, and that for each indicator there is an unacceptable (or terminal) level, D*, that represents the need for rehabilitation. For simplicity, it is assumed that all distress indicators have zero values at the beginning of any phase of the pavement's life cycle. Thus, level of service is represented symbolically by:

Acceptable Service Levels: $0 \leq D < D^*$

Unacceptable Service Levels: $D \geq D^*$

where it is understood that D represents one or more distress criteria such as cracking, faulting, or serviceability loss.

Amount of acceptable service will be defined as the number of load applications carried by the pavement during the period of acceptable service levels. If the loading characteristics (class 2211 in table 3) are constant for all applications, the symbol N will be used for the number of constant-stress applications that correspond to any acceptable level of D. The symbol N* will be used to denote the number of constant-stress applications that have accumulated when D reaches its terminal level, D*.

If, as in normal highway operations, stress levels (S) vary from vehicle to vehicle and from time to time for any given application, then one stress condition can be defined to be a standard stress level, S₀. The number of loading applications at stress level S₀ will be denoted by N₀ whenever D is less than D*. When D = D*, the corresponding number of standard stress applications is N₀* (S₀).

For any non-standard stress level, S_i, the number of applications at which D = D* is N_i* (S_i), and the stress equivalence ratio (SER) between standard and non-standard applications is defined as follows:

$$\text{For } D = D^*, \quad \text{SER}_i = N_0^* (S_0) / N_i^* (S_i) \quad (8)$$

If all stress determinants other than axle load (e.g., PCC thickness or roadbed soil modulus) are at the same levels for both S₀ and S_i, the corresponding SER is a load equivalence ratio (LER) defined as follows:

$$\text{For } D = D^*, \quad \text{LER}_i = N_0^* (\text{SAL}) / N_i^* (\text{AL}_i) \quad (9)$$

where SAL is a standard axle load and AL_i is the axle load for stress level S_i. Conventionally, SAL is taken to be an 18,000-lb (8,172 kg) single axle load, but other load factors such as tire pressure, lateral placement, etc., must also be specified for the standard loading.

Since highway traffic is comprised of many different axle loadings, when D = D* the pavement will have received N_i applications of axle load AL_i, for i = 1, 2,, but it is not expected that any N_i will have reached N_i*.

It is conventional to assume that any distress level D that is reached after N_i applications of AL_i would also be reached by some number of standard axle applications that is a multiple of N_i. The multiplier for N_i is called the load equivalence factor for AL_i and is assumed to be the load equivalence ratio given by equation 9. Thus, by definition,

$$\text{LEF}_i = N_0^* / N_i^* \quad (10)$$

and is relative to D, D*, SAL, AL_i, and other stress determinants.

For any particular axle loading (AL_i) and corresponding number of applications, N_i, the equivalent number of standard axle load applications (ESAL_i) is defined by:

$$\text{ESAL}_i = \text{LEF}_i * N_i = (N_0^* / N_i^*) * N_i \quad (11)$$

The total number of equivalent applications for N_i applications of AL_i, for i = 1, 2,, will be denoted by W and is given by either of the following:

$$W = \sum_i \text{ESAL}_i = \sum_i (N_o^*/N_i^*) * N_i \quad (12)$$

or

$$W/N_o^* = \sum_i (N_i/N_i^*) \quad (13)$$

Terms on the right side of equation 13 are often called load cycle ratios. It can be seen that $W = N_o^*$ when the summation of these ratios is unity. For this reason, the symbol W^* will be used to denote the number of equivalent standard axle load applications at which $D = D^*$.

Equation 13 is one form of Miner's hypothesis where terminal distress (D^*) will be reached when the load cycle ratio summation is unity. Because of the duality of equations 12 and 13, the use of Miner's hypothesis for aggregating mixed stress applications is algebraically identical to the use of load equivalence factors and equivalent load applications for the same purpose. It is therefore easy to show that Miner's original analyses of the fatigue failure of aluminum specimens would have produced the same results had he defined a standard stress level, then calculated equivalent applications for all other stress levels used in the studies.

One obvious flaw in the ESAL summation approach is that the defining relationship (equation 11) holds strictly only for relationships in which D increases linearly with N . For relationships that are quite non-linear, it must be supposed that there can be considerable divergence between W^* computed from mixed applications and the actual number of standard load applications (N_o^*) that would be observed when $D = D^*$.

Other uncertainties associated with the use of ESALs stem from the fact that LEFs are generally not the same for different distress indicators (D) and have generally unknown dependencies on the non-load determinants of stress levels. If, as is usually the case, LEFs are derived algebraically from distress prediction equations, then the LEF values can be highly dependent upon the form of the equation, i.e., the mathematical model that is used for D .

In spite of probable shortcomings of LEFs and ESALs, the accumulated equivalent axle load applications variable, W , and its terminal level, W^* , will be used as primary performance indicators for the derivation of performance-related specifications.

If D is any distress indicator in table 3, its relationship with distress predictors may be written generally as:

$$D = f(2211, W, 2213, 222, 223, 224, 225, 226) \quad (14)$$

where the predictor variables in function (f) are denoted by their table 3 codes, except for W (code 2212). At the terminal value of $D = D^*$, the corresponding value of W will be denoted by W^* . Thus, for D^* and W^* , equation 14 becomes:

$$D^* = f(2211, W^*, 2213, 222, 223, 224, 225, 226) \quad (15)$$

and may be called an implicit performance prediction equation for W^* . If equation 15 can be solved explicitly for W^* , then

$$W^* = f' (D^*, 2211, 2213, 222, 223, 224, 225, 226) \quad (16)$$

which is an explicit performance prediction equation, relative to distress indicator D and its terminal value, D^* .

A specific example of equation 16 is the AASHTO Design Guide rigid pavement performance equation that may be written as:^(86.3)

$$W^* = (\text{RHO}) [G^{*(1/\text{BETA})}] \quad (17)$$

where RHO and BETA are functions of distress predictors and $(1/\text{BETA})$ is the exponent for G^* . The variable G is defined by $G = (P_0 - P_W)/3$, where P_0 is the as-constructed serviceability (PSI) level, and P_W is the pavement's serviceability level after W equivalent standard load applications. Thus, G is a distress indicator for serviceability loss. When P_W reaches a specific terminal level P_W^* , then G^* is the corresponding terminal level for the distress indicator, G . For rigid pavements P_0 is generally in the neighborhood of 4.5, and P_W^* is often selected to be 2.5. Thus, for these values of P_0 and P_W^* , $G^* = 2/3$ in equation 17.

Nearly all table 3 relationships for distress indicators have been developed in the general form of equation 14 and from statistical analyses of particular data bases. Any of these distress prediction relationships can also be represented in the form of equation 15 or equation 16 and, thus, becomes either an implicit or an explicit prediction equation for the performance indicator W^* . Each such performance prediction equation is, of course, relative to a particular distress indicator, D , and its terminal level, D^* . The mathematical forms (models) for the distress relationship (f) and the performance relationship (f') have much bearing on the sensitivity of the the distress or performance indicators (D or W^*) to changes in the predictor variables.

A special class of performance prediction relationships arises when the distress indicator (D) is defined by only the presence or absence of its terminal level D^* , e.g., the presence or absence of fatigue cracking in a pavement section or laboratory specimen (1 = yes, 0 = no). In these cases there are no antecedent distress prediction relationships (equation 14), and the performance prediction relationships must be developed directly. The general form of these relationships does not include a term (D^*) for the distress indicator and may be written:

$$W^* = f'' (2211, 2213, 222, 223, 224, 225, 226) \quad (18)$$

In cases where either equation 16 or equation 18 has been derived to predict "applications to failure" at constant stress levels for all applications, then load equivalence factors (or load cycle ratio summations) must be used to apply the equations to mixed-traffic predictions. An example is represented by relationship C (appendix B) for predicting the number of loading cycles to fatigue failure in concrete beams. These prediction equations may be written as:

$$\log N^* = A - B(\text{STRS}/\text{STRG}) \quad (19)$$

where N^* is the number of stress applications to beam failure through fatigue, STRS is the constant flexural stress level for each application, and STRG is the beam's modulus of rupture. The graph of equation 19 is thus an S-N curve for plain PCC beams.

If it is desired to use equation 19 to predict fatigue failure after N_1 applications at stress level STRS₁, N_2 applications at STRS₂, etc., a standard stress level, STRS₀, can be defined, and all applications can be converted to equivalent number of stress applications. Thus, for STRS₀ and for STRS_i,

$$\log N_0^* = A - B(\text{STRS}_0/\text{STRG}), \quad (20)$$

$$\log N_i^* = A - B(\text{STRS}_i/\text{STRG}), \text{ for } i = 1, 2, \dots \quad (21)$$

The load equivalence factor for converting N_i applications to an equivalent number of N_0 application is:

$$\text{LEF}_i = (N_0^*/N_i^*) = \text{antilog}[B(\text{STRS}_i - \text{STRS}_0)/\text{STRG}] \quad (22)$$

Across all stress levels, the accumulated number of equivalent standard stress applications is:

$$W = \sum_i [(LEF_i) N_i] = (B/\text{STRG}) \sum_i [(\text{STRS}_i - \text{STRS}_0) N_i] \quad (23)$$

From equation 21, the predicted number of equivalent (standard) applications at failure is:

$$\log W^* = A - B(\text{STRS}_0/\text{STRG}) \quad (24)$$

Thus, failure is predicted whenever the right side of equation 23 is equal to the right side of equation 24. As has been stated, this equality condition is algebraically identical to the load cycle ratio condition that:

$$\sum_i (N_i/N_i^*) = 1 \quad (25)$$

ROLE OF PRIMARY RELATIONSHIPS IN THE DEVELOPMENT OF PERFORMANCE-RELATED M&C SPECIFICATIONS

The main role of primary relationships in the development and application of performance-related specifications is to provide a basis for predicting pavement distress and performance for different pavement structures within a given environment. For a given environment and design levels (target levels) for M&C pavement variables, the primary relationships will predict the extent of pavement distress after the pavement has reached any particular age and has received a particular number of load applications.

If the as-constructed pavement has levels for one or more M&C variables that differ from the corresponding target levels, the primary relationships can predict any differences in distress or performance that arise because the as-constructed M&C variables were not at their specified target levels. Thus, the primary relationships can be used as a basis for construction incentives or penalties that are associated with performance-related M&C specifications.

Based on an overall assessment, the following relationships from chapter 3 and appendix B were selected for use as initial input for algorithms that calculate distress/performance differentials associated with variations between as-designed pavements and as-constructed pavements.

- Stress Prediction : ELSYM5 (chapter 3)
- Pumping Prediction : COPEs (appendix B, relationship A)
- Cracking Prediction : COPEs (appendix B, relationship B)
- Faulting Prediction : COPEs (appendix B, relationship H)
- Joint Deterioration Prediction : COPEs (appendix B, relationship J)
- CRCP Distress Prediction : TXSDH (appendix B, relationship K)
- Serviceability Loss Prediction : COPEs (appendix B, relationship O)
- Performance Prediction : AASHTO (appendix B, relationship N)

The second role of primary relationships is to provide a basis for developing secondary relationships that relate M&C variables to one another and to primary relationship predictors that are also M&C variables. The development of secondary relationships is discussed in chapter 4.

A third role for primary relationships is to provide an objective basis for estimating the relative changes in distress and performance that are induced by changes in the primary predictors. These so-called sensitivity analyses can show, for example, the relative effects of load accumulations, environmental factors, roadbed strength, and structural variables.

Sensitivity analyses are discussed in chapter 5 for both primary and secondary relationships in connection with the secondary relationships that are developed through laboratory studies. The sensitivity analyses reflect not only the deterministic effects that are provided by prediction equations, but also the prediction errors that are associated with any primary or secondary relationship.

CHAPTER 4 SECONDARY RELATIONSHIPS AMONG M&C VARIABLES

This chapter begins with a review of M&C specifications that are in current use by a number of States. Some of these specifications are for variables that are predictors in primary relationships, but many are not. Variables that are not primary distress and performance predictors, however, may be predictors of the primary predictors. Such M&C variables will be called secondary predictors of distress and performance.

By definition, a secondary relationship among M&C variables is one that shows how the variables are related to one another and to at least one primary predictor. Also, by definition, any M&C variable that is a primary or secondary predictor is a performance-related variable. It follows that M&C variables that do not appear in established primary or secondary relationships are either not performance-related, or that the defining relationships have not yet been established.

A classification scheme is presented in this chapter for virtually all M&C variables that have been associated with the surfacing layer of concrete pavements. A major purpose for the scheme is to show simultaneously which variables appear (1) in State M&C specifications, (2) as predictors in primary relationships, (3) in established secondary relationships, (4) in existing data bases that might produce new secondary relationships, and (5) in a data base produced by a laboratory study.

In the final sections of this chapter, existing secondary relationships are discussed, and assessments are made of the potential for deriving new secondary relationships from two existing and available data bases.

SUMMARY OF CURRENT M&C SPECIFICATIONS

To evaluate and/or develop practical secondary prediction relationships between materials and construction (M&C) factors and stress/distress/performance (S/D/P) indicators, it was first necessary to review those M&C factors that are currently specified and controlled by State specifications. Using reviews of construction specifications from several States in different regions of the country, it can be seen which M&C factors are generally controlled by the States. Different types of specifications control these variables; upper and lower limits, qualitative, and AASHTO/ASTM standard specifications are some examples. Occasionally, penalties and incentives may accompany the specifications. The purpose of this section is to present and summarize M&C specifications that are presently in use and relate them as a subset of the M&C factors to be discussed in the next section.

The State specifications reviewed represent four different Strategic Highway Research Program (SHRP) climatic regions. Six States from the SHRP Wet/Freeze Zone are included; Illinois, Minnesota, New Jersey, New York, Ohio, and Pennsylvania. Two are included from the SHRP Wet/Non-freeze Zone; Georgia and Louisiana. Two States represent the SHRP Dry/Freeze Zone; Idaho and Colorado. The final two States, California and Texas, represent the SHRP Dry/Non-freeze Zone. Table 4 summarizes current M&C specifications for

Table 4. Summary of current M&C specifications in selected States.
 [Caution: table not precise (see text)].

M&C Specification	SHRP WET/FREEZE ZONE						SHRP WET NON-FREEZE ZONE		SHRP DRY FREEZE ZONE		SHRP DRY NON-FREEZE ZONE		# ST WITH SPEC
	IL	MN	NJ	NY	OH	PA	GA	LA	ID	CO	TX	CA	
SURFACING													
FUNCTIONAL CHARACTERISTICS													
Profile		LP	LPI			LP	U		U			U	6
Skid Resistance												L	1
STRUCTURE CHARACTERISTICS													
LAYER CHARACTERISTICS													
Thickness	LP	LP	LPI	LP	LP	LP	LP	LP	C	LP	LP	LPC	12
REINFORCEMENT FACTORS													
Reinforcement Type	QA	A	A	QB	QB	QA	AC	Q	AB	QC	BC		11
Amount of Reinforcement	C	Q	C				C	Q			C		6
JOINT FACTORS													
Load Transfer Devices	QA	QA	Q	C	QB	QA	AC	Q	QA		BC	QB	11
Joint Geometry			Q	C	C		C	C	C		C	QC	8
PCC CHARACTERISTICS													
STRENGTH CHARACTERISTICS													
Flexural Strength	L	L			L		L ^o		L		L	L	7
Compressive Strength	L	L	LPI	L		LP	LP	LP	L		L	C	10
Tensile Strength													0
STIFFNESS CHARACTERISTICS													
Elasticity Modulus													0
DURABILITY CHARACTERISTICS													
Air Content	UL		UL	ULB		ULP	UL	UL	ULA	ULP		U	9
Voids Ratio													0
Unit Weight				UL	B				A				3
MIX FACTORS													
Cement Content		L	L	L	L	UL	L	L	L	L	L	UL	11
Water/Cement Ratio			U	U	U	U	U	U	U	U	U	U ^{oo}	9
Slump	UC	UL	UL	UL	UL	UL	UL	UL	UL	ULP	UL	UL	12
Cement Type	QAB	QA	QB	QBC	QAB	QAB	QA	QA	QA	QB	QB	QB	11
Yield		Q			B				A				3
COARSE AGGREGATE FACTOR													
Aggregate Types	Q	Q	Q	Q	Q	Q	Q	Q		A		Q	10
Gradation	UL	UL	UL	ULB	ULAC	UL	UL	UL	UL	UL	UL	ULP	12
Wear	U		U	U	U	U	UL		U	U	U	U	10
FINE AGGREGATES													
Aggregate Type	Q	Q	Q	Q	Q	Q	Q				Q	Q	9
Gradation	UL	UL	UL	UL	UL	UL	UL	UL	UL	UL	UL	ULP	12
Sand Equivalent							L		L	L	L	L	5
Fineness Modulus		UL	UL		UL	UL			UL				5
ADDITIVES													
Air Entrainment	QA	QA	QUA	UL	ULA	ULA	QA	QC	ULA	QA	ULC	BC	12
Other	Q	Q	QUA				Q		B	QA	Q	Q	8
CONSTRUCTION CONSTRAINT													
Ambient Moisture			Q	Q			Q		Q				4
Ambient Pour Temperature	UL		UL	L	L	UL		QL	UL	UL	L		9
Curing Temperature	L	QL	QL	QL	L	L	L	L	L	QL	L	L	12
PCC PVT MILEAGE (1000 miles)	4.2	3.5	0.6	3.8	1.8	3.6	1.1	1.6	0.2	0.4	2.9	3.1	25.8
% OF TOTAL US PCC	6.7	4.0	1.0	6.3	2.9	5.9	1.9	2.5	0.3	0.6	4.8	5.0	41.9

A - AASHTO Specification (may involve Q, N, U, L below).
 B - ASTM Specification (may involve Q, N, U, L below).
 C - Specification as provided by Engineer and/or construction plans.
 Q - Qualitative Specification (e.g., presence or absence of dowels).
 N - Quantitative Specification for Target Level (no tolerance specified).
 U - Upper Tolerance Limit for Quantitative Specification.
 L - Lower Tolerance Limit for Quantitative Specification.

P - Specification accompanied by contractual penalty for failure to meet tolerance specifications.
 I - Specification accompanied by contractual incentive for meeting specification tolerance(s).

* Mix Design Requirement, but not a pavement acceptance criteria.
 ** Implied, since maximum water content and minimum cement content are specified.

these States. Note that the PCC pavement mileage in these States represents over 40 percent of all PCC pavements in the United States. Note also that table 4 was prepared to take a reading of what State practices have been recently in terms of M&C specifications. The table is not exact and may be incomplete for some factors.

The 12-State specification review considered only those specifiable M&C factors that relate to functional and structural characteristics for surfacing. Each of the M&C factors fit into at least one of four categories: (1) explicit predictor of S/D/P (figure 1, box B4), (2) surrogates for explicit predictors (figure 1, box C1), (3) control factors for explicit predictors and surrogates (figure 1, box C2), and (4) process control factors (figure 1, box C3). As shown in table 4, the structural characteristics are broken down into the two subgroups of layer and PCC characteristics.

Several types of specifications were noted during the review of State PCC pavement specifications. Many M&C factors were controlled by a combination of one or more specification types. The following specification classes are currently being used in the States reviewed:

- AASHTO Specifications.
- ASTM Specifications.
- Specifications as provided by Engineer and/or on construction plans.
- Qualitative specifications (e.g., presence or absence of dowels).
- Quantitative specifications for target levels:
 - upper tolerance limit.
 - lower tolerance limit.
 - upper and lower tolerance limit.
- Contractual penalty for failure to meet tolerance specifications.
- Contractual incentive for meeting specification tolerance(s).

Excluding a few special exceptions, all State PCC pavement specifications were made using one or more of the above specification types.

Of the specifications generally reviewed, profile and skid resistance are the only functional characteristic factors that are controlled by current State specifications. Profile, the relative amount of longitudinal roughness, has governing specifications in 6 of the 12 States. Three of these were accompanied by contractual penalties for failure to meet tolerances. Of the 12, only California places a lower limit specification on skid resistance, the resistance of a pavement surface to the sliding or skidding of a vehicle.

At present, there are five major factors controlled by State specifications relative to structural layer characteristics. The first and most obvious is PCC pavement thickness. All 12 States specify controls over thickness. With the exception of Idaho, thickness specifications are made

using lower quantitative limits and penalties for failure to meet the specified tolerances. In the case of New Jersey, contractual incentives are also specified for thicknesses exceeding the specified amount.

Most States have specifications controlling reinforcement type. A majority of the specifications are qualitative and usually accompanied by an AASHTO or ASTM specification. The amount of PCC reinforcement is controlled by the engineer and/or construction documents in six States while the remainder have no applicable specification.

Load transfer devices and joint geometry are two joint factors that are controlled in the reviewed States. Qualitative specifications with an AASHTO/ASTM companion are used in eight of the States for controlling load transfer. Another three States have controls using other specification formats. Seven of the 12 States specify joint geometry control through the engineer and/or construction documents.

The most widely controlled M&C factors are PCC characteristics (including mix factors). In the 12 State specifications reviewed in this study, approximately two thirds of the factors fit into this category. In studying table 4, it is noted that current PCC specifications are relatively consistent across all of the four climatic regions. The States are currently using PCC characteristics as the primary controlling factors for rigid pavements.

Although not reflected in table 4, aggregate quality control is also an M&C specification that is applied by most States. Generally, this is accomplished through certifying/approving sources of aggregate which meet abrasion, soundness, alkali and D-cracking requirements, etc.

Compressive and flexural strength are two factors controlled by a majority of the States, while none of the 12 States in this review have specifications regulating tensile strength. Nine States specify lower limits on compressive strength with California allowing the engineer and/or construction plans to set limits. Four States have penalties for compressive strengths below tolerance levels with New Jersey adding incentives for superior strengths. Seven set forth lower limits on flexural strength. Again, these specifications are consistent across all four SHRP climatic regions. Stiffness characteristics (i.e., elastic modulus) have no controlling specifications in the 12 States reviewed in this study.

Durability characteristics are primarily controlled by setting quantitative limits on air content. Eight States place upper and lower limits with California setting only an upper limit. Three States control unit weight by three different specification types; AASHTO, ASTM, and upper and lower limits. None control the voids ratio, a measurement of bulk density.

Mix factors make up the largest single subset of M&C factors for PCC characteristics. These factors are routinely controlled in most of the 12 States. Eleven States set lower limits on cement content. Nine specify upper limits on water/cement ratios. All of the States set upper and lower limits on slump. Cement type is specified with standard AASHTO/ASTM specifications along with qualitative type specifications. All States use

this type of control. Yield is specified in three States by three differing specification types.

Coarse and fine aggregates both have a variety of specifications controlling their use. Eight States use qualitative controls for both coarse and fine aggregate type. All States placed upper and lower limits on the gradation of both coarse and fine aggregates. Wearing of the coarse aggregate was given an upper limit in ten States. Sand equivalent of the fine aggregate was given a lower limit in five while fineness modulus was controlled by upper and lower limits in five States.

The category of PCC additives is headed by air entrainment additives. All 12 States in the study placed controls over the use of air entrainment. The types of specifications varied greatly from State to State. Other PCC additives (e.g., high-range water reducers, flyash, etc.) were given qualitative controls in 7 of the 12 States.

Three construction constraints were made in the States within the study. Ambient moisture has qualitative limits in four of the States. Quantitative limits are placed on ambient placement temperature in nine. The final characteristic, curing time, is given a lower limit in all of the States with four adding qualitative constraints to the limit.

The State specifications for M&C factors represent much collective knowledge and years of experience on the importance of these factors to the construction and performance of concrete pavements. For the present study, the information shown in table 4 provides a substantive basis in the selection of M&C factors for the development of secondary relationships among M&C variables.

SECONDARY RELATIONSHIP VARIABLES

The purpose of this section is to identify and classify all M&C variables that appear in existing secondary relationships and/or that are candidates for useful relationships that have not yet been developed.

In the general research framework of figure 1, any secondary relationships among M&C variables must include at least one primary relationship predictor (boxes B3 and B4) and should contain one or more other M&C factors that are represented by box C. To provide scope commensurate with the project resources, secondary relationships in this study will be restricted to only those M&C variables that are directly related to the surfacing layer of rigid pavements. Relative to figure 1, this restriction excludes primary predictors associated with either roadbed soil properties (box B3) or base/subbase properties (box B4). At least for relationships derived from laboratory studies, other excluded M&C variables are those relating to reinforcement, load transfer, joint geometry, and shoulder construction.

Table 5 contains a detailed list of M&C variables that are associated with the surfacing layer of concrete pavements. The table includes all variables that are candidates for the secondary relationships that will be discussed in the remainder of chapter 4 and in chapter 5. As shown in the

Table 5. Classification and cross references for PCC M&C variables.

M & C VARIABLES ASSOCIATED WITH THE SURFACING LAYER OF CONCRETE PAVEMENTS		A. Number of States having Specifications for Variable (see table 4)	B. Primary Relationships Containing Var as a Predictor (see app. B)	C. Secondary Relationships Containing Variable (see app. C)	D. Variables Occurring in RI Data Base (see app. D)	E. Variables Occurring in COPEs Data Base (see app. E)	F. Variables Selected for Experimental Study * (see chapter 5)
CLASS	SUBCLASS CODES AND VARIABLES						
Variables for PCC Surfacing Layer	Surface Profile (As-Constr.)	6/12	N				(M)
	Surfacing Thickness	12/12	ALL		N18,N48	D41	(F)
	Reinforcement Variables	11/12	B,G,I				(F)
	Joint Geometry Variables	8/12	B,H,J,O,R		N15		(F)
	Load Transfer Variables	11/12	H,J,N				(F)
	Shoulders Variables		H				
Properties of Hardened PCC	Flexural Strength	7/12	B-F,N-Q	A-F	N38-N41	D102A	M
	Compressive Strength	10/12	R	A-P,U	N42-N46		M
	Tensile Strength		G	G			M
	Elastic Modulus		C-G,I,N-Q	N,O,P,Q			M
	Freeze-Thaw Resistance			M,T			(M)
	Shrinkage		G	R			(M)
	Thermal Coefficient		G	S			(M)
	Air Bubble Distribution						(M)
	Gel-Space Ratio (Porosity)			H			(M)
	Scaling Resistance						(M)
	Abrasion Resistance						(M)
	Permeability						(M)
PCC Placement and Curing	Placement Temperature	9/12	R				
	Curing Time	12/12					
	Curing Temperature	?					
	Curing Humidity	4/12					
Properties of Plastic PCC	Slump	12/12		U	N32		M
	Air Content	9/12	R	H,K,M,T,U	N33-N34	D104A	M
	Unit Weight	3/12		N	N35	D107A	M
	Yield				N37		M
	Mixing Time						M
	Time of Set						
	Heat of Hydration						
Coarse Aggregates Properties	Type	10/12		B,D,Q,S		D110	F
	Gradation	12/12		C,G,L		D109	F
	Soundness	10/12					
	Reactivity	?	J,O				
	Quantity			Q,R		D101A	F
	D-Cracking Potential						
Fine Aggregates Properties	Type	9/12				D112	
	Gradation	12/12					
	Sand Equivalent	5/12					
	Fineness Modulus	5/12					F
	Soundness						
	Reactivity	?	J,O				
Quantity			Q,R		D101B	M	
Cement Propert.	Cement Type	11/12				D105	
	Cement Factor (Content)	11/12		I,K,L		D101C	F
	Cement Alkali Content					D106	
Water Variable	Water Chemical Composition						
	Water/Cement Ratio	9/12		D,H,I,J,R,U		WTOC	M
	Water Content						F
Additives Properties	Air Entrainment Agent/Type	12/12					F
	Fly ash Type/Content						
	Superplasticizer Type/Content						
	Accelerator Type/Content						
	Retarder Type/Content						

* Note: F = experimental factor
M = measured variable
() = large scale study only

first column, the surfacing variables are listed in nine classes that range from layer properties through additive properties.

The remaining columns of table 5 show which variables (column A) have State M&C specifications that were shown in table 4, which variables (column B) appear either explicitly or implicitly as primary predictors in one or another of the primary relationships shown in appendix B, which variables (column C) appear in one or another of the secondary relationships given in appendix C, which variables (column D) occur in the five-state Resource International Inc. (RII) data base that is presented in appendix D, which variables (column E) occur in Concrete Pavement Evaluation System (COPES) data base presented in appendix E, and which variables (column F) were chosen in chapter 5 for inclusion in the laboratory study.

Thus, table 5 serves not only as a classification scheme for relevant M&C variables, but also provides cross-references for the occurrence of each variable in the various elements of the study.

Comparison of columns A and B shows that, with few exceptions, State specifications provide good coverage of predictor variables in primary relationships. As will be discussed in the next section, column C shows that many secondary relationships exist for relating primary predictors to one or two secondary predictors, but none of the available secondary relationships provides a comprehensive coverage of all the secondary M&C factors that are included in State specifications. The same can be said for the sets of variables that appear in the RII and COPES data bases (columns D and E), as will be discussed later in this chapter.

The last column (column F) of table 5 indicates which M&C variables are included in the laboratory study that is documented in chapter 5 on the development of new and more comprehensive secondary relationships. Variables enclosed in parentheses are not included in the proposed initial study, but are additional candidates for the larger-scale laboratory and/or field studies that are discussed in chapter 8.

ASSESSMENT OF AVAILABLE SECONDARY RELATIONSHIPS

As previously discussed, a secondary relationship is one that relates a primary predictor of pavement performance to one or more M&C variables. When combined with primary performance prediction relationships, these secondary relationships provide the necessary link between recognized measures of pavement performance and various M&C factors that have not traditionally been used to predict pavement performance. The literature review for this project uncovered many useful equations that may be classified as secondary relationships. Appendix C provides a list of the select relationships that are of interest to this study.

This section of the report is provided to assess those available secondary relationships based on their utility in developing a PRS system. Consequently, it is useful at this point to identify the assessment criteria:

- Is the dependent variable in the equation a primary predictor that is commonly found in the available primary relationships? If not, it is

not much use in a PRS system. [Concrete flexural strength (modulus of rupture), elastic modulus and slab thickness are the most commonly used primary predictors, however, there are other factors such as shrinkage and thermal coefficient that are now finding their way into newer mechanistic models.]

- How many other M&C factors are considered by the relationship in estimating the value of the primary predictor? It is certainly desirable to use an equation that accounts for several key M&C factors that have an effect on the primary predictor, particularly if the factors have interacting effects on the predictor.
- Is the relationship accompanied by pertinent statistical measures (i.e., coefficient of determination, standard error of estimate and number of cases used in derivation)?

The primary predictors of PCC pavement performance fall under five categories: (1) PCC strength, (2) PCC bending stiffness, (3) PCC shrinkage, (4) PCC durability, and (5) slab physical characteristics. The first four of these are all measurable properties of the hardened concrete which are not directly controllable during the design/construction process and are, therefore, amenable to being correlated with controllable M&C factors. Slab physical characteristics (i.e., thickness, joint spacing, load transfer, etc.) are also important primary predictors, however, they are directly controllable from the design/construction standpoint and, therefore, do not require secondary relationships. Thus, only the first four categories are addressed here.

PCC Strength

Based upon the amount of effort that has been directed towards developing performance prediction relationships that consider concrete strength, it is logical to conclude that strength is a very important material-related property. Strength has been characterized under three modes of loading: compression, tension and flexure. Because of concrete's great capacity to carry load in compression, compressive strength is the factor that has been given the most attention. This is reflected by the fact that of all the strength relationships (A through M in table 6 and appendix C), the six related to compression (H through K) consider the most number of independent variables. It is further demonstrated by the fact that six of the seven remaining strength relationships include compressive strength as an independent variable. Unfortunately, compression is not the mode through which most concrete pavements develop cracks. For this reason, compressive strength will be regarded as a surrogate for the primary flexural strength predictor.

In general, concrete pavements develop cracking when externally or internally induced tensile stresses exceed the tensile strength of the concrete. It is also generally recognized that cracking may develop due to the cumulative fatigue effects of multiple applications of stress that may be well below the tensile strength of the concrete. For this reason, tensile stress is the most commonly used concrete pavement behavioral response in developing performance prediction relationships. Unfortunately, because of the complexity of test procedures designed to measure concrete tensile

Table 6. Comparison of available secondary prediction relationships.

App. C Cross- Ref.	Dependent Variable (Primary Predictor in PPR)	Independent Variables That Are Also Primary Predictors of Pavement Performance										Other Independent Variables ¹						Statistics				
		SF	ST _d	ST _i	SC	EC	KS	KT	KD	UW	SL	GR	WC	CC	AC	AE	CA	MS	VA	R ²	SEE	n
A	SF [] ²	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
B	SF []	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	-	61
C	SF []	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	71
D	SF [28]	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	#	-	-	-	-	8
E	SF []	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
F	SF []	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
G	ST _i []	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	46
H	SC []	-	-	-	-	-	-	-	-	-	-	#	#	-	-	-	-	-	-	-	-	17
I	SC [07] ³	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	-	-	-	-	12
J	SC [t] ³	-	-	-	-	-	-	-	-	-	-	-	#	-	-	#	-	-	-	-	-	-
K	SC [28]	-	-	-	-	-	-	-	-	-	-	-	-	#	-	#	-	-	-	-	-	-
L	SC [28]	-	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	#	-	-	-	-
M	SC []	-	-	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	-	-	-
N	EC	-	-	-	[]	-	-	-	-	#	-	-	-	-	-	-	-	-	-	-	-	-
O	EC	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	EC	-	-	-	[]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Q	EC	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	#	-	#	-	-	-
R	KS	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	-	#	-	-	-
S	KT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	14
T	KD	-	-	-	-	-	-	-	-	-	-	-	-	-	#	-	-	-	-	-	-	-
U	WC	-	-	-	[]	-	-	-	-	-	#	-	-	-	-	#	-	-	-	-	-	-

Variable Descriptions:

SF = PCC Flexural Strength or Modulus of Rupture
 SC = PCC Compressive Strength
 ST_d = PCC Tensile Strength (Direct Tension)
 ST_i = PCC Tensile Strength (Indirect Tension)
 EC = PCC Elastic Modulus
 KS = PCC Shrinkage Coefficient
 KT = PCC Thermal Coefficient
 KD = PCC Durability

UW = PCC Unit Weight
 SL = PCC Slump
 GR = Gel-Space Ratio
 WC = Water/Cement Ratio
 CC = Cement Content
 AC = Air Content
 AE = Entrained Air Content
 CA = Coarse Aggregate Type
 VA = Volume of Aggregate
 MS = Maximum Size Aggregate

R² = Coefficient of Determination
 SEE = Standard Error of Estimate
 n = Number of Cases (Samples)

¹ # indicates independent variable considered in relationship.

² Numbers in brackets [] represent age (days) of specimen when tested.

³ For this variable, strength correlations are provided for 1, 3, 7 and 28 days.

strength, it has not received much attention as a potential primary predictor in concrete pavement performance prediction relationships. The advent of the indirect or splitting tension test has provided a more attractive method to measure concrete tensile strength; however, it has had very little effect on the most commonly used measure of concrete strength for rigid pavement performance prediction, i.e., flexural strength.

The flexural strength (or modulus of rupture) test has achieved its preferred status because of its relative simplicity and the fact that it does simulate the kind of bending stresses that are experienced in concrete pavements. One other aspect of the flexural test is that besides the measurement of ultimate strength, a number of studies have been conducted to study the effects of cyclic flexural loads on the fatigue of the concrete. These studies have resulted in primary prediction relationships that have been used to predict the field performance of concrete pavements.

In terms of the usefulness of the strength equations for use in developing secondary prediction relationships, several observations can be made:

- In addition to the fact that compressive strength is not found in many of the available concrete pavement performance prediction relationships, there is a problem with all six of the secondary prediction relationships studied (H through M) in that none considers more than two of the "other" independent variables. The fact that many of the other independent variables have been studied and included in one or another of these relationships implies that they must have some significance in terms of their effect on concrete strength. Unfortunately, there is no one relationship that accounts for all the effects of the independent variables as well as their interactions.
- Both tensile and flexural strength relationships (A through G) suffer from the same problem discussed above. In fact, these relationships consider even fewer of the other independent variables.
- Another problem with the flexural and tensile strength relationships is that six of the seven depend on some knowledge of the compressive strength of the concrete. For the needs of this project, it is much more preferable to have secondary prediction relationships that are a function of M&C factors that are directly controllable (i.e., those listed under the other independent variables columns).

PCC Bending Stiffness

The primary measure of concrete bending stiffness is its Young's modulus (or modulus of elasticity). Elastic modulus is a very common primary predictor in mechanistically derived primary prediction relationships. It has appeared directly in at least one equation but is more frequently used as a factor for predicting the critical concrete tensile stress due to wheel load.

Four secondary prediction relationships that relate elastic modulus to other M&C factors were discovered in the literature review phase of the

project. Unfortunately, these relationships suffer from the same kind of problems that were discussed for the strength prediction relationships, that is, they rely on another primary predictor (concrete compressive strength) and/or they do not consider many of the other independent variables that can affect a concrete's resistance to bending.

PCC Shrinkage

This refers to the drying shrinkage that occurs in portland cement concrete once its moisture condition is allowed to vary with the environment. Drying shrinkage is affected primarily by the unit water content of the concrete. Other factors affecting drying shrinkage include cement composition, cement content, quantity and quality of paste, mixture proportions, amount of reinforcing steel, maximum size of aggregate, and curing conditions.

Shrinkage is a key factor in continuously reinforced concrete (CRC) pavements or concrete pavements that have a long joint spacing. As shrinkage occurs and is restrained by friction along the underside of the slab, internal stresses build up that can exceed the strength of the concrete. When this happens, a crack pattern is established that will certainly have an effect on the long-term performance of the pavement. Shrinkage is normally controlled by maximizing the amount of aggregate in the mix. In jointed pavements, it is further treated by the selection of an appropriate joint spacing. In CRC pavements, shrinkage is treated in the design process by controlling the strength (and therefore crack spacing) and by providing enough steel reinforcement to ensure that the cracks that do develop do not become very wide.

Only one shrinkage relationship was uncovered during the literature review phase for this project. This relationship does seem to cover two of the key factors that are known to affect shrinkage, (i.e., water/cement ratio and volume of aggregate); however, it would be desirable to conduct a laboratory study where the effects of the other potential independent variables are considered.

PCC Thermal Coefficient

Concrete thermal coefficient can have a significant impact on concrete pavement performance in that it has an effect on the amount of horizontal movement a slab will undergo as it is subjected to changes in temperature. When combined with other loading mechanisms, slab contraction (due to low temperatures) can result in mid-slab cracking. On the other hand, when a slab expands (due to higher temperatures), it can result in severe joint distress such as compression failures and blowups, particularly when the joint becomes filled with incompressible materials.

Thermal coefficient is primarily a function of the coarse aggregate used in the mix. From that standpoint, a secondary relationship that considers only coarse aggregate type may be sufficient for use in a PRS system. However, it would be desirable to have a relationship that accounts for a wider cross section of coarse aggregate types along with some other key characteristics of the coarse aggregate.

PCC Durability

Concrete durability refers to the ability of a given concrete to withstand the freeze-thaw cycles that occur in northern environments. The typical type of distress that has been associated with concrete durability is D-cracking. Air content, air entrainment and air bubble size and distribution have been found to be the factors that most affect freeze-thaw durability. Other significant factors that can affect durability are the D-cracking potential of the coarse aggregate and the reaction of the coarse aggregate to alkali and/or sulfate attack. For the relationship shown in table 6 (labelled T), only the effect of air content is reflected. Based on this, it would be desirable to have a relationship in which other key factors are considered.

Summary

In general, the following observations can be made that essentially assess the usefulness of the secondary relationships studied:

- Not one relationship considered all the potential independent variables. Even if a given factor is considered to be insignificant, it is desirable to have the experimental results to support it.
- Many of the equations included terms that consisted of other primary predictors. This causes problems in a PRS system in that although these other primary predictors are significant, they are not directly controllable M&C factors.
- None of the equations has the important statistics (i.e., coefficient of determination and standard error of estimate) attached to them. In order to consider the variability effects of the individual factors within the system, it is important to have these kinds of statistics.

The assessment of the secondary prediction relationships (in terms of their usefulness in developing a PRS system) indicates the strong need for a statistically designed laboratory experiment to study the effects of the directly controllable M&C factors on selected primary predictors of pavement performance. This assessment was used as a basis for designing the small-scale laboratory study discussed in chapter 5. It is also used as a basis for designing the large-scale laboratory and field experiments that are addressed in chapter 8.

ASSESSMENT OF EXISTING DATA BASE POTENTIAL

While attempting to develop secondary relationships among M&C factors and performance predictors, the researchers hoped to draw upon any useful information contained in existing data bases. After an extensive literature review, two data bases seemed sufficiently comprehensive to deserve further investigation. These were the Resource International Inc (RII) and the COPES data bases. Extensive statistical analyses were performed on each data base and secondary relationships were derived using multiple regression analysis. The detailed results of these studies are presented in appendixes D and E. In this section, the findings and conclusions of these analyses

will be summarized and the potential for developing secondary relationships from existing data bases will be evaluated.

RII Data Base

The focus of the RII study was the development of primary prediction relationships for PCC pavements based on historical data collected in five different States.^(84.2) This study focused on the development of secondary prediction relationships utilizing the same comprehensive RII data base. Using standard stepwise multiple regression, two primary and three secondary relationships were derived. The primary relationships were developed to help gauge the reasonableness of the RII data and to ensure compatibility between any derived secondary relationships and generally accepted primary relationships. Prior to the regression analyses, an extensive study of the individual variables and their interrelations was undertaken. A Pearson correlation matrix, frequency histograms and scatter plots were all generated for this purpose. The details of these statistical analyses are presented in appendix D along with supporting tables and figures. In the course of the investigation, several observations were made which question the validity of the resulting regression equations. The chief topic of discussion here will be the shortcomings of the RII data base for producing secondary prediction relationships.

The early stages of the statistical analyses showed that several key variables possessed a substantial number of missing observations. In the original RII study, steps were taken to fill some empty cells by developing relationships from the existing data. No such effort could be made in this study. The difficulties arising from missing observations were greatly magnified when two or more variables were studied simultaneously.

Several unsettling associations among the variables were exposed during the investigation. The strong negative correlation between water/cement ratio and concrete slump conflicts with generally accepted engineering principles. Obviously, a lower slump value is not consistent with a higher water/cement ratio. It was noted, however, that slump was strongly correlated with pavement age. The latter variable was included in the analyses to help detect changes in common construction techniques and/or technology over time. This strong correlation may indicate that such a change has occurred and could have been the cause of the inconsistent correlation discussed above. For example, the introduction of slip-form pavers would push slump values down but not necessarily alter the water/cement ratios or flexural strengths. It is vital to identify these types of phenomena yet very difficult to do so. For this reason, observational data bases are often questionable sources of statistically-based secondary relationships. Some of the correlations coincided with general expectations but the difficulties, such as these mentioned above, overshadowed their significance.

Scatter plots are perhaps the best way to visualize the relationships between two variables. Appendix D presents a wide assortment of these plots. A majority of the graphs display reasonable associations between the variables, but two produced entirely unexpected results. One depicts a strong relationship between 7-day flexural strength and concrete slump that is inconsistent with expectations. Typically, lower concrete slump predicts

higher strength; not the lower values predicted by this scatter plot. The second questionable relationship presents a graph that predicts sharply higher strengths with increasing air contents. Within the range of the air content values in the RII data base, such a drastic effect is not expected. Again, these inconsistencies could be attributed to differences in mix design and/or construction practices between the States.

Three pavement performance predictors were selected to serve as dependent variables for multiple regression analyses. The stepwise approach for entering and deleting variables was employed in the development of these secondary relationships. Secondary prediction equations were developed for 7-day flexural strength, 7-day compressive strength and concrete core strength. Unfortunately, after a brief review, each of these derived equations displayed questionable attributes.

The equation for 7-day flexural strength was based on only 53 of the 733 sections because of the missing value problem. When the stepwise regression was complete, only slump and percent air remained as independent variables in the equation. Contrary to expectations, however, the coefficient on slump was positive indicating that an increase in slump would, likewise, increase flexural strength. As mentioned previously, slump is strongly correlated with time and this could provide the source for this inconsistency. Only 42 percent of the variability in 7-day flexural strength was accounted for by slump and percent air (i.e., $R^2 = 0.42$).

The only independent variable that entered the secondary equation for predicting 7-day compressive strength was water/cement ratio. A negligible 8 percent of the variability in compressive strength was accounted for. The analysis was performed on 219 sections.

Perhaps the best of the three derived secondary relationships was for concrete core strength. Unfortunately, core strength has not generally been used as a direct predictor of pavement performance. Only 59 percent of the core strength's variability was accounted for by the water/cement ratio, the only independent variable to enter the equation.

Two primary relationships were developed; one to predict the pavement condition rating, and the other to predict ride quality index. Both of these equations contained inconsistencies similar to those mentioned previously. The relationship for ride quality had a positive coefficient for slump, contrary to general expectations. A term for pavement age entered the equation for pavement condition rating and possessed a positive coefficient. It is not expected that an increase in the age of a pavement will produce higher ratings of pavement condition.

In summary, missing data, non-representative observations and inconsistent correlations are causes for concern that lead to the conclusions that the RII data base would not serve to develop useful and reasonable secondary relationships for PCC pavements.

COPES Data Base

The extensive amount of information available in the COPES data base made it a likely source of statistically-based secondary relationships.^(85.2)

One illustrative secondary relationship was developed using multiple regression analysis. The equation makes reasonable predictions of 28-day modulus of rupture using three M&C factors as the independent variables. The details of the relationship are presented in appendix E. Prior to developing this equation with regression analysis, extensive effort was directed toward studying the variables themselves. Specific variable characteristics reviewed were arithmetic means, standard deviations and distributions (histograms). The interrelations between the variables were studied by producing a Pearson correlation matrix and several scatter plots. During this in-depth study of the data, several problematical observations were made that question the validity of the derived secondary relationship.

An item of concern was Minnesota's overwhelming majority of test sections in the data base. Of 1182 observations, 994 were made in Minnesota. The balance of the sections were located in California, Georgia, Louisiana, and Utah. Initially, it was hoped that the COPES data base would supply data collected from six different States evenly dispersed within a wide range of geographical locations. Since this was not the case, the available data may be nonrepresentative of the M&C variables necessary for reasonable and widely accepted secondary relationships. Equations built upon this information would be based mostly on Minnesota highway sections.

While developing the Pearson correlation matrix for key variables in the COPES data base, it was observed that several were subject to high missing value counts especially when considered pairwise with other variables. The 28-day modulus of rupture was targeted as an explicit predictor of performance for which a secondary relationship would be derived. Unfortunately, only 199 of the test sections had a modulus value present. Therefore, any correlation with modulus could contain, at best, only 199 pairs of the variables and in most cases fewer. A list of the variables and the number of available cases of each is presented in appendix E along with the correlation matrix which shows the pairwise counts.

Another notable missing value count exists for present serviceability index (PSI). It was hoped that this measure of pavement performance could be used to derive a primary relationship to measure the reasonableness of the available COPES data. The analysis necessary to produce this primary relationship was conducted, but the extremely low number of pairwise sets did not provide an adequate basis for the analysis.

A second variable, concrete slump, was chosen to serve as the primary predictor for another secondary relationship. It was discovered, however, that the distributions of several key variables, including slump, displayed modal values which contained the vast majority of the observations. Thus, these variables were essentially constant. The correlations between these variables and others had little meaning since a reasonable range of values was not available and, furthermore, their inclusion in a derived relationship could lead to invalid conclusions. As a case in point, the histogram of mean concrete slump displayed a mode of 1.5 in (38.1 mm). This mode represents 1011 out of 1125 total observations. Engineering reasoning would expect more variation in the measured slumps from 1125 sites. In appendix E, this problem is discussed further, and other variables with modal tendencies are identified.

Steps were taken to ensure the compatibility of any successfully derived secondary relationships with generally accepted primary relationships. Developed in tandem, the primary and secondary relationships could provide a plausibility test for the data. Two measures of PCC pavement performance were the focus of the primary relationships; PSI and transverse cracking. Once again, the analyses produced disappointing results. As mentioned above, only 26 observations of PSI were present which removed it from any practical use. Next, transverse cracking displayed negligible linear correlation with any of the other variables in the data base making further investigation meaningless.

Because of the potentially significant effects of changes in common practice and/or technology over time, a variable relating to the "time opened to traffic" was included in the correlation matrix to identify such occurrences. Pavement thickness, water/cement ratio and several other concrete mix parameters exhibited reasonably strong correlation with the age of the pavement, a variable that is directly related to the date opened to traffic. One might conclude that common practice has changed over time, but further investigation would be required to support this logic. These types of phenomena can have significant effects on the results of statistical analyses and must be considered during the formulation of secondary relationships. The difficulty in analyzing these is one argument for the use of controlled laboratory data in lieu of these observational data bases.

Although the original researchers at the University of Illinois had successfully developed primary pavement performance prediction relationships, the effort to derive secondary relationships from the COPES data base would have to be characterized as unsuccessful. Missing data, low pairwise variable counts, geographic bias, and inability to check the reasonableness of the information with primary relationships from the same data base created a formidable barrier. The primary result of this investigation was the conclusion that a properly designed laboratory testing program would produce secondary relationships that would be far superior to any that could be produced from either the RII or COPES data bases.

CHAPTER 5 LABORATORY STUDIES FOR DEVELOPMENT OF SECONDARY RELATIONSHIPS

This chapter gives details of the experimental design, implementation procedures, and analytical results of the laboratory study for the development of secondary relationships among primary predictors of PCC distress and key M&C factors that may be controlled during the construction process. The initial experimental design and implementation of the study are presented in the next two sections, respectively. The rationale for the analyses of laboratory study data is then presented followed by the actual analyses. The chapter is closed by summarizing the results of the laboratory study.

INITIAL EXPERIMENT DESIGN FOR LABORATORY STUDY

Through a process involving literature reviews and meetings among engineers, PCC mix design specialists, and statisticians, seven M&C variables were selected as experimental design factors for the initial laboratory study. Each factor was controlled at two levels, so that the total number of factorial combinations was 2^7 or 128. The seven factors selected are as follows:

1. Coarse Aggregate Type (CAT): Soft and Hard.
2. Coarse Aggregate Maximum Size (CAM): Small and Large.
3. Fine Aggregate Fineness Modulus (FAM): Low and High.
4. Air Entraining Agent Quantity (AEQ): None and Some.
5. Coarse Aggregate Quantity (CAQ): Low and High.
6. Cement Quantity (CEQ): Low and High.
7. Water Quantity (WAQ): Low and High.

The first three factors determined the nature of the aggregates used in the PCC mixes. Two coarse aggregate types were chosen to cover a wide range of hardness. Two coarse aggregate sizes were selected to examine their impact, both individually and in interaction with coarse aggregate type. Two levels of fine aggregate modulus were chosen to determine the effect on air content and slump.

The fourth factor provided for two levels (amounts) of the air entraining agent. At the first level for this factor, no agent was to be used. For the second level, a fixed amount of agent was apportioned to target a broad range of air contents typical of paving concrete.

The remaining controlled factors determined the relative quantities of coarse aggregates, cement, water and fine aggregates that occur in each PCC batch. For the fifth factor, coarse aggregate factor levels of 60 and 75 percent were targeted for bulk (dry-rodded) quantities per cubic yard. These levels were estimated to produce true volume percentages in the neighborhood of 33 to 42 percent for the experimental batches.

Low and high levels for quantities of cement and water (the sixth and seventh factors) were to be specified as percents of PCC batch volume. The four combinations of cement and water quantities were intended to produce four different water/cement ratios (WCR) by volume. As indicated in figure 6, the four WCR values (shown with rectangles) were initially selected to provide levels for percent water and percent cement to not only satisfy the practical limiting conditions for WCR but also enclose those WCR values that are within the realm of currently utilized mix designs that are based on strength criteria and other generally accepted concrete mix proportioning characteristics such as workability.

Thus, figure 6 implies that the desired low and high levels for water quantity might be 13 and 16 percent, and that levels for cement quantity might be 8 and 12 percent. Final selections of these levels were not made until the mix criteria were assessed and several test batches were mixed and tested.

The fine aggregate quantity was the uncontrolled factor in the experiment. Therefore, no indication was given as to the quantities (levels) of fine aggregate used in each batch. After all the levels for a given batch were identified, enough fine aggregate was added to produce a cubic yard.

Although there were 128 factorial combinations of levels for the 7 factors, only 64 of the combinations were actually implemented in the study. Using principles of experimental design, a one-half fraction of the complete factorial was used to identify the 64 primary combinations or cells of the factorial. This fractional factorial approach made it possible to collect information on as many variables as possible, while staying within the available funding for specimen preparation and testing. It did produce constraints on the extent of the statistical relationships that could be derived (i.e., the confounding of higher-order interactions between the factors), but these were considered acceptable.

To obtain estimates of "between batch" error, eight factorial combinations were replicated. The eight replicate cells represent a further fraction of the complete factorial and provide one replicate set of data for each of the eight aggregate combinations given by the first three factors.

To summarize, the laboratory study required a total of 72 batches, 64 of which represent distinctive combinations of the 7 experimental factors and 8 of which represent replication of distinctive combinations. The 72 batches and associated concrete specimens were produced in a completely randomized order.

Table 7 is an illustrative table that contains identification data, factorial design specifications, and calculations to produce mix weights for each mix component. In addition to factorial specifications, the calculations require moisture contents and specific gravities for the two aggregates and assumptions on the air volumes that will be produced. It is noteworthy to mention that the percent volume and weight of the fine

PERCENT CEMENT	PERCENT WATER																
	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
7	0.36	0.41	0.45	0.50	0.54	0.59	0.64	0.68	0.73	0.77	0.82	0.86	0.91	0.95	1.00	1.04	1.09
8	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.60	0.64	0.67	0.71	0.75	0.79	0.83	0.87	0.91	0.95
9	0.28	0.32	0.35	0.39	0.42	0.46	0.49	0.53	0.56	0.60	0.64	0.67	0.71	0.74	0.78	0.81	0.85
10	0.25	0.29	0.32	0.35	0.38	0.41	0.44	0.48	0.51	0.54	0.57	0.60	0.64	0.67	0.70	0.73	0.76
11	0.23	0.26	0.29	0.32	0.35	0.38	0.40	0.43	0.46	0.49	0.52	0.55	0.58	0.61	0.64	0.66	0.69
12	0.21	0.24	0.26	0.29	0.32	0.34	0.37	0.40	0.42	0.45	0.48	0.50	0.53	0.56	0.58	0.61	0.64
13	0.20	0.22	0.24	0.27	0.29	0.32	0.34	0.37	0.39	0.42	0.44	0.46	0.49	0.51	0.54	0.56	0.59
14	0.18	0.20	0.23	0.25	0.27	0.29	0.32	0.34	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.52	0.54

1. Cell entries are WCR values computed by $0.3175 (\% \text{ Water}) / (\% \text{ Cement})$ where $0.3175 = 1 / (\text{Specific Gravity of Cement}) = 1/3.15$.
2.  = Combinations for which $(\text{WCR}) < 0.34$
3.  = Combinations for which $(\text{WCR}) > 0.65$
4. WCR values in rectangles represent one option for levels of percent water and percent cement that cover practical ranges of WCR and that contain mix designs that might be used in practice.

Figure 6. Rationale for selections of factorial levels for water and cement quantities.

Table 7. Batch input data for the laboratory study.

A. IDENTIFICATION

1. Batch Number _____ 2. Sequence Number _____ 3. Date _____

B. AGGREGATES & AIR ENTRAINMENT CHARACTERISTICS OF BATCH

- 1. Coarse Aggregate Type _____
- 2. Coarse Aggregate Max Size _____
- 3. Fine Aggregate Fineness Modulus _____
- 4. Air Entrainment Level for Batch _____

C. MIX QUANTITIES

Coarse Agg.	Cement	Water	Air	Fine Agg.	Total
-------------	--------	-------	-----	-----------	-------

1. FACTORIAL LEVELS

1.1 % of Total Volume					
1.2 CF/CY					

2. BATCH CALCULATIONS

2.1 Specific Gravity					
2.2 LB/CF					
2.3 CF in Batch					
2.4 LB in Batch					
2.6 W/C Ratio (WCR)					

aggregate result from what remains after the coarse aggregate, cement, water and air quantities have been taken into account. Thus, table 7 represents a form sheet that was completed for each of the 72 batches. In essence, table 7 provides a completely defined "recipe" for each of the experimental mixes.

Finally, table 8 lists the characteristics that were assessed for each PCC batch, and the various tests and measurements that were performed for both the plastic mix and the hardened concrete. Plastic batch measurements include slump, air content, unit weight and yield.

Two 6-in by 6-in by 21-in (152.4-mm by 152.4-mm by 533.4-mm) beams and six 6-in by 12-in (152.4-mm by 304.8-mm) cylinders were produced from each batch. Each batch required approximately 3 ft³ (0.085 m³) of concrete to form all of the necessary specimens.

As indicated in table 8, two of the beams were used for testing 7-day flexural strengths (ASTM C-78). Two cylinders were used for testing 7-day compressive strengths (ASTM C-39). Two cylinders were used to obtain both 28-day compressive strengths and static modulus of elasticity (ASTM C-469). The two remaining cylinders were earmarked to produce 7-day splitting tensile strengths (ASTM C-496).

The expected data base for the laboratory study can be inferred from tables 7 and 8. It can be seen that the data base contains information for 72 batches times 8 specimens per batch, or 576 data cells in all. Each line represents one batch and contains values for all factorial specifications, batch measurements, and strength tests that are associated with the batch.

IMPLEMENTATION OF LABORATORY STUDY

The initial experiment design described in the previous section was slightly modified when put into practice. This section details these differences and outlines the entire laboratory testing process from start to finish. Final material selections, factorial levels, mixing processes, testing procedures, reporting, and quality assurance measures are reviewed.

Material selections (i.e., concrete mix ingredients) for the laboratory study were made with specific goals in mind. First, it was desirable to select materials that would represent a broad spectrum of typical materials used in industry. This proved challenging given the limited available funds. A second goal, critical from the statistical point of view, was to secure a source for each material that was consistent throughout the project. Finally, it was necessary for all selected materials to meet the "standards" that are common to concrete mix design in current practice. For example, the gradation of the coarse aggregate was required to meet ASTM C-33 specifications. In a nutshell, the final material selections were made to fix the uncontrolled material characteristics according to currently accepted industry standards.

Most of the project's material selections were related to the aggregates to be used in the study. During preliminary design, it was decided that two types of coarse aggregates would be studied; a crushed limestone (soft) and a siliceous river gravel (hard). Also, for each of these aggregate types, two maximum aggregate sizes, 3/4-in (19.05-mm) and 1/2-in (12.7-mm), would

Table 8. Output specimens and measurements for each PCC batch of the laboratory study.

A. BATCH MEASUREMENTS

1. Slump (inches) (ASTM C-143)
2. Air Content (% by volume) (ASTM C-231)
3. Unit Weight (lbs/cuft) (ASTM C-138)
4. Yield (cuft PCC/sack cement) (ASTM C-138)

B. BEAMS (Two 6"x6"x21" beams per batch)

1. Flexural Strength at 7 days (ASTM C-78)
Beams B1 and B2

C. CYLINDERS (Six 6"x12" cylinders per batch)

1. Compressive Strength at 7 days (ASTM C-39)
Cylinders C1 and C2
2. Compressive Strength at 28 days (ASTM C-39)
Cylinders C3 and C4
3. Static Modulus of Elasticity (ASTM C-469)
Cylinders C3 and C4
4. Splitting Tensile Strength at 7 days (ASTM C-496)
Cylinders C5 and C6

be included. Several trips to local aggregate suppliers were made, and samples for the four coarse aggregate types were collected. The aggregate suppliers possessed fairly detailed data on each of their aggregates, which accelerated the selection process. Once candidates were identified, additional testing was conducted to verify the accuracy of the supplier's data. Once the aggregates were selected, quantities large enough for the entire project were set aside to ensure a consistent material throughout the experiment.

Similar searching and testing techniques were used to locate two fine aggregate sources with the desired wide range in fineness modulus for the factorial experiment. Table 9 presents a summary data table for the six aggregates used during the study.

Other project materials included cement, water, and an air entraining agent. A type I cement common to the area was selected for the project and enough cement for the entire project was set aside from a single lot to ensure consistency. Regular city-supplied tap water was used as the water source. No additional testing of the cement or water was performed. Finally, a commonly used air entraining agent was selected for the testing program. A certification that the air entraining agent met the requirements of ASTM C-260 was provided by the manufacturer.

After all of the materials were selected and acquired, several test batches were mixed to refine initial factorial levels for water quantity, cement quantity, coarse aggregate quantity, and the amounts of air entraining admixture to be added. Two levels for each of these factors were identified to supply a broad range for each controlled variable. The test batches were extremely beneficial for adjusting all of the mix levels so that the "driest mix" was not unworkable and that the "wettest mix" was not unrealistically fluid. The two factorial mixes corresponding to these theoretical boundary mixes were tested repeatedly until the adjusted mix levels produced acceptable plastic characteristics. This cyclic process also included adjustments for air entraining agent and the quantity of concrete necessary to prepare all of the desired specimens. Table 10 displays the final levels used for each of the experimental variables within their factorial framework. As mentioned in the previous section, the fine aggregate quantity was not controlled but allowed to fill the remaining portion of a fixed volume of concrete for each mix. The paragraphs below will discuss, briefly, the rationale used to make initial estimates for the controlled mix levels.

Given the material characteristics, several concrete mix designs were studied using the American Concrete Institute (ACI) design procedure.^(82.9) The intent of the study was not to simply follow existing design proportions and procedures since they are typically biased toward optimizing the concrete's strength. The design procedures merely provided a starting point for "typical" mix makeup as well as appropriate techniques for classifying mix characteristics (e.g., volume of coarse aggregate per unit volume of concrete). One requirement for this laboratory study was to stretch beyond the limits of typical mix designs and study the effects (i.e., performance considerations) of straying from common practice. For example, five- and six-sack mixes (i.e., five or six 94-lb (42.68-kg) cement sacks per cubic yard) represent typical upper and lower bounds for paving concrete. The

Table 9. Aggregate data used in laboratory study.

Coarse Aggregate Properties:

Property	Coarse Aggregate Type			
	Siliceous River Gravel	Siliceous River Gravel	Crushed Limestone	Crushed Limestone
Maximum Size (in)	1 1/2	3/4	1 1/2	3/4
Specific Gravity	2.615	2.621	2.521	2.554
Absorption(%)	1.29	1.53	3.94	3.42
Dry Rodded Weight (pcf)	99.9	99.7	90.9	90.2
Gradation (% Passing)				
2"	100	100	100	100
1 1/2"	100	100	100	100
1"	-	100	-	100
3/4"	54.9	89.5	60.3	100
1/2"	-	-	-	-
3/8"	18.2	25.8	12.0	44.8
No. 4	2.5	3.2	3.3	2.8
No. 8	-	0.2	-	1.4
No. 16	-	-	-	-

Fine Aggregate Properties:

Property	Fine Aggregate Type	
	Finish Sand	Concrete Sand
Specific Gravity	2.612	2.615
Absorption (%)	1.14	1.12
Dry Rodded Weight (pcf)	97.6	103.9
Fineness Modulus	2.10	2.84
Gradation (% Passing)		n/a
1/2"	100	
3/8"	100	
No. 4	100	
No. 8	99.8	
No. 16	97.8	
No. 30	74.3	
No. 50	17.1	
No. 100	0.8	
No. 200	0.1	

Conversion Factors: 1 in = 25.4 mm

1 pcf = 16.0 kg/m³

selected cement levels for the factorial are slightly beyond these normal limits. The ACI mix procedure also shows practical limits on water to cement ratios and coarse aggregate volume. The logic behind allowing the fine aggregate quantity to fill the remaining concrete volume also originated with the ACI mix design procedure. This demonstrates the ACI method's assumption that concrete strength is not highly correlated to the fine aggregate quantity. Using mix design as a backdrop, engineering judgement, common practice, and sample test batches provided for the selection of appropriate levels for each of the factors.

Batch mixing and specimen creation constituted a large portion of the effort expended on the project. It was vital to have extremely efficient flow of information between the laboratory and managing engineers and to minimize any errors that would require a batch to be re-mixed. Several measures were taken to achieve these goals.

Each concrete batch required extensive pre-calculation prior to mixing. One characteristic of mixing concrete is the need to adjust the amount of water to be added to the mix by the current moisture condition of the coarse and fine aggregates. Every morning that a batch was to be prepared, the moisture condition of all aggregates was measured. Specific gravities and absorption levels were measured on a weekly basis to catch any "through the pile" variation. The results of these tests were immediately telefaxed to the managing engineer for review and preparation of that day's mix designs. A computerized spreadsheet was prepared to minimize the occurrence of calculation errors. Hard copies of the mix design were telefaxed back to the laboratory for review by the lab. An example mix design spreadsheet is presented in table 11. This technique provided excellent job control, minimized errors, and provided a natural quality control chain. The laboratory technicians simply read the exact amount of each material needed directly off the design spreadsheet. Once the day's batches were recorded, the lab returned the completed standard reporting form shown in table 12. All of the designs and reporting forms were kept in a single notebook and updated as the 7- and 28-day test results became available.

Standard ASTM testing equipment and procedures were employed for all plastic concrete testing and for cylinder and beam specimen formation, handling and testing. To add consistency to the batching process, a single gasoline powered mixer was used for all of the batches and a single technician was in charge of all the mixing and testing. As mentioned previously, the mixes were randomly ordered to avoid confounding with any systematic changes in laboratory conditions. ASTM C-192 procedures for mixing order and time were strictly followed. Slump, air content, and unit weight were all measured in accordance with their respective ASTM specifications. Beam and cylinder specimens were carefully created and marked for future testing according to ASTM C-192. A vibrator was used to consolidate the freshly molded specimens. Finishing and curing were also performed according to specification.

The hardened concrete beams and cylinders were tested at 7 and 28 days; compressive strength, flexural strength, splitting tensile strength, and static modulus of elasticity were all measured. Table 8 summarizes the planned hardened concrete testing program and the respective ASTM specifications that were followed for each procedure. All equipment met the

Table 11. Mix design spreadsheet.

MIX DESIGN FOR FH-231 FACTORIAL CELLS

 Batch No. : 56
 Sequence No. : 1
 Description of Batch :
 A) Siliceous River Gravel
 B) 1.50" Maximum C.A. Size
 C) Fineness Modulus of F.A. is 2.10
 D) 6.5 ounces per cubic yard of Air Entraining Agent
 E) Coarse Aggregate of 1850 pounds
 F) Cement Quantity is 584 pounds
 G) Water Quantity is 270 pounds

Background Information Necessary to Complete the Mix Design:

Bulk Specific Gravity of F.A. = 2.61 using SSD condition
 Bulk Specific Gravity of C.A. = 2.62 using SSD condition
 Fineness Modulus of F.A. = 2.10
 Dry-rodded Unit Weight of C.A. = 99.90 LB per CF
 Absorption of F.A. = 1.00%
 Absorption of C.A. = 1.50%
 Specific Gravity of Cement = 3.15
 Specific Gravity of A.E. Agent = 1.04
 Amount of concrete required = 2.80 CF

 Total Moisture Content of F.A. = 1.50%
 Total Moisture Content of C.A. = 0.12%

Calculations for Mix Design:

Amount of Water per Cubic Yard of Concrete = 270 Pounds
 Amount of Cement per Cubic Yard of Concrete = 584 Pounds
 Calculated Water to Cement Ratio by Weight = 0.46
 Amount of AE Agent per Cubic Yard of Concrete = 6.50 oz
 Amount of Coarse Aggregate per Cubic Yard (SSD) = 1850.00 Pounds
 Total Estimated Percent Air Content = 6.0%

Volumetric Mix Quantities for One Cubic Yard:

Volume of Water = 4.33 cf 16.03%
 Solid Volume of Cement = 2.97 cf 11.00%
 SSD Volume of C.A. = 11.33 cf 41.96%
 Est. Volume of Air = 1.62 cf 6.00%
 Volume of AE Agent = 0.01 cf 0.03%
 SSD Volume of F.A. = 6.75 cf 24.99%
 Mix Totals -----> 27.00 cf 100.00%

Weight Mix Quantities for One Cubic Yard (not adjusted for moisture):

Weight of Water = 270.00 pounds 7.10%
 Weight of Cement = 584.00 pounds 15.36%
 Weight of C.A. (SSD) = 1850.00 pounds 48.64%
 Weight of Air = n/a pounds n/a
 Weight of AE Agent = 0.44 pounds 0.01%
 Weight of F.A. (SSD) = 1098.75 pounds 28.89%
 Mix Totals -----> 3803.18 pounds 100.00%

Unit Weight of Concrete = 140.86 PCF

Weights for Small Test Batch (adjusted for moisture):

Weight of Added Water = 30.08 pounds
 Weight of Cement = 60.56 pounds
 Weight of C.A. (wet) = 189.20 pounds
 Weight of Air = n/a pounds
 Weight of AE Agent = 0.05 pounds (or 0.67 ounces)
 Weight of F.A. (wet) = 114.51 pounds
 Mix Totals -----> 394.40 pounds

Table 12. Standard batch testing report form.

LAB NO. 17503

BATCH MEASUREMENTS

Batch No. 56
Sequence No. 1
Plastic Mix: (Date 9/21/88)
1. Slump = 2"
2. Unit Weight = 142.51 PCF
3. Yield = 26.7 CF
4. Air Content = 5%
5. Temperature = 87°F
6. Technician = EF

Hardened Specimens:

1. Flexural Strength @ 7 days for Beam 1 = 640 PSI
Flexural Strength @ 7 days for Beam 2 = 675 PSI
2. Compressive Strength @ 7 days for Cylinder 1 = 3740 PSI
Compressive Strength @ 7 days for Cylinder 2 = 3640 PSI
3. Splitting Tensile Strength @ 7 days for Cylinder 1 = 415 PSI
Splitting Tensile Strength @ 7 days for Cylinder 2 = 455 PSI
4. Compressive Strength @ 28 days for Cylinder 3 = 4820
Compressive Strength @ 28 days for Cylinder 4 = 4620
5. Static Modulus @ 28 days for Cylinder 3 = 5075489.5
Static Modulus @ 28 days for Cylinder 4 = 4613282.5

A GOOD-LOOKING, WORKABLE MIX.

specifications outlined by ASTM. As testing was completed, the standard reporting forms were updated and forwarded to the engineer in charge of testing. Any comments or problems encountered by the laboratory during testing were noted on the report form.

As the testing results were accumulated, the data were entered into an electronic spreadsheet data base. The spreadsheet was programmed to perform simple error checking and quality assurance on the data entered. The computerized data bases were very flexible and easy to manipulate and enhance. Other advantages to using a spreadsheet were: simple and readily available statistical analysis procedures, easy exportation of data to more advanced statistical programs, and the ability to create various summary reports sorted by various parameters.

The laboratory testing results for the entire project are summarized in appendix F. The complete data base including all duplicate specimens and replicate batches (data base C) is presented in this appendix. The data base excluding replicate mixes (data base A) is also presented. Each batch had a variety of post-calculations performed to allow for a more in-depth study and analysis, so appendix F also contains only those batches used for replicate analysis (data base B).

RATIONALE FOR ANALYSES OF THE LABORATORY STUDY DATA

This section presents rationale for the variables and procedures that are used in analyses of the laboratory study data that are contained in appendix F, data base C. Purposes and outputs for each procedure are discussed, but specific implementation and results of the procedures are reserved for a later section.

Classification of Data Base Variables

As an aid to the discussion of analytical procedures, it is useful to categorize the entire set of data base variables in six classes, class T through class Y, as shown in the first column of table 13.

Class T: Identification Variables

Four identification variables are used for the 72 mixes that were prepared in the lab study. First is the design sequence number (T1 = DES SEQ) for each of the 64 cells of the factorial design given in table 10. Since replicate mixes were made for eight of the factorial cells, a second identification variable (T2 = REP) is needed to distinguish between the two replicates.

No replicate code letter is used for the 56 unreplicated design sequence numbers. The eight replicated design sequence numbers contain mix codes A and mix B. Taken together, the design sequence number and the replicate mix code provide a unique identification for each of the 72 mixes.

The third identification variable is the randomized sequence (T3 = MIX SEQ) in which the mixes were prepared and ranges from 01 through 72. The final identification variable is the date (T4 = DATE) upon which each mix was prepared.

Table 13. Classification and definition of data base variables.

Class	Subclass or Definition	Variables Observed in the Laboratory Study
IDENTIFICATION VARIABLES (T)	Experimental Design (T1, T2)	DES SEQ = Factorial Cell Sequence (1-64) DES REP = Factorial Cell Replicate (A or B)
	Time of Mix (T3, T4)	MIX SEQ = Random Order of Mix (1-72) MIX DATE = Day/Month/Year of Mix
CONTROLLED DESIGN FACTORS (U)	Aggregate Properties (U1, U2, U3)	CAT = Coarse Aggregate Type ___ (0 = Crushed Stone, 1 = Siliceous Gravel) ___
	Mix Quantities (U4, U5, U6, U7)	CAM = Coarse Aggregate Maximum Size ___ (0.75", 1.50") ___
DESIGN FACTOR FUNCTIONS (V)	Variables Determined by Class U Factor Values (V1, V2, V3, ...)	FAM = Fine Aggregate Modulus (2.10, 2.84)
		AEQ = Quantity of Air Entrainment ___ Agent (oz/cy) (0, 6.5) ___
CO VARIABLES (W)	Uncontrolled but Measured (W1, ...)	CAQ = Coarse Aggregate Quantity (lb/cy) (1542, 1850)
		CEQ = Cement Quantity (lb/cy) (426, 584)
PLASTIC CONCRETE PROPERTIES (X)	Uncontrolled but Measured or Calculated from Mix Data (X1, X2, X3, X4)	WAQ = Water Quantity (lb/cy) (236, 270)
		FAQ = Fine Aggregate Quantity ___ (By subtraction of CAQ, CEQ, WAQ) ___
HARDENED CONCRETE PROPERTIES (Y)	Flexural Strength (Y1) (ASTM C78-84, psi)	WCR = Water Cement Ratio (WAQ/CEQ) ___ (0.40, 0.46, 0.55, 0.63) ___
	Compressive Strength (Y2, Y4) (ASTM C39-84, psi)	ACR = Aggregate Cement Ratio (CAQ + FAQ)/CEQ
	Split Tensile Strength (Y3) (ASTM C496-85, psi)	TEMP = Mix Temperature (deg. F)
	Modulus of Elasticity (Y5) (ASTM C469-83, psi)	SLUMP (ASTM C143-78, inches) ___
		UNIT WEIGHT (ASTM C138-81, lb/cf) ___
		YIELD (ASTM C138-81, cu ft, calculated) ___
		AIR CONTENT (ASTM C231-82, percent) ___
		fr7 = Flexural Strength at 7 days [fr71 and fr72] ave. & Diff.
		fpc7 = Compressive Strength at 7 days ___ [fpc71 and fpc72] ave. & diff. ___
		fpc28 = Compressive Strength at 28 days [fpc281 and fpc282] ave. & diff.
		ft7 = Split Tensile Strength at 7 days [ft71 and ft72] ft7 ave., ft 27 diff.
		Ec28 = Elastic Modulus at 28 days [Ec281 and Ec282] Ec 28 ave., Ec 28 diff.

Class U: Controlled Design Factors

The second class of data base variables is comprised of the seven two-level factors that were specified by the experimental design (table 10) and controlled during mix preparations. Three of these factors are aggregate properties: coarse aggregate type (U1 = CAT), coarse aggregate maximum size (U2 = CAM), and fine aggregate modulus (U3 = FAM). The remaining four factors are for mix quantities by weight of air entrainment agent (U4 = AEQ), coarse aggregate (U5 = CAQ), cement (U6 = CEQ), and water (U7 = WAQ). During the course of analysis, it was found useful to convert the mix quantities to related variables such as percent of mix volume.

Class V: Design Factor Functions

The third class of data base variables includes design factor functions that are determined by the design factor levels of any particular mix. The first of these variables is the quantity of fine aggregate (V1 = FAQ) that was added to the coarse aggregate, cement, and water quantities to produce a given total mix quantity. The remaining functions are water/cement ratio (V2 = WCR), and aggregate/cement ratio (V3 = ACR). The following definitions show how each V is related to two or more Us.

$$V1 = FAQ = 4320.6 - 1.041 CAQ + 0.032 CAT - 0.828 CEQ - 2.569 WAQ - 27.862 AEQ \quad (26)$$

$$V2 = WCR = WAQ/CEQ = U6/U5 \quad (27)$$

$$V3 = ACR = (CAQ + FAQ)/CEQ = (U4 + V1)/U5 \quad (28)$$

The foregoing equations show that the Vs are completely defined by the Us and may therefore have high correlations with the Us. The design factor functions are included as alternative and/or supplemental explanatory variables in analyses of the engineering properties of the PCC mixes and hardened concrete specimens.

Class W: Covariables

A covariable is defined to be an uncontrolled independent variable that is measured during the course of mix preparation or specimen testing. It may have a significant influence on the dependent variables of the study. The only true covariable in the present study is mix temperature at the time of mix (W1 = TEMP).

Class X: Plastic Concrete Properties

Properties of the plastic concrete (class X) at the time of mix include slump (X1 = SLMP), unit weight (X2 = UNWT), yield (X3 = YLD), and air content (X4 = AIR). Each of these variables is a dependent variable relative to variables in classes U, V, and W and is measured or calculated through ASTM procedures that are referenced in table 13.

Class Y: Hardened Concrete Properties

The main dependent variables in the laboratory study are the properties of the hardened concrete specimens (class Y) that were made from each mix. Four of these variables are strength properties: flexural strength at 7 days ($Y1 = fr7$), compressive strength at 7 days ($Y2 = fpc 7$) and 28 days ($Y4 = fpc28$), and split tensile strength at 7 days ($Y3 = ft7$). The fifth property is the PCC elastic modulus at 28 days ($Y5 = Es28$). ASTM procedures for measuring these five properties are also referenced in table 13. Each hardened concrete property is measured for two duplicate specimens from each mix. Thus, the data base contains Y values for each specimen, for the mean value of Y, for the two specimens (Y_{ave}), and for the Y difference between the two specimens (Y_{diff}).

Classification and Scope of Data Analyses

As shown in the middle column of table 14, five types of analyses will be performed. Variables that enter into each analysis type are shown in the first column; output tables and figures are shown in the third column. The details and actual results of these analyses for the laboratory data are presented in the next section entitled, "Analyses of the Laboratory Study Data."

Type 1 Analyses: Two-Variable Relationships

Initial analytical procedures are to examine all relationships that may exist between pairs of variables in appendix F, data base A. These procedures enable familiarization with all the experimental data and provide a means for identifying data that may be anomolous.

A standard statistical program is used to produce a table of means and standard deviations for all individual variables and a table of simple correlation coefficients between pairs of variables. The program identifies all correlations that are significant at either the 0.01 level or the 0.001 level.

Results are tabulated in a triangular matrix whose diagonal cells contain means and standard deviations for all variables, and whose off-diagonal cells contain the simple correlation coefficients for all pairs of variables.

Correlations of special interest include those between the covariable ($W1 = TEMP$) and other variables. If, for example, none of these correlations is significant, it is not likely that the covariable can be used to explain variations in the dependent variable.

Another set of useful correlations are those between specimen differences (Y_{diff}) and specimen means (Y_{ave}). If these correlations are significant, it is likely that the Y values should be transformed (e.g., to $\log Y$) in order to have more homogeneous error variation.

Next a two-way plot is made for each pair of data base variables in classes U through Y, exclusive of Y_{diff} . Each plot is accompanied by simple linear regression statistics that include the intercept and slope of

Table 14. Classification and scope of analyses for the lab study data.

VARIABLES		ANALYSIS	OUTPUTS
Dependent	Independent	Type and sequence	Tables and figures
X = Mix Properties Y = PCC Properties	U = Design Factors V = Functions of U _i W = Covar.	1. Two - Variable Relationships Among X, Y, U, V, & W	a. Means, Standard Deviations & Correlation Coefficients c. Two - Variable Graphs with simple regression outputs.
X diff & Y diff between Replicate Mixes & Duplicate Specimens Y diff between duplicate specifications		2. Analysis of Variance between Replicate Mixes & Duplicate Specimens	a. Between - Specimen Variance and Homogeneity b. Between - Replicate Variance and Homogeneity
X _i by U only X _i U with V Y _i U only Y _i U & V		3. Analysis of Variance & Covariance to Estimate Factor Effects & Residual Variances	a. ANOVAs and Significant Effects of Design Factors on Mix Properties b. ANOVAs and Significant Effects of Design Factors on PCC Properties
X _i on U & V Y _i on U & V Y _i on X Y _i on X, U, & V Y _i on Y _j		4. Regression Analyses for Prediction of Mix & PCC Properties, and for the Assessment of Prediction Reliability	a. Regression of Mix Properties on Significant Design Factors b. Regression of PCC Properties on Significant Design Factors c. Regression of PCC Properties on Mix Properties d. Regression of PCC Properties on Significant Design Factors & Mix Properties e. Interrelations Among PCC Properties
X _i f(U,V) + res Y _i f(U, V) + res Y _i f(X, U, V) + res		5. Sensitivity Analyses for Predictors of Mix & PCC Properties	a. Sensitivity of Mix Properties to Design Factors b. Sensitivity of PCC Properties to Design Factors c. Sensitivity of PCC Properties to Mix Properties & Design Factors

the least-squares line, standard errors for these coefficients, R-square, and the standard error of estimate (SEE).

Finally, all relevant two-variable equations or graphs among the secondary relationships are plotted on the corresponding two-way graphs of the data base variables. This step enables visualization of similarities and differences between the study relationships and those that have been derived elsewhere.

Type 2 Analyses: ANOVA for Duplicate Specimens and Replicate Mixes

The second type of analysis is to determine variances in X and Y values between the eight pairs of replicated mixes, and to determine variances in Y values between the duplicate specimens that were made from each of the 72 mixes. Both of these variances represent chance variation or experimental error. Since both replicates have the same nominal levels of all design factors (U) and design factor functions (V), differences (in X or Y) between replicates reflect chance variation that is not attributable to the design factors, but instead, represent the net effects of all extraneous variables that are at work when two independent mixes are made and when two sets of specimens are made and tested for a given mix. The replication variance is, therefore, the appropriate basis for all tests of the statistical significance of design factor effects on the X and Y variables.

Differences in Y values between two duplicate specimens from the same mix reflect the net effects of only those extraneous variables that are associated with specimen preparation and testing and do not reflect variations between separate mixes. The duplicate variance is, therefore, expected to be considerably smaller than the replicate variance and is not an appropriate basis for testing the statistical significance of factors that change from mix to mix.

The two duplicate values for each Y from a given mix serve three useful purposes. The first is that the mean value of Y (Y ave) for the two duplicates provides a more precise value for subsequent analysis than would be provided by a single specimen. Secondly, the duplicate differences (Y diff) can be tested for statistical homogeneity. If certain isolated differences are non-homogeneous relative to the remaining differences, objective rules can be used to decide that one or the other of the two specimens has an extreme Y value and should not be used in subsequent analyses. Finally, both the replicate differences and the duplicate differences can be tested for dependence upon the corresponding mean values.

If replicate differences in X or Y have a significant relationship with the corresponding replicate means, or if specimen differences (Y diff) are systematically related to their corresponding means (Y ave), it may be possible to eliminate or reduce the dependency by transforming the original values of X and/or Y, perhaps through the use of logarithms. The reason for such transformations is that subsequent analyses of variance and regression analyses are all based on the assumption of homogeneous chance variation at all levels of the dependent variable being analyzed.

The first step is to construct distributions of duplicate differences for each Y. Differences in the tail of any distribution can be tested for

"extreme values" relative to the remainder of the distributions. If, however, the correlation table shows significant correlation between Y diff and Y ave for any Y, then Y transformations should be made to determine whether the correlation can be thereby reduced. If so, the corresponding distribution of Y diff should be reconstructed, and extreme values retested.

The duplicate difference analysis thus identifies (1) transformations that may be needed for Y variables, and (2) extreme values (for individual Ys) that may be excluded from further analyses.

The analysis of replication differences begins with a table that is produced from data base B and that contains all replicate X and Y values. Replicate means and replicate differences are calculated, and replicate variances are calculated from the replication differences. As was stated earlier, these replication variances will be used in both analyses of variance and regression analyses.

Just as for the duplicate differences, homogeneity tests are made for replicate differences to detect needs for transformations and/or for exclusion of extreme values.

Type 3 Analyses: Analysis of Variance and Covariance

The third type of analysis consists of analyses of variance (ANOVA) and covariance (COVAR) to determine which design factors (U) and design factor functions (V) have statistically significant effects on the mix properties (X) and hardened PCC properties (Y). The ANOVA and COVAR thus serve to identify factors and factor functions that are candidate independent variables for subsequent regression analyses and to screen out variables that should not enter the regression analyses.

Independent variables for each ANOVA are the 7 design factors (U) and the 21 cross products of U_i with U_j , for a total of 28 independent variables. Three-factor and higher order interactions cannot be used since these interactions were confounded with one another by the choice of the half-factorial for the experimental design (table 10). Each ANOVA covers the 64 mixes in data base A and therefore has a total of 63 degrees of freedom (df). Since the independent variables account for 28 df, there remain $63 - 28 = 35$ df for residual variation that is not explained by the design factor effects.

Each ANOVA shows the total sum of squares (Total SS) for all 63 df; the SS for each factor and interaction effect (one df each) and the unpooled residual SS (Res SSu). The unpooled residual mean square (Res MSu) is given by $\text{Res SSu}/35$, and its square root is the unpooled root-mean-square residual (RMSu Res).

If the SS for any particular independent variable is divided by the total SS and multiplied by 100, the result is the percent (SS%) of total SS that is explained by the independent variable. The total SS% for all 28 independent variables is the maximum percent of the variation in X or Y that can be explained by the design factors and corresponds to the multiple R-square in regression analyses.

The ANOVA includes a ratio comparison (F ratio) of the SS for each independent variable with the Res MSu and shows the significance level (SL) that is associated with the given effect. If (SL) is sufficiently small, it is inferred that the effect is real and, therefore, is a valid explanatory variable for X or Y. A 10 percent significance level, $SL = 10$ is used throughout the analyses to separate significant effects from those which are not significant and, therefore, belong to the set of residual effects. If the SS for all non-significant factor effects are added to the Res SSu, the result is the pooled residual SS (Res SSp) and has pooled degrees of freedom (dfp) equal to 35 plus the number of non-significant effects that were pooled. Division of Res SSp by the dfp thus produces the pooled residual mean square (Res MSp) and corresponding pooled root-mean-square residual (Res RMSp).

All of the foregoing ANOVA results for each X and Y variable are displayed in the next section, "Analyses of the Laboratory Study Data." In addition, the last two summary lines for each analysis show whether or not the Res MS is significant relative to the corresponding replication mean square (Rep MS) that was previously derived.

The Rep MS represents variation that cannot be explained by any of the design factor variables and, if Res MSp is not significant relative to Rep MS, it is inferred that the ANOVA has identified all explanatory variables. Otherwise, it is inferred that the Res MSp may contain factor effects (three-factor and higher order interactions) that are significant. In such cases, it may be that one of the design factors (U_i) has neither a significant main effect nor significant interactions with any other design factor ($V_i V_j$). If so, the ANOVA can be rerun with the full factorial that is represented by the six remaining design factors and significant higher order interactions may thereby be identified.

After ANOVA has been performed for any X or Y, the next step is to introduce the design factor functions (V) and covariables (W) in a covariance analysis that also contains all U_i and $U_i U_j$.

Although the variables in class V are not true covariables, they are treated as such in the COVAR analyses. These analyses show the extent and manner in which significant U_i and $U_i U_j$ effects may be transferred to (or subsumed by) significant effects of the V variables. Since by equations 26, 27 and 28, the V_i have mathematical dependencies on the U variables, it is not expected that any COVAR will produce a greater amount of explained variation (or less residual variation) than was produced by the corresponding ANOVA. Rather, the COVAR analyses should show the ways in which V variables can be substituted for U variables as alternative inputs to subsequent regression analyses.

If the linear associations among U and V variables are quite high, the variance-covariance matrix for COVAR may be singular (or nearly so) and it will not be possible to perform the covariance analysis. In such cases, only a partial set of the V_i can be used. The usable V_i can be identified by successive runs with different subsets of the V_i .

The computer program for COVAR provides a number of options for determining the V_i effects. The first option is to calculate the V effects,

adjust the dependent variables for the V effects, then determine the U effects. A second option is to determine the U effects first, then calculate the V effects. Through the use of both options, it is possible to identify alternatives for input variables to the regression analyses. For each X and Y, ANOVA and COVAR results are tabulated and are used to identify the most promising sets of independent variables for the regression analyses.

The ANOVA and COVAR programs produce tables of mean values for the U_i and V_i effects including interaction effects of the U cross-products. From these tables, significant effects of U and V on the X_i and Y_i are identified.

Type 4 Analyses: Multiple Regression Analyses

The fourth analysis type consists of multiple regression analyses for the derivation of equations that predict mix properties (X) from various combinations of design factors (U and V), and that predict hardened PCC properties (Y) from various combinations of design factors and mix properties (U, V, and X). The resulting equations are the secondary relationships that were set forth in the objectives for the laboratory study.

Prior to each regression analysis it is necessary to decide what mathematical model to use for the relationship. Considerations for model selection include boundary conditions, transformations of variables, and mathematical form. For some models, it may be necessary to use non-linear regression analysis. It is assumed, however, that models can be selected that are linear in the coefficients (perhaps after transformations) and that linear regression analyses will suffice for the laboratory study data. The discussion which follows illustrates only the case where the untransformed dependent variable is a linear combination of the untransformed independent variables as they exist in the data base.

As shown in table 14, the first set of regressions are for the prediction of mix properties (X) from design factor variables (U and V) whose effects on X were found to be significant in the variance and covariance analyses. Stepwise multiple regression is used with an input probability criterion of 10 percent ($PIN = 0.10$). At each step, an additional independent variable (U_i , $U_i U_j$, or V_i) is introduced only if it will make a contribution to the prior-step regression that is significant at the 10 percent level. The procedure ends when there are no remaining candidate variables whose inclusions would make a significant contribution at the specified (10 percent) level.

If only the significant U_i and $U_i U_j$ were candidate variables, the derived regression equation should include all these variables. When the V_i are also included as candidates however, it is expected that some of the U_i and $U_i U_j$ that were significant in the ANOVA will be supplanted by one or more of the V_i , and that the final regression equations will contain only a subset of the candidate input variables.

The display of regression results for each X is given in the next section and lists the coefficient for each significant predictor of X_i (U_i ,

U_iU_j, and V_i) and for the constant term (i.e., the X intercept of the equation's graph). The significance level for each candidate input term is shown, but no coefficients are given for candidates that were not retained in the final equations.

Following the regression equations, the next set of regression results include the total sum of squares for X which has 63 degrees of freedom and the regression sum of squares (Reg SS) that is explained by the equations independent variables and that has df equal to the number of variables in the equations. The ratio of Reg SS to Total SS is the R-square for the regression, i.e., the fraction of total variation in X that is explained by the predictor variables. The difference between Total SS and Reg SS is the residual sum of squares (Res SS) whose df are 63 minus the df for Reg SS. The residual mean square (Res MS) is Res SS divided by its df, and the standard error of estimate (SEE) is the square root of Res MS.

As was done in the analyses of variance and covariance, an F ratio is calculated by dividing the Res MS by the replicate mean square (Rep MS). If this ratio is not significant (say at the 10 percent level), it is inferred that deviations of the observed X values from corresponding predicted values (i.e., regression residuals) are compatible with X differences between replicate mixes. If the F ratio is significant however, it can be said that there is a significant lack-of-fit. Lack-of-fit generally arises whenever one or more of the following is true (1) the input data contains "extreme values" for one or more dependent and/or independent variables, (2) significant predictors have been omitted from the input independent variables, and (3) the model for the regression analysis does not have the correct mathematical form for the regression relationship. If either case (1) or case (2) is true for the regression analysis, the same was true for the prior ANOVA and COVAR analyses, and would be evidenced by significant ratios of Res MS to Rep MS in those analyses.

To assist in the detection of extreme values, the regression analysis includes a display of all 64 residuals, both numerically and graphically. Using this display the regression results end with a tabulation of those cases whose residuals are greater than 2.8 times SEE. For 64 cases, only about one such residual is expected, and if any absolute residual exceeds 3 times SEE, it is almost certain that extreme values are present in the input data.

Examination of all input data for the extreme residuals may reveal previously unnoted errors or may provide no explanations. It must thus be decided whether or not to rerun all analyses (ANOVA, COVAR, and regression) with corrected data or in the absence of the extreme data. If lack-of-fit cannot be identified with extreme residuals, then an effort should be made to find transformations (for the input variables) that will reduce the Res MS to a level that is no longer significant relative to the Rep MS. The nature of needed transformations can be investigated by regressing residuals from the initial equation on U and V. If the residual regressions produce significant effects, then the initial forms of the U and V variables were not appropriate for explaining the effects of U and V upon X.

A final step in each regression analysis is to plot observed X values versus predicted X values (as shown in the next section). These graphs not

only identify extreme values but may reveal systematic departures from the expected linear trend with unit slope.

All of the foregoing discussion for regressions of X_i on U and V variables is equally applicable to the remaining regression analyses. The second set of regressions is for each Y_i on U and V variables. Results are given in the next section.

The third set of regressions are for Y_i versus only the X variables to determine the manner and extent to which hardened PCC properties can be predicted from only the observed properties of the plastic mix. The results of these are also given in the next section.

The next set of regressions are for the prediction of Y_i from the complete set of available independent variables (U, V, and X). For each of these regressions, the selection of candidate input variables is guided by the results of all previous analyses of Y_i , including ANOVA, COVAR, and regression analyses. The regression results include the major secondary relationships that were sought in the laboratory study.

The last set of regressions are for pairwise associations among selected Y_s . Of particular interest are regression equations for predicting flexural strength ($Y_1 = fr_7$) from compressive strength ($Y_2 = fpc_7$) and for predicting PCC modulus ($Y_5 = Ec_{28}$) from either flexural or compressive strength. In these and other $Y_i Y_j$ regressions, various transformations of the variables (e.g., $\log Y_1$ and $\log Y_2$) may be introduced to provide for boundary conditions and non-linearity that are not given by simple linear regressions for the untransformed variables. Results for the $Y_i Y_j$ regressions are presented in the next section.

Type 5 Analyses: Sensitivity Analyses

The term sensitivity refers generally to the amount of change that is produced in a dependent variable by specified changes in one or more independent variables. Sensitivity is an important criterion in the development of performance-related specifications since greater effects on performance will be produced by M&C variables to which performance predictors (e.g., PCC strength) are more sensitive than by M&C variables to which the predictors are relatively insensitive. It follows that strict quality control and acceptance criteria are much more important for M&C factors to which the performance predictors are highly sensitive.

It appears that standard definitions have not been developed for the quantification of sensitivity, and that different definitions are used by different researchers. For the present report, it will be assumed that the sensitivity of a dependent variable to independent variables is relative to one or more functional relationships for the prediction of a given dependent variable from specific independent variables. Moreover, for the laboratory study, the functional relationships of most concern are the regression equations that have been developed for the prediction of X_i from U and V, for predicting Y_i from U and V, and for predicting Y_i from U, V, and X.

Two types of sensitivity will be considered, local and global. Local sensitivity refers to the amount of change in Y that is brought about by

unit changes in the independent variables at a point that is defined by values for the independent variables. Local sensitivity is thus defined by the partial derivatives of Y with respect to the independent variables, when all are evaluated at the point of interest.

Local sensitivity has its application in the setting of tolerance limits for M&C specifications. In this case, the set of target levels for the specified variables gives the point at which the partial derivatives are evaluated and the evaluated derivatives show the relative degrees of control that are needed for the variables.

Global sensitivity refers to the change in Y that is brought about over the full experimental range of the independent variables. To narrow the definition of global sensitivity, it is customary to calculate the percentage change in Y that results from percentage changes in a given independent variable when all remaining independent variables are at their mean values. From these calculations the relative importance of the independent variables can be assessed.

Global sensitivity has its application in the selection of M&C variables for quality control and acceptance sampling. Variables associated with low sensitivity may be by-passed and control may be exercised on only those variables to which the dependent variable is highly sensitive.

Further discussion and details for sensitivity analyses will be given after regression equations have been developed from the laboratory data.

ANALYSES OF THE LABORATORY STUDY DATA

This section presents results from the five types of data analyses for which rationale was discussed in the previous section.

Pairwise Associations Between Study Variables

The classification of variables in table 13 shows 7 design factors (U1 = CAT through U7 = WAQ), 3 design factor functions (V1 = FAQ, V2 = WCR, and V3 = ACR), 1 covariable (W1 = TEMP), 4 mix properties (X1 = SLUMP through X4 = AIR), and 5 properties of the hardened PCC (Y1 = fr7AVE through Y5 = Ec28AVE) for a total of 20 independent variables.

Four of these variables represent mix quantities that can be expressed either by weight in pounds per cubic yard (i.e., U5 = CAQ, U6 = CEQ, U7 = WAQ, and V1 = FAQ), or as percents of total volume (i.e., U5 = CAP, U6 = CEP, U7 = WAP, and V1 = FAP). It is useful to know how both versions of these four variables are associated with any of the remaining independent variables. Thus, the total number of independent variables is 24 and the total number of pairwise associations among these variables is $(24 \times 23)/2 = 276$.

Two methods are used to infer the nature and degree of each of the pairwise associations. One is to calculate the simple linear correlation coefficient between the two variables in each pair; the second is to plot the two-way scatter diagram for each pair of variables. Since values for each of the 24 variables were observed (or calculated) for each of the 64

mixes in data base A (appendix F), each correlation coefficient is based on 64 pairs of values and each two-way graph contains 64 data points. The correlation coefficient quantifies the direction and degree of linear association between the two variables while the two-way graph indicates possible non-linearities, the degree of scatter, and which points (if any) appear to be extreme values relative to the remaining points.

Pairwise Correlations

Means and standard deviations for the 24 primary independent variables are shown in the diagonal cells of the first 24 rows of table 15. Cells below the diagonal entries contain the pairwise correlation coefficients for these variables. Coefficients marked with a single asterisk are statistically significant at the 0.01 level; those marked with two asterisks are significant at the 0.001 level. Except for the expected high correlations between mix quantities by weight and by volume, all correlations between pairs of design factors (U_i and U_j) are essentially zero because of the orthogonal experimental design.

Fine aggregate quantity ($V_1 = \text{FAQ or FAP}$) is correlated to a significant degree with the other mix quantities, especially with coarse aggregate quantity ($U_5 = \text{CAQ or CAP}$). Water cement ratio ($V_2 = \text{WCR}$) has a very high correlation with cement quantity ($U_6 = \text{CEQ or CEP}$) and a less high correlation with water quantity ($U_7 = \text{WAQ or WAP}$). The third design factor function, aggregate cement ratio ($V_3 = \text{ACR}$), has an extremely high correlation with cement quantity ($U_6 = \text{CEQ or CEP}$). As will be discussed later, if two independent variables have pairwise correlation greater than about 0.92 in absolute value, then only one should be candidate for any regression analysis. Thus, either CEQ/CEP or ACR, but not both, are candidate elements of multiple regression equations.

Table 15 shows that mix temperature ($W_1 = \text{TEMP}$) is not correlated significantly with any of the remaining independent variables. This fact is due to the randomization of mix designs and implies that no temperature adjustment need be introduced in any regression analysis.

The next four lines in table 15 are for mix properties and show that the highest correlations are between AEQ and SLMP or UNWT, between CEQ/CEP or WCR and UNWT or YLD, and between $X_3 = \text{YLD}$ and $X_2 = \text{UNWT}$.

Correlations between PCC properties (Y_1 through Y_5) and the independent variables are generally higher for volumetric quantities of coarse aggregate and cement (CAP and CEP) than for weight quantities (CAQ and CEQ). Although Y_1 through Y_5 are highly correlated with $V_3 = \text{ACR}$, still higher correlations exist for correlations of Y_1 through Y_5 with $U_6 = \text{CEQ or CEP}$.

Of considerable interest is the fact that all five hardened PCC properties (Y_1 - Y_5) have high correlations with one another as shown in table 15. The bottom lines show the extent of correlation mix sequence numbers ($T_1 = \text{MIX SEQ}$) and Y_1 through Y_5 differences between specimens with the remaining independent variables. It can be seen that the randomized mix sequence numbers are uncorrelated with all independent variables except for

Table 15. Means, standard deviations, and pairwise correlation coefficients for variables in data base A.

		U1	U2	U3	U4	U5		U6		U7															
		CAT	CAM	FAM	AEQ	CAQ	CAP	CEQ	CEP	WAQ	WAP	V1		V2	V3	W1	X1	X2	X3	X4	Y1	Y2	Y3	Y4	Y5
												FAQ	FAP	WCR	ACR	TEMP	SLMP	UNWT	YLD	AIR	fr 7AVE	fpc 7AVE	ft 7AVE	fpc28AVE	Ec28AVE
U1	CAT	0.50 / 0.50																							
U2	CAM	0.00	1.13 / 0.38																						
U3	FAM	0.00	-0.00	2.47 / .37																					
U4	AEQ	0.00	0.00	-0.00	3.25 / 3.28																				
U5	CAQ	0.03	-0.03	-0.03	-0.03	1700 / 155																			
	CAP	0.03	0.00	.05	-.02	.97**	38.6 / 3.99																		
U6	CEQ	0.00	0.00	-0.00	0.00	.03	.17	505 / 79.6																	
	CEP	.01	.01	.05	.01	.08	.24	.99**	9.29 / 1.60																
U7	WAQ	0.00	0.00	-0.00	0.00	-0.03	-0.03	0.00	0.00	253 / 17.1															
	WAP	0.00	.03	.11	0.00	.11	.18	.20	.23	.93**	14.64 / 1.06														
V1	FAQ	.11	-.01	.02	-.42**	-.79**	-.82**	-.36*	-.40**	-.20	-.39*	1421 / 198													
	FAP	.12	0.00	.09	-.45**	-.78**	-.77**	-.27	-.30*	-.21	-.32*	.98**	31.55 / 4.04												
V2	WCR	.02	.02	.02	.02	-.06	-.18	-.90**	-.90**	.42**	.21	.24	.17	0.51 / 0.09											
V3	ACR	.05	-.01	0.00	-.16	-.03	-.16	-.98**	-.98**	-.08	-.27	.45**	.37*	.85**	6.35 / 1.15										
W1	TEMP	.12	.01	-.04	-.11	-.03	-.04	-.14	-.14	.14	.10	.11	.10	.20	.16	78.12 / 5.56									
X1	SLMP	-.08	.07	.16	.53**	.21	.14	-.19	-.20	.47**	.37*	-.47**	-.54**	.39**	.05	-.03	2.03 / 2.44								
X2	UNWT	.19	.01	.23	-.61**	.27	.41**	.48**	.54**	-.19	.08	-.04	.10	-.53**	-.36*	.01	-.61**	140 / 5.08							
X3	YLD	.02	-.08	-.32	-.03	-.37*	-.57**	-.55**	-.65**	-.02	-.37*	.51**	.34*	.49**	.55**	.07	-.18	-.71**	27.73 / 0.73						
X4	AIR	.08	-.03	-.16	-.71**	-.19	-.31**	-.32*	-.38*	.04	-.18	-.05	-.17	.32*	.20	-.04	.55**	-.87**	.58**	5.59 / 2.83					
Y1	fr 7 AVE	-.11	-.12	.16	-.20	.21	.37*	.79**	.82**	-.14	.13	-.32*	-.20	-.78**	-.74**	-.20	-.39**	.72**	.73**	-.61**	535 / 125				
Y2	fpc 7 AVE	-.18	-.06	.13	-.23	.22	.36*	.81**	.84**	-.23	.03	-.31*	-.19	-.84**	-.76**	-.21	-.41**	.72**	-.68**	-.57**	.91**	3565 / 1134			
Y3	ft 7 AVE	-.15	.02	.15	-.19	.22	.37*	.71**	.75**	-.20	.07	-.31*	-.19	-.74**	-.67**	-.24	-.40**	.70**	-.71**	-.58**	.84**	.90**	381 / 90.7		
Y4	fpc 28 AVE	-.21	-.11	.14	-.23	.23	.37*	.79**	.82**	-.18	.08	-.33*	-.21	-.74**	-.74**	-.21	-.38*	.70**	-.69**	-.58**	.89**	.98**	.88**	4377 / 1255	
Y5	Ec 28 AVE	-.01	-.02	.14	-.37**	.28	.42**	.60**	.65**	-.19	.08	-.21	-.08	-.62**	-.52**	-.10	-.49**	.84**	-.73**	-.71**	.84**	.83**	.77**	.84**	4.33 / 0.71
T1	MIX SEQ	-.03	-.03	.15	.02	.19	.19	.07	.07	.06	.09	-.21	-.21	-.05	-.08	-.75**	.14	.01	-.08	0.00	.14	.12	.09	.15	.11
Y1D	fr 7 DIFF	-.01	-.09	-.19	-.10	-.16	-.15	-.01	-.02	-.07	-.08	.20	.21	-.03	.03	.27	-.23	.06	.03	.08	-.09	-.06	-.09	-.05	0.00
Y2D	fpc 7 DIFF	-.15	-.11	.03	-.25	.13	.17	.24	.25	-.11	-.02	-.07	-.02	-.26	-.20	-.01	-.29	.34*	-.23	-.32**	.34*	.45**	.39**	.45**	.38*
Y3D	ft 7 DIFF	.05	.02	.04	.20	.13	.14	.12	.12	-.03	.01	-.21	-.20	-.14	-.15	-.34*	.10	-.02	-.12	.13	.15	.12	.08	.09	.14
Y4D	fpc 28 DIFF	.03	.12	.06	-.24	.12	.17	.31*	.32*	-.02	.09	-.09	-.04	-.31*	-.27	-.18	-.25	.38**	-.27	-.24	.33*	.38**	.36*	.34*	.37*
Y5D	Ec 28 DIF	.21	-.10	-.03	-.14	.09	.10	.06	.07	.09	.12	-.02	0.00	-.02	-.03	.24	-.06	.18	-.10	-.15	.09	0.00	-.02	0.00	.26

- Diagonal Cells Contain Mean/Standard Deviation for Variable
- Below Diagonal Cells Are Pairwise Linear Correlation Coefficients
- All Values are for N = 64 Mixes

* 1-Tailed Significant: -0.01
 ** 1-Tailed Significant -0.001

mix temperature ($W_1 = \text{TEMP}$). This fact shows that the randomization was effective, and that temperature might have had a systematic influence on the dependent variables had the mix order not been randomized.

Correlations of the differences between specimens (Y_1 diff through Y_5 diff) with independent variables are generally of low magnitude. Correlations of Y_i diff with Y_i are not large enough to suggest that the Y variables need be transformed to induce greater homogeneity of variance.

Two-Way Graphs For the Study Variables

The rows of table 15 represent 30 different variables of which 9 are design factors (U_i), 4 are design factor functions (V_i), 1 is a covariable (W_1), 4 are mix properties (X_i), 5 are PCC properties (Y_i AVE), 1 is mix sequence (T_1), and 5 are specimen differences (Y_i diff). The body of table 15 contains 480 correlation coefficients for pairwise linear associations of the 30 variables. Although a two-way graph (scatter diagram) could be plotted for each coefficient, only certain of these graphs have direct bearing on the analyses to be presented in later sections. As was explained in the preceding section, the correlation coefficients for $T_1 = \text{MIX SEQ}$, for the five Y_i diff, and for W_1 suffice to decide that none of these variables needs to be considered further in the data analysis. Correlations between pairs of design factors (U_i and U_j) are virtually zero, so graphs for these pairs are not useful for the analysis. Finally, essentially the same association exists between either the by-weight or by-volume versions of the a mix variables and any of the remaining variables. Since graphs need be plotted for only one of the two versions, the by-weight versions (CAQ, CEQ, WAQ, and FAQ) will be used.

With the foregoing exclusions, the correlations of interest involve 19 variables and 150 two-way graphs as shown in the following abstract from the middle portion of table 15.

		7Uj	3Vj		
Vi	FAQ	21 ViUj	3 ViVj		
(3)	WCR	graphs	graphs		
	ACR			4 Xj	
Xi	SLMP				
(4)	UNWT	28 XiUj	12 XiVj	6 XiXj	
	YLD	graphs	graphs	graphs	
	AIR				5Yj
Yi	fr7AVE	35 YiUj	15 YiVj	20 YiXj	10 YiYj
(5)	fpc7AVE	graphs	graphs	graphs	graphs
	ft7AVE				
	fpc28AVE				
	Ec28AVE				

The 150 two-way graphs listed above were prepared and examined in the following order:

Set YiYj: (5 x 4 / 2 = 10 graphs)
Set YiUj + YiVj + YiXj: (5 x 14 = 70 graphs)
Set XiXj: (4 x 3 / 2 = 6 graphs)
Set XiUj + XiVj: (4 x 10 = 40 graphs)
Set ViVj: (3 x 2 / 2 = 3 graphs)
Set ViUj: (3 x 7 = 21 graphs)

The computer routine for plotting each two-way graph also produces regression statistics for the least squares line through the data points. These statistics were also examined as part of the analysis.

At this stage of analysis, there were three inferences to be drawn from each graph, namely:

1. The degree of association and closeness of fit, as given by R-squared and the standard error of estimate (SEE).
2. Indications of non-linearity, as judged by the presence of systematic (non-random) scatter of data points from the regression line.
3. Indications of outliers (extreme values) as evidenced by points that deviate by (say) more than 2.5 x SEE from the line.

In the event of case (2), the term "non-linear" was printed on the graph. For case (3) the outlier points were circled and identified by their mix sequence numbers.

Inspection of all 150 two-way graphs showed that only four mixes produced outliers for any of the graphs, namely, mix numbers 34, 37, 49, and 64. All four of these mixes were made from the river gravel aggregate ((V1 = CAT = 1) and all contained the lower level of water quantity (V7 = WAQ = 236 lb (107 kg)).

Mix 49 produced outliers for virtually all graphs and has unusually low values for all Yi, particularly for Y5 = Ec28AVE at 1.80 million psi (127,000 kg/cm²). In spite of the fact that this mix received no air entrainment (U4 = AEQ = 0), its air content (W4 = AIR) was 10 percent and its yield (X3 = YLD = 30.5) was exceptionally high.

Mix 64 appears to be extreme only with respect to the tensile strength of both cylinders which produced Y3 = ft7AVE = 665 psi (46.8 kg/cm²). All other Y AVE values for this mix appear to be consistent with those from the remaining mixes.

Mix 34 produced outliers on most graphs involving flexural strength since its Y1 = fr7AVE was low [544 psi (38.2 kg/cm²)] relative to its other Y values [e.g., Y2 = fpc7AVE = 5660 psi (398 kg/cm²) and Y5 = Ec28AVE = 5.45 million psi (383,000 kg/cm²)]. Another aberration for mix 34 was its high air content (X4 = AIR = 8.2 percent) in view of there being no air entrainment (U4 = AEQ = 0) for this mix.

Finally, mix 37 produced an extreme point on a few two-way graphs, but with no particular pattern such as evidenced by mixes 49, 64, and 34.

The four graphs (figures 7, 8, 9, and 10) in the pages that follow have been selected to illustrate the nature of outliers produced by mixes 34, 37, 49, and 64. Two additional graphs (figures 11 and 12) are also included to show that some of the pairwise associations may not be linear. Figure 11, for example, shows that the relationship between compressive strength ($Y2 = \text{fpc7AVE}$) and slump ($X1 = \text{SLMP}$) is probably non-linear since more points lie above the regression line than below for low slump, and more points lie below than above for high slump values.

Figure 12 (and other two-way graphs for V3) shows that $Y1$ increases with $V3 = \text{ACR}$ when $U5 = \text{CEQ}$ is fixed at either 584 lb or 426 lb (265 kg or 193 kg), but that $Y1$ decreases as the aggregate/cement ratio moves from values around 5 to values around 7. This phenomenon arises because both strength and ACR decrease with decreasing cement content when other variables are fixed. Since ACR is highly correlated with CEQ (see table 15), the regression line in figure 12 does not show the true relationship between $Y1$ and $V3$. For this reason, $V3 = \text{ACR}$ was not used in regression analyses that also involve $U5 = \text{CEQ}$.

Figure 12 also serves as a reminder that the points in any two-way graph of the study data differ from one another on all remaining variables. Thus, any regression line for a two-way graph will ordinarily be an average regression for the separate regressions that represent different levels of the remaining variables.

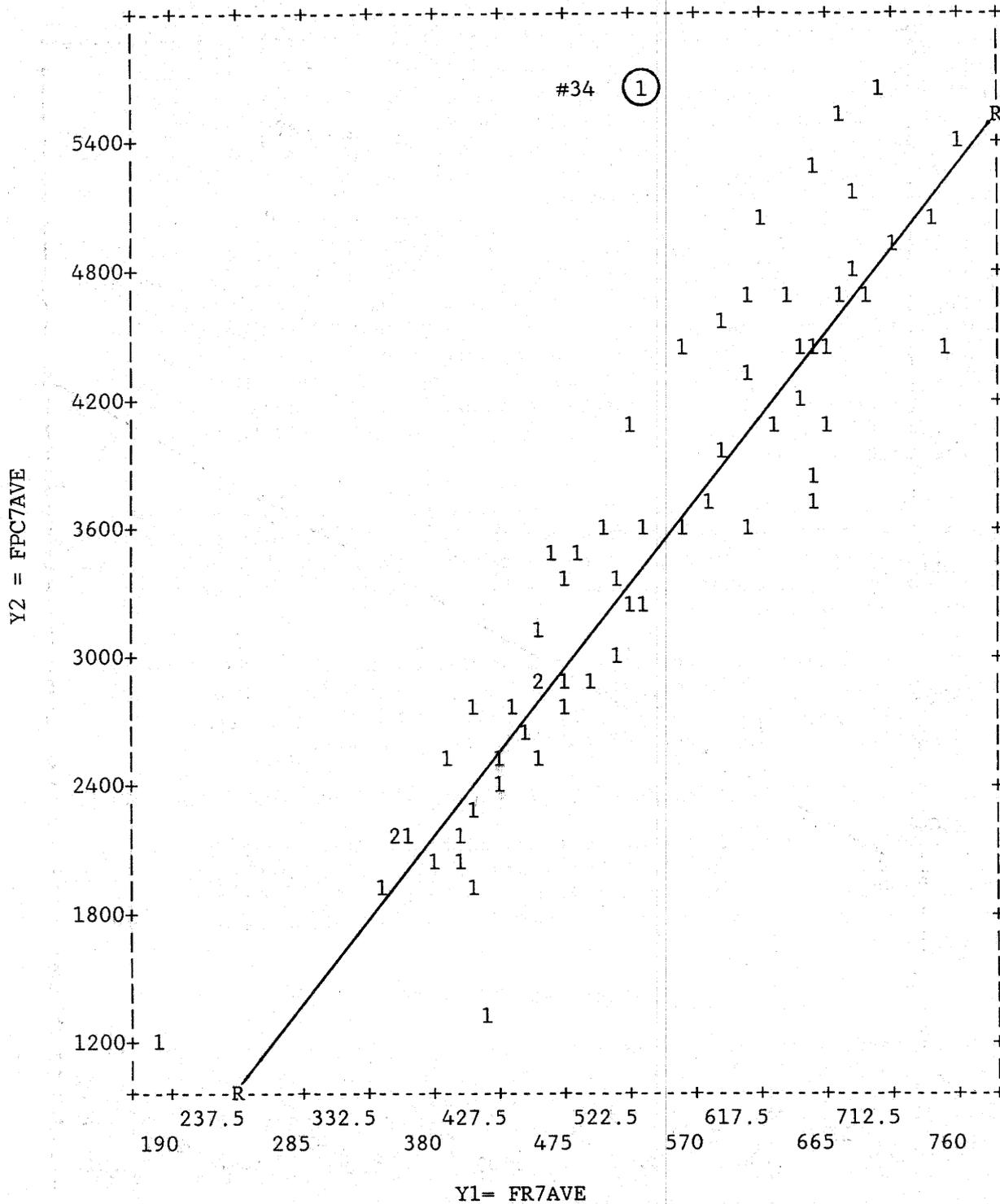
Another use for the two-way graphs is for comparison with corresponding pairwise associations that have been developed in previous studies. Most of the latter are identified in appendix C, but comparisons with results of this study are also made in later sections of this report.

Analyses of Variance Between Duplicate Specimens and Replicate Batches

Two levels of chance variation (experimental error) were observed in the laboratory study. The first level was quantified by differences in PCC properties between the specimens that were made from the same mix. The second level of chance variation was between corresponding data for two replicate mixes.

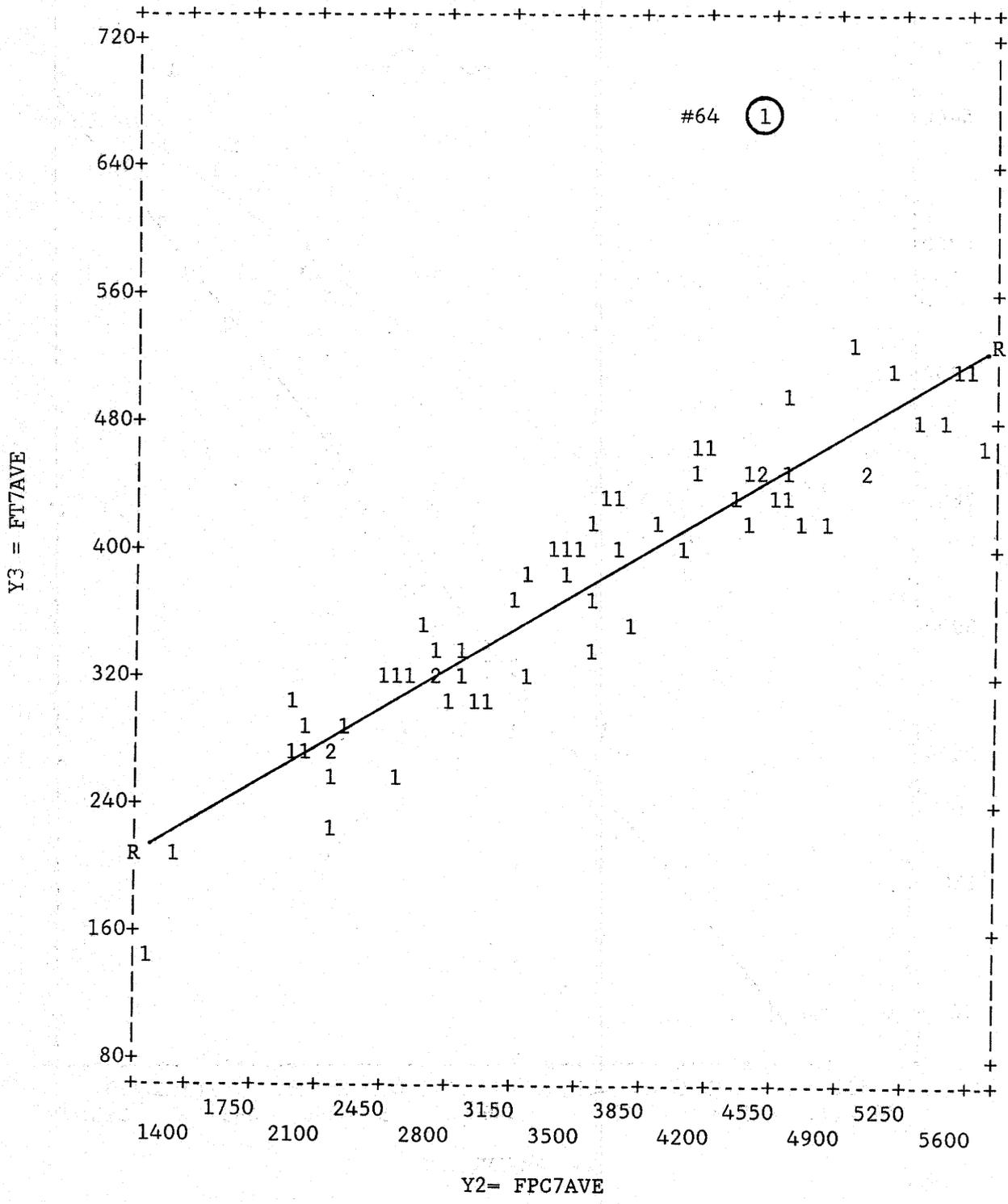
Variance Between Duplicate Specimens

Specimen differences are denoted by $Y1 \text{ diff} = \text{fr7diff}$, etc. for the 72 mixes in data base C. The frequency distributions for the absolute values of $Y1 \text{ diff}$ through $Y5 \text{ diff}$ are shown at the top of table 16. Means and standard deviations are shown for each of the five distributions. If the differences in any distribution all reflect the same set of chance causes, it can be expected that none of the 72 differences will deviate more than (say) 3 standard deviations from the distribution mean. Table 16 shows, however, that one difference in the $Y5 \text{ diff}$ distribution lies nearly 13 standard deviations from the mean, that one difference in both the $Y1 \text{ diff}$ distribution (mix 37) and the $Y4 \text{ diff}$ distribution (mix 16B) lies more than 8 standard deviations from its mean, and that one difference in the $Y2 \text{ diff}$ distribution (mix 38B) lies more than 7 standard deviations from its mean. It was inferred that these four differences are not homogeneous with their

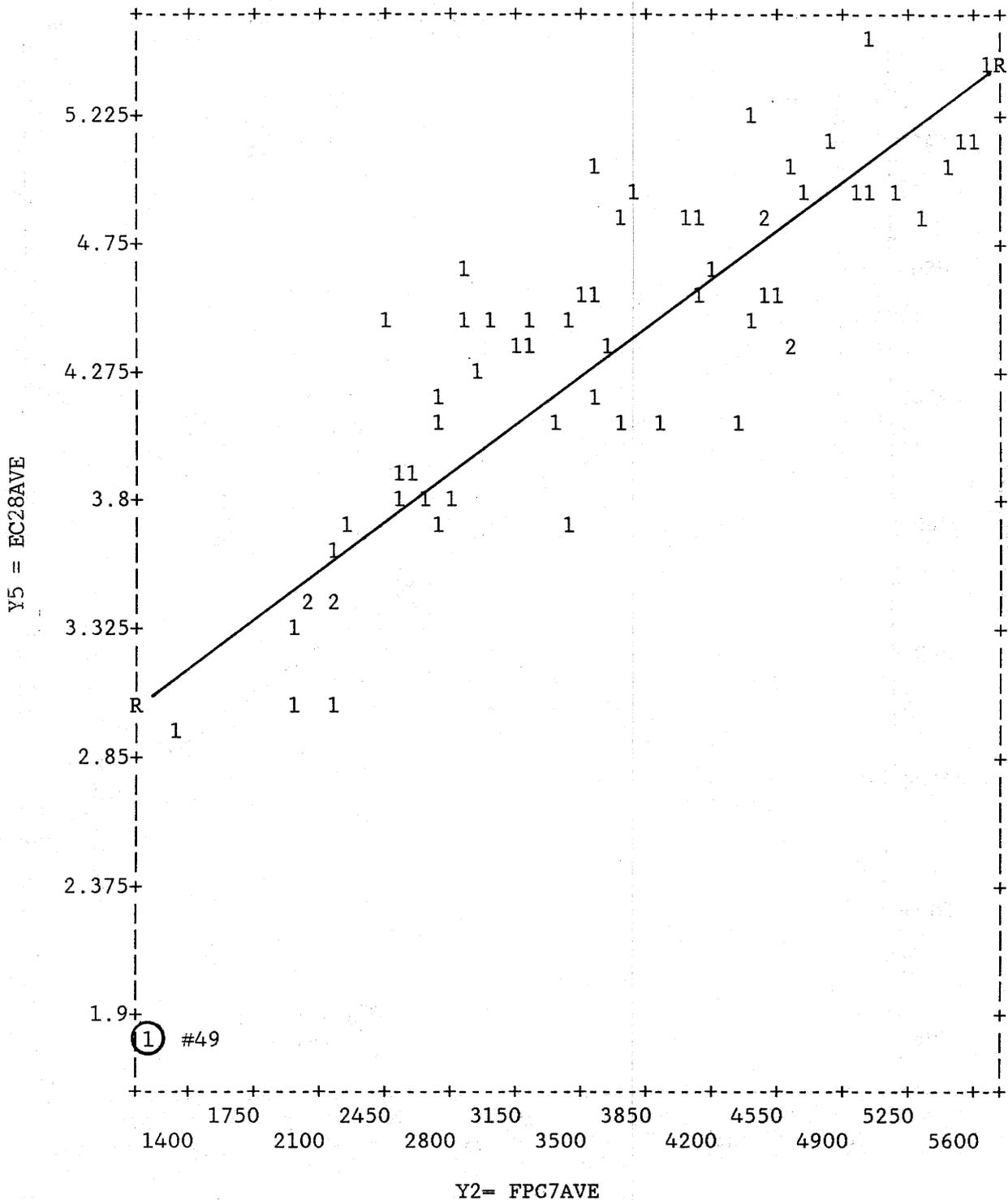


64 cases plotted. Regression statistics of FPC7AVE on FR7AVE:
 Correlation .89730 R Squared .80515 S.E. of Est 504.79354 Sig. .0000
 Intercept(S.E.) -852.67708(283.12506) Slope(S.E.) 8.23353(.51439)

Figure 7. Two-way plot of FPC7AVE (Y2) versus FR7AVE (Y1).

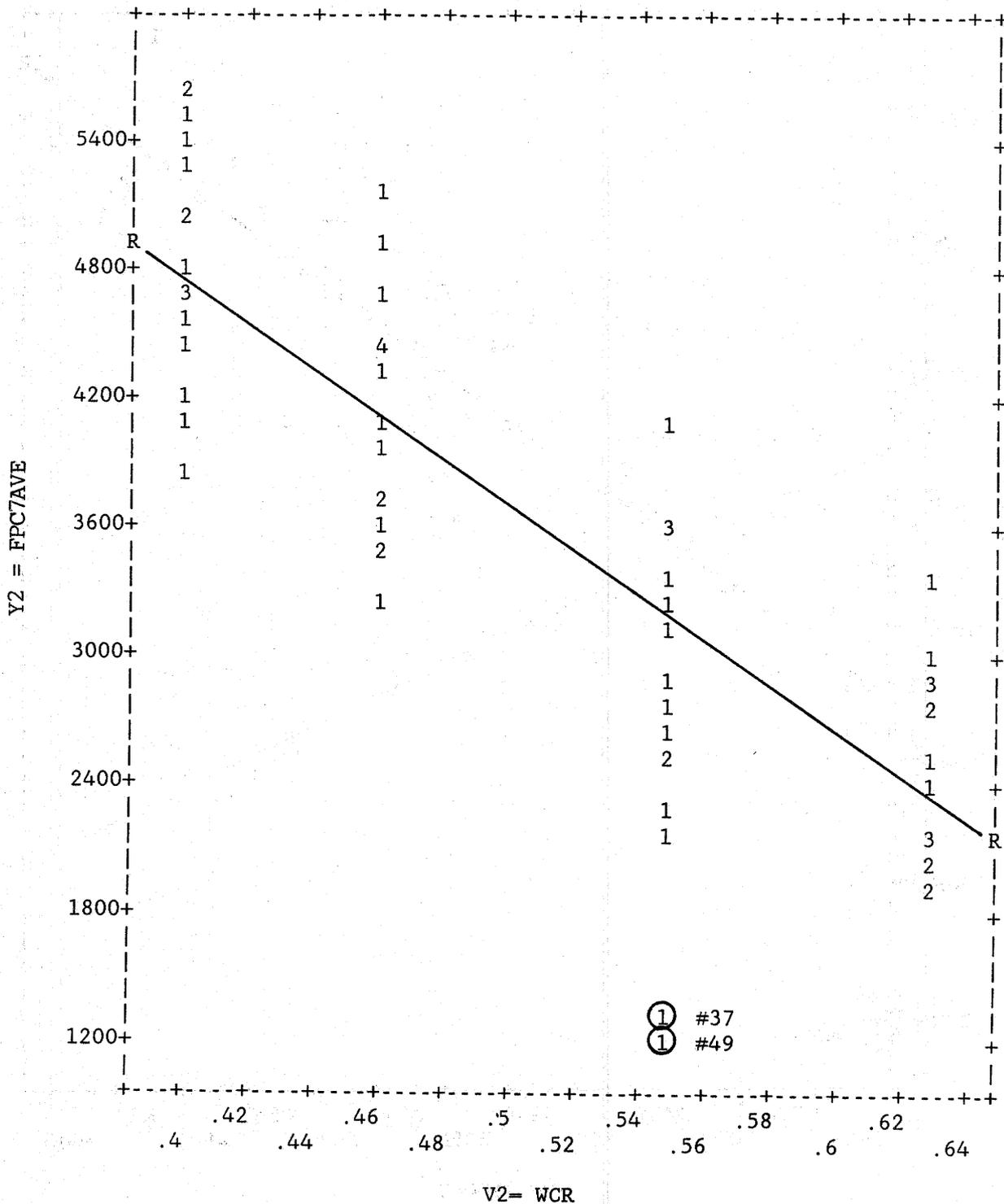


64 cases plotted. Regression statistics of FT7AVE on FPC7AVE:
 Correlation .89492 R Squared .80088 S.E. of Est 40.78979 Sig. .0000
 Intercept(S.E.) 125.99217(16.93535) Slope(S.E.) .07153(.00453)
 Figure 8. Two-way plot of FT7AVE (Y3) versus FPC7AVE (Y2).



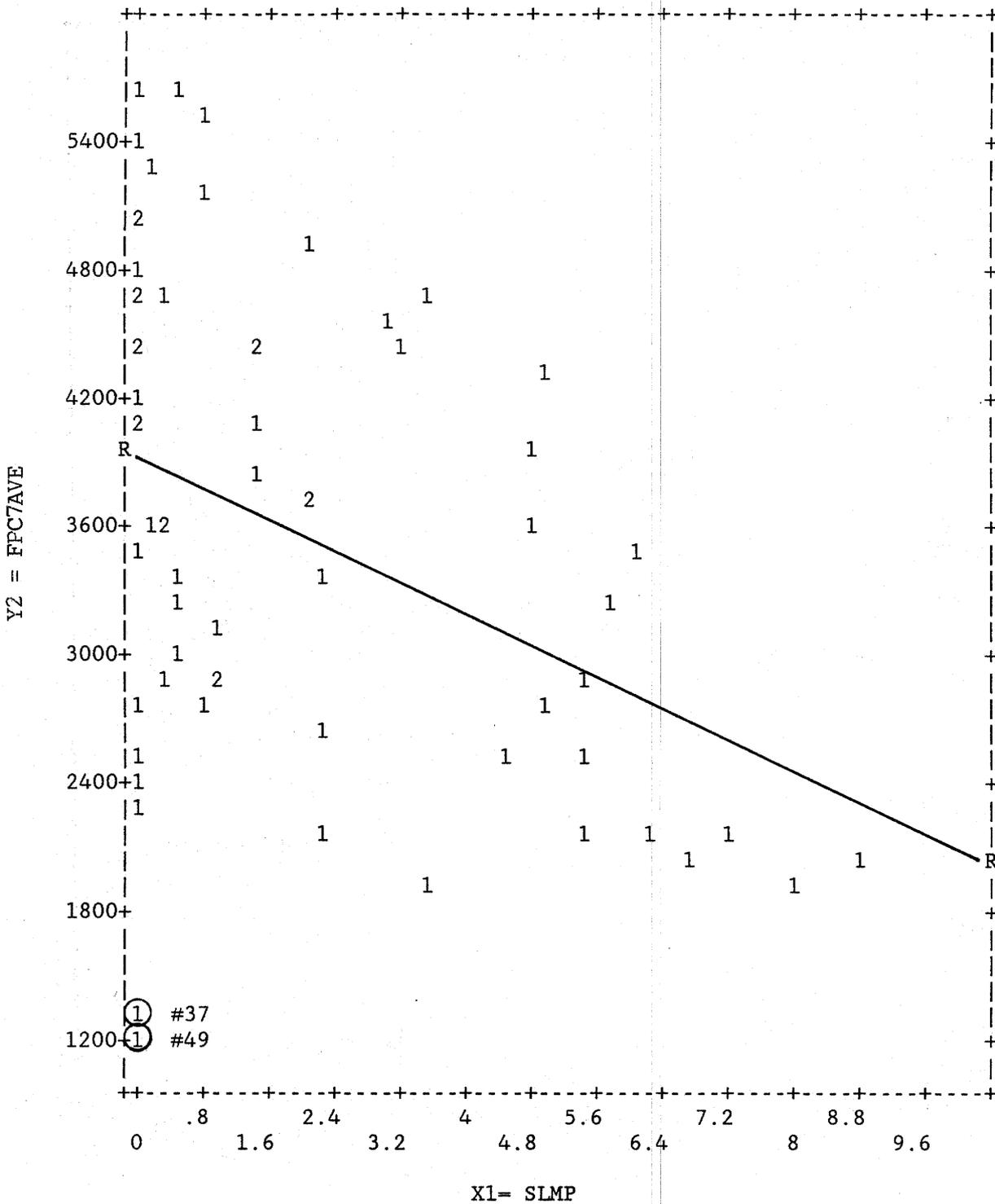
64 cases plotted. Regression statistics of EC28AVE on FPC7AVE:
 Correlation .85173 R Squared .72544 S.E. of Est .36256 Sig. .0000
 Intercept(S.E.) 2.47351(.15053) Slope(S.E.) .00052(.00004)

Figure 9. Two-way plot of EC28AVE (Y5) versus FPC7AVE (Y2).



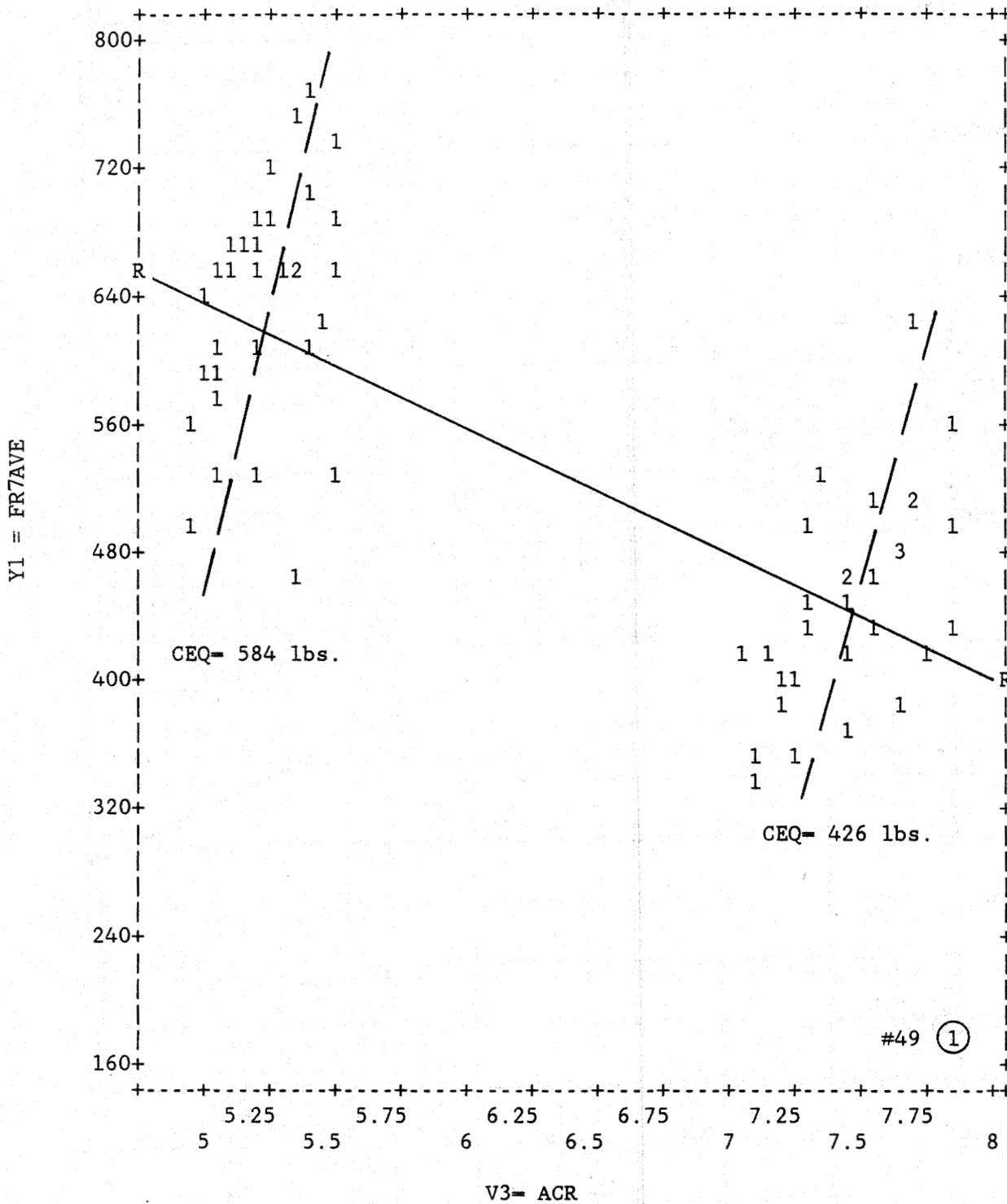
64 cases plotted. Regression statistics of FPC7AVE on WCR:
 Correlation -.83150 R Squared .69139 S.E. of Est 635.29362 Sig. .0000
 Intercept(S.E.) 9022.39583(469.80597) Slope(S.E.) -10700.531(907.93298)

Figure 10. Two-way plot of FPC7AVE (Y2) versus WCR (V2).



64 cases plotted. Regression statistics of FPC7AVE on SLMP:
 Correlation -.41168 R Squared .16948 S.E. of Est 1042.18407 Sig. .0007
 Intercept(S.E.) 3952.69302(169.83385) Slope(S.E.) -191.15562(53.74151)

Figure 11. Two-way plot of FPC7AVE (Y2) versus SLMP (X1).



64 cases plotted. Regression statistics of FR7AVE on ACR:
 Correlation $-.73996$ R Squared $.54754$ S.E. of Est 83.83215 Sig. $.0000$
 Intercept(S.E.) $1040.71030(59.13814)$ Slope(S.E.) $-79.34650(9.16031)$

Figure 12. Two-way plot of FR7AVE (Y1) versus ACR (V3).

Table 16. Variations in PCC properties between duplicate specimens.

Duplicate Diff. (psi)	Y1	Y3
	f _r 7 freq.	f _t 7 freq.
0 - 20	29	37
21 - 40	24	20
41 - 60	11	8
61 - 80	4	5
81 - 100	2	1
101 - 120	1	0
121 - 140	0	0
141 - 160	0	0
Subtotal	71	71
Mean Diff.	30.6 psi	28.4 psi
Std. Dev. Diff.	22.6 psi	20.3 psi
Extreme Diff.	220 psi	200 psi
Dist. from Mean	8.38 S.D.	8.45 S.D.
Des. Seq. No.	37	16B

Dup. Diff. in/in x 10 ⁻⁶	Y5
	E _s 28 freq.
.00 - .10	30
.11 - .20	20
.21 - .30	9
.31 - .40	6
.41 - .50	2
.51 - .60	1
.61 - .70	2
.71 - .80	0
.81 - .90	1
.91 - 1.00	0
1.01 - 1.10	0
Subtotal	71
Mean Diff.	0.189 psi
Std. Dev. Diff.	0.178 psi
Extreme Diff.	2.45 psi
Dist. from Mean	12.70 S.D.
Des. Seq. No.	36

Duplicate Diff. (psi)	Y2	Y4
	f _{pc} 7 freq.	f _{pc} 28 freq.
0 - 40	221	17
41 - 80	9	17
81 - 120	16	11
121 - 160	10	4
161 - 200	5	9
201 - 240	2	4
241 - 280	3	3
281 - 320	2	2
321 - 360	1	3
361 - 400	1	0
401 - 440	0	1
441 - 480	0	0
Subtotal	70	71
Mean Diff.	113.8 psi	119.9 psi
Std. Dev. Diff.	88.0 psi	99.5 psi
Extreme Diff.	540 & 740	610 psi
Dist. from Mean	4.84 & 7.11	4.93 S.D.
Des. Seq. No.	28 & 38B	60

Data adjustments based on duplicate difference extremes. Values in parentheses are replacements for observed values in data bases A and B.

DES. SEQ.	Data	Variable	Dup. 1	Dup. 2	Dup. Diff.	Dup. Mean	Comments
16A	Obs. Adj.	Y3 = f _t 7	515 "	510 "	5 "	513 "	No Adjustment. Used to Compare With 16B
16B	Obs. Adj.	Y3 = f _t 7	500 "	300* (459)	200 (41)	400 (480)	*Dup. 2 is Low Extreme. Replace with Dup. 1 - 2 SD Diff.
28	Obs. Adj.	Y2 = f _{pc} 7	5330 "	5870 "	540* "	5600 "	Dup. Diff is Extreme. Can't Tell Which Way. Leave Unadj.
36	Obs. Adj.	Y5 = E _c 7 (x 10 ⁻⁶)	7.10* (5.06)	4.65 "	2.45 (0.41)	5.88 (4.83)	Dup. 1 is High Extreme. Replace With Dup. 2 + 2 SD Diff.
37	Obs. Adj.	Y1 = f _r 7	220* (395)	440 "	220 (45)	330 (418)	Dup. 1 is Low Extreme. Replace With Dup. 2 - 2 SD Diff.
38A	Obs. Adj.	Y2 = f _{pc} 7	3720 "	3640 "	80 "	3680 "	No Adjustment. Used to Compare With 38B
38B	Obs. Adj.	Y2 = f _{pc} 7	3450 "	2710* (3274)	740 (176)	3080 (3362)	*Dup. 2 is Low Extreme. Replace With Dup. 1 - 2 SD Diff.
60	Obs. Adj.	Y4 = f _{pc} 28	5340 "	4730 "	610* "	5035 "	*Dup. Diff. is Extreme. Can't Tell Which Way. No Adj.

comparison differences and that the Y value of one or the other of the two specimens in each case is an outlier relative to the set of chance causes that produce specimen differences.

As shown in the bottom part of table 16, it was decided that the outlier specimen was unusually low for Y1 in mix 37, for Y2 in mix 38B, and for Y3 in mix 16B. The outlier for Y5 in mix 36 was extremely high [$E_{c28} = 7.10$ million psi ($499,000$ kg/cm²)] relative to all remaining E_{c28} values. As shown in the comments column at the bottom of table 16, values for the four outliers were each adjusted (arbitrarily) by two standard deviations in the direction of the value of its companion specimen.

The distribution of Y2 diff also includes one difference (mix 28) that is more than four standard deviations from the distribution mean, as does the Y4 diff distribution (mix 60). As shown at the bottom of table 16, however, it was concluded that there is not sufficient evidence for adjustment of specimen values associated with these somewhat less extreme differences.

It is noted that the adjustments described in table 16 were incorporated in the analysis data base (data base A) but not in the complete data base (data base C). Moreover, all two-way graphs described previously were plotted from data base A after the specimen values were adjusted for outliers.

Variance Between Replicate Batches

The experimental design (table 10) identifies eight factorial combinations of the U_i for which two independent mixes (replicate A and replicate B) would be prepared. Data for the eight pairs of replicate mixes are given in appendix F. For each pair, the data for replicate A also appears in the analysis data base. Data for replicate B are used strictly for comparisons with the A replicates. As can be seen in the mix sequence and data columns, all 16 mixes were prepared in a random order relative to each other and to the remaining mixes.

Data from data base B are reproduced in table 17, including adjustments that were shown at the bottom of table 16 for Y3 in mix 16B and for Y2 in mix 38B.

For each pair of mixes, table 17 shows differences between the two replicate X values, and differences between the two duplicate means for the five Y variables. The mean square of the eight mean differences for each variable is called Rep MS in the bottom portion of table 17, and its square root (Rep RMS) is the standard deviation for variability between replicates. Since those mean squares (or standard deviations) represent all chance variation in X and Y values that is not attributable to the controlled factors (U and V), they are measures of the variation that cannot be explained by the controlled factors of the study.

As will be explained later, the standard deviation of residuals from a regression analysis for an X_i or Y_i will be divided by the corresponding Rep RMS for the X_i or Y_i . If this ratio is relatively small, the regression equation was assumed to be a good fit for the data points. If the ratio is

Table 17. Variations in PCC properties between replicate mixes.

Replicate Mixes		Plastic Properties (x)				Hardened PCC Properties (V)									
DES SEQ	REP	X1 SLMP (in)	X2 UNWT (pcf)	X3 YLD (cy)	X4 AIR (%)	Y1 = fr7(psi)		Y2 = fpc7(psi)		Y3 = fr7(psi)		Y4 = fpc28(psi)		Y5 = Es28(10 ⁻⁶)	
						Dup Mean	Dup Diff	Dup Mean	Dup Diff	Dup Mean	Dup Diff	Dup Mean	Dup Diff	Dup Mean	Dup Diff
3	A	1.00	140.1	28.0	3.4	460	70	2910	160	300	20	3925	170	4.30	0.10
	B	.75	141.4	27.6	3.1	508	15	3015	150	308	15	4210	180	4.35	0.20
	Mean	.88	140.8	27.8	3.2	484	43	2962	155	304	18	4068	175	4.32	0.15
	Diff	.25	1.3	0.4	0.3	48	55	105	10	8	5	285	10	.05	0.10
16	A	.75	144.3	26.4	4.0	675	10	5525	190	513	5	6505	10	5.13	0.05
	B	1.75	141.1	27.6	5.4	673	35	5105	10	(480)	(41)	6030	40	4.75	0.40
	Mean	1.25	142.7	27.0	4.7	674	22	5315	100	(496)	(23)	6268	25	4.94	0.22
	Diff	1.00	3.2	1.2	1.4	2	25	420	180	(33)	(36)	475	30	0.38	0.35
21	A	.25	133.7	28.2	6.6	498	35	2830	60	298	25	3500	100	3.81	0.30
	B	.75	133.0	28.4	7.0	468	45	2400	20	288	35	3070	100	3.88	0.85
	Mean	.50	133.4	28.3	6.8	483	40	2615	40	293	30	3285	100	3.84	.58
	Diff	.50	0.7	0.2	0.4	30	10	430	40	10	10	430	0	.07	.45
26	A	.75	143.0	27.4	2.8	683	45	5165	150	515	10	6125	250	4.93	0.35
	B	2.50	142.4	27.5	4.1	690	10	4660	40	450	0	5340	200	4.75	0.20
	Mean	1.62	142.7	27.4	3.4	686	28	4912	85	482	5	5732	225	4.84	0.28
	Diff	1.75	0.6	0.1	1.3	7	35	505	110	65	16	785	50	0.18	0.15
38	A	2.00	137.6	27.6	7.6	578	5	3680	80	405	30	4820	20	4.05	0.05
	B	2.50	136.9	27.4	7.6	573	25	(3362)	(176)	323	65	4140	40	4.20	0.15
	Mean	2.25	137.2	27.5	7.6	576	15	(3521)	(128)	364	48	4480	30	4.12	0.10
	Diff	0.50	0.7	0.2	0.0	5	20	(318)	(96)	82	35	680	20	.15	.10
41	A	0	140.1	28.7	3.4	433	35	2730	120	313	15	3475	70	4.20	0.00
	B	0	143.2	28.0	3.8	445	30	2955	390	360	35	4020	160	4.50	0.10
	Mean	0	141.6	28.4	3.6	439	32	2842	255	340	25	3748	195	4.35	0.15
	Diff	0	3.1	0.7	0.4	12	5	225	270	55	20	545	90	.30	0.10
52	A	0	146.4	27.6	3.2	680	50	4815	130	418	65	5140	240	5.00	0.20
	B	.25	146.2	27.6	4.0	663	25	4376	40	420	40	5395	50	5.20	0.35
	Mean	.12	146.3	27.6	3.6	672	38	4596	85	419	52	5268	145	5.10	0.28
	Diff	.25	0.2	0	0.8	17	25	445	90	2	15	255	190	.20	.15
63	A	6.75	(143.1)	27.8	8.5	383	15	2055	30	283	15	2790	0	3.45	0.06
	B	7.75	145.6	27.8	8.6	375	20	2010	0	313	15	2935	50	3.75	0.15
	Mean	7.25	140.5	27.8	8.6	379	18	2032	15	298	15	2862	25	3.60	0.08
	Diff	1.00	2.5	0	0.1	8	5	45	30	30	0	145	50	.30	.15
Overall Mean	1.734	140.6	27.72	5.19	549	29	(3599)	(109)	(374)	(27)	4464	105	4.39	.216	
Rep MS	.234	(1.823)	.136	.294	236.2	573	61197	10490	1008	535	122090	8769	.0267	.0440	
df	8	(8)	8	8	8	16	(8)	(16)	(8)	(16)	8	16	8	16	
Rep RMS	.484	(1.35)	.369	.542	15.4	23.9*	(247)	(102)	(31.8)	(23.1)	349	93.6	.163	.210	
Coeff. Var. *	27.9%	1.0%	1.3%	10.4%	2.8%		6.9%		8.5%		7.9%		3.7%		

Note: Numbers in parentheses reflect adjustments given at bottom of Table 16.
 * Coeff. Var. = Rep RMS / Overall Mean x 100%.

relatively large, it was inferred that a better-fitting regression model should be used.

The last line of table 17 gives coefficients of variation (CV) that are given by dividing the RepRMS values by the respective overall means. It can be seen that X2 = UNWT and X3 = YLD have extremely small CVs of the order of 1 percent. This is because both X2 and X3 are governed by the mix quantities for aggregates, cement, and water, and are, therefore, closely controlled at the same levels for the two replicate mixes. Air content (X4), on the other hand, has a replicate CV of about 10 percent, and the CV for slump (X1) is nearly 30 percent.

Thus, both slump and air content replicate mixes exhibit a relatively large degree of chance variation. Replicate variation for compressive strength (Y2 and Y4) and tensile strength (Y3) are characterized by CVs that range from about 7 to 8.5 percent. The replicate CV for PCC modulus (Y5) is about 4 percent, and for flexural strength about 3 percent. It is believed that all five CVs for the Ys are relatively small and, therefore, reflect a high degree of control for the laboratory study as a whole.

Analysis of Variance and Covariance for Mix Properties and PCC Properties

This section presents analyses of variance (ANOVA) and covariance analyses (COVAR) for the four mix properties (X1 through X4) and the five hardened PCC properties (Y1 through Y5). The ANOVA were made possible by the orthogonality of the experimental design (table 10) and serve to identify which design factors (U1 through U7) and cross-products thereof (U_i x U_j) have statistically significant effects on any X or Y variable. In a sense, ANOVA is a precursor analysis for the regression analyses to follow, and provides a rational basis for inclusion or exclusion of independent variables in the regression models.

Analyses of Variance

The orthogonality of the experimental design ensures that all main effects (e.g., U1 = CAT or U6 = CEQ) and two-factor interactions (e.g., U2 x U3 = CAM x FAM) are additive and independent of one another. Because only a one-half fraction of the complete factorial was used, each three-factor effect (e.g., U1U2U3 = CAT x CAM x FAM) is confounded with a particular four-factor effect (e.g., U4U5U6U7 = AEQ x CAQ x CEQ x WAQ). This means that any significant three-factor effect is identically equivalent to its companion four-factor effect and that the two effects cannot be analytically separated.

The companion three-factor and four-factor interactions can be identified from the following display of all seven design factors:

U1 = CAT, U2 = CAM, U3 = FAM, U4 = AEQ, U5 = CAQ, U6 = CEQ, U7 = WAQ

If any three of the factors are selected as a three-factor combination, then the remaining factors represent the four-factor combination with which the three-factor combination is confounded. The same rule holds for two-factor combinations, since each is confounded with its five-factor companion.

Similarly, each single factor is confounded with the remaining six-factor combination.

Past experience with ANOVAs for experimental data show that, like for successive terms in a convergent series, the magnitude of effects generally decrease from one-factor effects to two-factor effects to three-factor effects, etc, and that at some point all higher-order effects are of the same size as replication error.

Each ANOVA for the study data is based on 64 mixes and, therefore, represents 64 degrees of freedom (df). One degree of freedom (df) is for the mean value of the X or Y being analyzed, seven df are for the one-factor main effects, $7 \times 6 / 2 = 21$ are for two-factor combinations and $7 \times 6 \times 5 / (3 \times 2) = 35$ df are for the three-factor combinations.

Table 18 shows ANOVA results for each of the four mix properties (X1 through X4). The total sum of squares (SS) for each variable is shown in the first line of the summary box and is the sum of the 64 X deviations from the X mean. The ANOVA procedure begins by calculating how much of the total is attributable to each of the 7 main effects (U1 = CAT through U7 = WAQ), how much is attributable to each of the 21 two-factor effects (CAT x CAM through CEQ x WAQ), and what level of significance is attained by each of these effects relative to the remaining (residual) part of the total SS. At this point, the residual sum of squares has 35 degrees of freedom (i.e., $63 - 7 - 21 = 35$). Division of the residual SS by 35 gives the unpooled residual mean square (Res MSu). F ratios are calculated by dividing the SS for each individual effect by Res MSu, and F tables are used to determine the level of statistical significance attained by each effect. The significance levels are shown in parentheses for all main effects and two-factor effects and each represents the probability that the effect SS is really a residual effect rather than the effect of the variable to which it is attributed.

For the study ANOVAs, the 10 percent level (SL = 10) has been used as the criterion for separation of significant effects from residual effects. Thus, effects having SL < 10 percent are considered to be significant; effects with SL > 10 percent are considered to be within the realm of residual variation.

For each X, table 18 shows significance levels for each U effect, and if SL < 10, the table shows the percent of total SS that corresponds to the SS for the significant effect. For X1 = SLMP, for example, the first significant effect is for U3 = FAM. This effect is shown to be significant at the 3 percent level (SL = 03) and has SS percent of 2.55. Thus, the actual SS for FAM was $0.0255 \times 376.07 = 9.59$. Although actual SS might have been shown in table 18 for each effect, it was assumed that the SS percent gave a more useful indication of the size of the effect than does the actual SS.

For X1 = SLMP, table 18 shows that five main effects (ME) were significant at the 10 percent level, and (in the summary box) that these five effects explain 62.05 percent of the total variation in X1. Only 4 of the 21 two-factor effects (2FI) are significant at the 10 percent level, and account for an additional 11.12 percent of the total variation. Thus, for

Table 18. Analysis of variance and covariance for mix properties (X).

Depend. Var. Indep. Var.		X1 = SLMP (in)		X2 = UNWT (PCF)		X3 = YLD (cy)		X4 = AIR (%)		
		ANOVA	COVAR	ANOVA	COVAR	ANOVA	COVAR	ANOVA	COVAR	
		SS % (SL)	SS % (SL)	SS % (SL)	SS % (SL)	SS % (SL)	SS % (SL)	SS % (SL)	SS % (SL)	
Design Factors (U)	U1	CAT	(30)	(31)	3.4 (00)	3.5 (00)	(90)	(83)	(19)	(17)
	U2	CAM	(34)	(34)	(87)	(87)	(60)	(29)	(67)	(65)
	U3	FAM	2.6 (03)	2.6 (04)	5.3 (00)	5.3 (00)	10.1 (00)	10.1 (00)	2.7 (01)	2.7 (01)
	U4	AEQ	28.1 (00)	28.1 (00)	37.1 (00)	37.1 (00)	(50)	(69)	50.0 (00)	50.0 (00)
	U5	CAQ	5.6 (00)	5.6 (00)	5.8 (00)	5.8 (00)	14.2 (00)	14.2 (00)	4.9 (00)	4.9 (00)
	U6	CEQ	3.5 (01)	3.5 (02)	23.4 (00)	23.4 (00)	30.2 (00)	30.2 (00)	10.4 (00)	10.4 (00)
	U7	WAQ	22.3 (00)	22.3 (00)	3.7 (00)	3.7 (00)	(90)	(83)	(49)	(46)
Design Factor Cross-Products (Uj Uj)	CAT • CAM	(16)	(26)	(47)	(82)	(20)	(56)	(66)	(54)	
	CAT • FAM	(14)	(17)	0.6 (10)	(52)	1.7 (07)	(42)	(92)	(96)	
	CAT • AEQ	(61)	(70)	(54)	(29)	(70)	(24)	(71)	(86)	
	CAT • CAQ	1.5 (10)	(37)	1.1 (03)	1.6 (01)	1.6 (07)	2.5 (03)	1.3 (08)	1.0 (10)	
	CAT • CEQ	(60)	(47)	0.8 (08)	(50)	(11)	(43)	(13)	(51)	
	CAT • WAQ	(64)	(54)	(18)	(12)	(25)	1.3 (11)	1.4 (06)	0.9 (11)	
	CAM • FAM	(29)	(38)	(98)	(76)	(95)	(81)	(67)	(87)	
	CAM • AEQ	(57)	(37)	(39)	(70)	(55)	(64)	(18)	(18)	
	CAM • CAQ	(46)	(53)	1.1 (04)	(12)	2.2 (03)	(13)	(18)	(13)	
	CAM • CEQ	(61)	(58)	(88)	(42)	(80)	(50)	(85)	(57)	
	CAM • WAQ	(54)	(37)	(13)	(24)	(30)	(25)	(64)	(53)	
	FAM • AEQ	(21)	(29)	(14)	(29)	(35)	(37)	(17)	(49)	
	FAM • CAQ	(28)	(32)	(67)	(27)	(80)	(31)	(35)	(35)	
	FAM • CEQ	2.0 (06)	1.7 (09)	(80)	(35)	(95)	(40)	(72)	(71)	
	FAM • WAQ	(52)	(67)	(40)	(50)	(70)	(71)	(33)	(72)	
	AEQ • CAQ	(95)	(99)	(51)	(26)	(80)	(33)	(100)	(92)	
AEQ • CEQ	2.9 (02)	(91)	2.1 (00)	0.7 (09)	4.2 (01)	1.6 (08)	5.5 (00)	1.2 (07)		
AEQ • WAQ	4.7 (01)	3.1 (02)	4.5 (00)	4.7 (00)	9.2 (00)	9.6 (00)	5.0 (00)	4.4 (00)		
CAQ • CEQ	(21)	(44)	(52)	(85)	(65)	(72)	(55)	(54)		
CAQ • WAQ	(28)	(25)	(56)	(36)	(70)	(38)	(87)	(66)		
CEQ • WAQ	(79)	(68)	(12)	(30)	(30)	(25)	(78)	(21)		
UFns (V)	V1	FAQ	• • •	(29)	• • •	(14)	• • •	(11)	• • •	(52)
	V2	WCR	• • •	1.64 (09)	• • •	(16)	• • •	(14)	• • •	(52)
	V3	ACR	• • •	2.08 (06)	• • •	(11)	• • •	(12)	• • •	1.61 (04)
ANOVA / COVAR Summary	Total SS / DF		376.07 / 63		1625.45 / 63		33.607 / 63		505.73 / 63	
	SS %	ME	62.1 / 5	62.1 / 5	78.7 / 6	78.7 / 6	54.5 / 3	54.5 / 3	67.9 / 4	67.9 / 4
	/ df	2 FI	11.1 / 4	4.8 / 2	10.3 / 6	7.0 / 3	19.0 / 5	15.0 / 4	13.3 / 4	7.5 / 4
	@ (10)	VE		3.7 / 2						1.6 / 1
	Total / df		73.2 / 9	70.5 / 9	88.9 / 12	85.6 / 9	73.5 / 8	69.5 / 7	81.2 / 8	77.0 / 9
	Res MS p / dfp		1.58 / 54	2.05 / 54	3.53 / 51	4.33 / 54	.162 / 55	.183 / 56	1.73 / 55	2.15 / 54
	Res RMS p		1.26 in.	1.43 in	1.88 pcf	2.08 pcf	0.40 cy	0.43 cy	1.31%	1.47%
	Rep MS / df		0.234 / 8		1.823 / 8		0.136 / 8		0.294 / 8	
EMSR / (SL)		6.8 (00)	8.8 (00)	1.9 (NS)	2.4 (10)	1.2 (NS)	1.3 (NS)	5.9 (01)	7.3 (00)	
PROBABLE 3FI/4FI and (SS%)	CAT • FAM • AEQ (1.6)		CAT • FAM • AEQ (0.7)		CAT • FAM • AEQ (1.5)		CAT • CAQ • WAQ (1.6)			
	AEQ • CAQ • WAQ (1.9)		FAM • AEQ • CEQ (1.0)		FAM • AEQ • CEQ (2.0)		CAT • FAM • AEQ • WAQ (1.1)			
	CAT • FAM • CAQ • CEQ (1.4)		AEQ • CAQ • CEQ (0.7)		AEQ • CAQ • WAQ (1.7)		CAT • FAM • CAQ • WAQ (2.3)			
CAT • FAM • CEQ • WAQ (1.7)		AEQ • CAQ • WAQ (0.7)				FAM • AEQ • CAQ • WAQ (1.6)				
CAT • AEQ • CAQ • WAQ (2.7)										
Max SS %/df		82.4 / 14		92.0 / 16		78.7 / 11		87.7 / 12		
Min Res MS/df		1.35/49		2.77/47		0.138/52		1.22/51		

slump, the summary box shows that 73.17 percent of the total SS is explained by nine effects, of which five are main effects.

The next step is to pool the X1 sum of squares for the 2 non-significant ME and the 17 non-significant 2FI with the unpooled residuals SS and form a pooled residual SS that is based on $35 + 2 + 17 = 54$ df. For X1, the ResMSP is shown to be 1.58, and its square root (ResRMSp) to be 1.26 in (32.0 mm).

The next step was to compare the pooled residual mean square with the replicate mean square for X1 that was given in table 17 and that is reproduced in table 18. The ratio of these two mean squares is called the error mean square ratio (EMSR) and for X1 is $1.58/0.234 = 6.75$. From tables of the F ratio, it was found that this EMSR (for 54 and 8 df) is significant at the 0.5 percent level, shown as (00) in table 18. It was, thus, inferred that the pooled residual mean square contains significant effects that were not accounted for by the main effects and two-factor effects of the design factors.

ANOVAs for X2 = UNWT and X3 = YLD in table 18 show that the significant main effects and two-factor effects account for about 89 percent and 74 percent of the total variations in X2 and X3, respectively, and that the EMSR is not significant for either variable. On the other hand, the fact that EMSR for X4 = AIR is significant at the 1 percent level implies that main effects and two-factor effects ($SS\% = 81.2$) did not provide an adequate explanation for X4 variations.

The ANOVAs for Y1 through Y5 in table 19 show that main effects and two-factor effects account for about 90 percent of the total SS for Y1 = fr7AVE, 87 percent of the variation in Y2 = fpc7AVE, 72 percent of the Y3 = ft7AVE variation, 85 percent of the variation in Y4 = fpc28AVE, and 75 percent of the total variation in Y5 = Ec28AVE. However, the EMSR is significant at the 10 percent level for all five variables, suggesting that higher order interactions may be needed as further explanatory variables.

Since additional explanatory variables must come from three-factor and/or four-factor interactions, each ANOVA for X1 through X4 and Y1 through Y5 was expanded to show the effects associated with each of the 35 degrees of freedom for the unpooled residual sum of squares. As was explained earlier, each of these effects is attributable to a specific three-factor interaction or to its companion four-factor interaction. As shown across the bottom of tables 18 and 19, from three (for X3) to eight (for Y1 and Y3) three/four-factor interactions were thus identified as being significant at the 10 percent level relative to the pooled residual mean square.

To assist in deciding whether each higher order effect represented a 3FI or 4FI, it was noted that one design factor (U2 = CAM) did not produce a significant main effect for any of X1 through X4, and was significant for only two of Y1 through Y5 (Y1 and Y4). Furthermore, of all two factor interactions of U2 = CAM with the remaining six design factors, the 10 percent significance level was reached only by CAM x CAQ for X2 and X3, by CAM x AEQ for Y3 through Y5, and for CAM x CEQ for Y1. It was, therefore, inferred that 3FI and 4FI involving CAM were highly unlikely, and that ANOVA

Table 19. Analysis of variance and covariance for hardened PCC properties (V).

Depend. Var. Indep. Var.		Y1 = fr 7 AVE		Y2 = fpc 7 AVE		Y3 = ft 7 AVE		Y4 = fpc 28 AVE		Y5 = Es 28 AVE		
		ANOVA SS % (SL)	COVAR SS % (SL)	ANOVA SS % (SL)	COVAR SS % (SL)	ANOVA SS % (SL)	COVAR SS % (SL)	ANOVA SS % (SL)	COVAR SS % (SL)	ANOVA SS % (SL)	COVAR SS % (SL)	
Design Factors (U)	U1 CAT	1.3 (06)	1.3 (03)	3.2 (00)	3.2 (00)	2.2 (08)	2.2 (06)	4.4 (00)	4.4 (00)	(90)	(90)	
	U2 CAM	1.5 (01)	1.5 (02)	(29)	(18)	(82)	(81)	1.1 (08)	1.1 (04)	(78)	(77)	
	U3 FAM	2.4 (01)	2.4 (00)	1.8 (01)	1.8 (00)	2.2 (08)	2.2 (07)	1.9 (02)	1.9 (01)	1.9 (07)	1.9 (06)	
	U4 AEQ	3.9 (00)	3.9 (00)	5.5 (00)	5.5 (00)	3.4 (03)	3.4 (02)	5.4 (00)	5.4 (00)	13.9 (00)	13.9 (00)	
	U5 CAQ	4.6 (00)	4.6 (00)	2.5 (00)	2.5 (00)	3.7 (03)	3.7 (02)	3.0 (01)	3.0 (00)	5.3 (00)	5.3 (00)	
	U6 CEQ	62.4 (00)	62.4 (00)	66.2 (00)	66.2 (00)	51.0 (00)	51.0 (00)	62.2 (00)	62.2 (00)	36.4 (00)	36.4 (00)	
	U7 WAQ	2.0 (01)	2.0 (00)	5.4 (00)	5.4 (00)	3.8 (02)	3.8 (02)	3.3 (00)	3.3 (00)	3.6 (01)	3.6 (01)	
Design Factor Cross-Products (U _i U _j)	CAT • CAM	(91)	(39)	(58)	(38)	(88)	(19)	(45)	(68)	(80)	(66)	
	CAT • FAM	(54)	(20)	0.8 (07)	(36)	(38)	(81)	0.7 (15)	(41)	(31)	(62)	
	CAT • AEQ	(21)	(21)	(52)	(97)	(68)	(28)	(73)	(24)	(86)	(60)	
	CAT • CAQ	2.4 (01)	2.1 (01)	(15)	1.1 (02)	2.0 (10)	3.5 (02)	(52)	(17)	3.2 (02)	4.0 (01)	
	CAT • CEQ	(31)	(86)	(19)	(46)	(67)	(16)	(26)	(82)	3.5 (02)	(59)	
	CAT • WAQ	1.0 (10)	1.2 (04)	(37)	0.6 (08)	(70)	(40)	(24)	1.1 (04)	(22)	(14)	
	CAM • FAM	(69)	(71)	(99)	(67)	(87)	(68)	(90)	(53)	(69)	(90)	
	CAM • AEQ	(88)	(78)	(19)	(31)	1.2 (09)	(42)	0.9 (11)	(17)	1.2 (14)	(34)	
	CAM • CAQ	(32)	(72)	(52)	(85)	(76)	(69)	(78)	(67)	(45)	(68)	
	CAM • CEQ	0.7 (10)	(40)	(49)	(94)	(46)	2.0 (08)	(61)	(99)	(31)	(61)	
	CAM • WAQ	(77)	(99)	(79)	(87)	(42)	(48)	(62)	(95)	(29)	(55)	
	FAM • AEQ	2.2 (00)	1.6 (02)	(25)	(11)	(50)	(72)	(19)	1.0 (05)	(51)	(37)	
	FAM • CAQ	1.6 (04)	2.0 (01)	(29)	0.9 (03)	(65)	(62)	(59)	(17)	1.5 (10)	2.1 (05)	
	FAM • CEQ	(46)	(37)	(80)	(57)	(99)	(36)	(37)	(79)	(68)	(99)	
FAM • WAQ	(39)	(36)	(78)	(41)	1.2 (09)	(21)	(65)	(22)	1.6 (10)	2.3 (04)		
AEQ • CAQ	1.3 (02)	1.6 (02)	(20)	0.9 (03)	(85)	(62)	0.9 (12)	1.5 (02)	(25)	(15)		
AEQ • CEQ	(92)	(46)	(93)	(87)	(45)	1.4 (15)	(61)	(77)	(55)	(76)		
AEQ • WAQ	0.9 (04)	0.8 (08)	(38)	(12)	(27)	1.5 (13)	(51)	(26)	3.0 (02)	3.7 (01)		
CAQ • CEQ	(72)	(84)	(30)	(48)	(43)	(77)	(30)	(47)	(44)	(79)		
CAQ • WAQ	1.5 (04)	1.8 (01)	0.7 (09)	1.4 (01)	1.3 (08)	2.4 (06)	0.7 (16)	1.4 (02)	(29)	(18)		
CEQ • WAQ	(83)	(48)	0.9 (07)	(81)	(49)	(21)	0.7 (16)	(52)	(30)	(97)		
UFns	V1 FAQ	• • •	(24)	• • •	0.5 (10)	• • •	(36)	• • •	1.1 (04)	• • •	(88)	
	V2 WCR	• • •	(63)	• • •	(19)	• • •	1.3 (15)	• • •	(46)	• • •	(69)	
	V3 ACR	• • •	(71)	• • •	(64)	• • •	(44)	• • •	(37)	• • •	(69)	
ANOVA / COVAR Summary	Total SS / DF	991,514 / 63		81,082,943 / 63		518,053 / 63		99,375,898 / 63		31,829 / 63		
	SS %	ME	78.1 / 7	78.1 / 7	84.5 / 6	84.5 / 6	66.4 / 6	66.4 / 6	81.2 / 7	81.2 / 7	61.1 / 5	61.1 / 5
	/ df	2 FI	11.5 / 8	11.0 / 7	2.4 / 3	4.8 / 5	5.7 / 4	10.6 / 5	3.9 / 5	5.0 / 4	14.0 / 6	12.0 / 4
	@ (10)	VE		---		0.5 / 1		1.3 / 1		1.1 / 1		
	Total / df		89.6 / 15	89.1 / 14	86.8 / 9	89.8 / 12	72.1 / 10	78.3 / 12	85.1 / 12	87.2 / 12	75.2 / 11	73.2 / 9
	Res MSp / dfp		2159 / 48	2210 / 49	197,602 / 54	162,961 / 51	2716 / 53	2200 / 51	290,528 / 52	249,609 / 51	.152 / 52	.158 / 54
	Res RMS p		46.5 psi	47.0 / psi	445 psi	404 psi	52.1 psi	46.9 psi	539 psi	500 psi	0.39	0.40
	Rep MS / df		236.2 / 8		61,197 / 8		1008 / 8		122,090 / 8		0.0267 / 8	
EMSR / (SL)		9.1 (00)	9.4 (00)	3.2(03)	2.7(08)	2.7(08)	2.2(NS)	2.4 (09)	2.0 / (NS)	5.7(01)	5.9(01)	
PROBABLE 3FI & (SS%)		CAT • FAM • AEQ (0.5) CAT • FAM • CAQ (0.6) CAT • CEQ • WAQ (0.5) FAM • AEQ • CEQ (0.5) FAM • CEQ • WAQ (0.8) AEQ • CAQ • WAQ (1.6)		FAM • CEQ • WAQ (1.0) CAT • FAM • CEQ (0.7) FAM • CAQ • CEQ (0.7) AEQ • CAQ • CEQ (0.6) CAT • CAQ • CEQ • WAQ (0.8)		CAT • CEQ • WAQ (1.8) FAM • AEQ • WAQ (1.3) FAM • CAQ • CEQ (1.9) AEQ • CAQ • CEQ (2.0) FAM • CAQ • CEQ • WAQ (3.3)		CAT • FAM • CEQ (1.2) CAT • CAQ • CEQ (0.9) FAM • CAQ • CEQ (0.9) FAM • CEQ • WAQ (1.1)		CAT • FAM • CEQ (1.2) FAM • AEQ • CEQ (1.6) FAM • CEQ • WAQ (2.4) CAQ • CEQ • WAQ (1.4)		
	PROBABLE 4FI & (SS%)	FAM • AEQ • CEQ • WAQ (0.5) CAT • FAM • AEQ • CAQ (0.5)		CAT • CAQ • AEQ • WAQ (0.8) FAM • CAQ • CEQ • WAQ (0.6)		CAT • AEQ • CAQ • WAQ (1.1) CAT • FAM • CEQ • WAQ (2.0) CAT • FAM • HEQ • CAQ (1.7)		FAM • CAQ • CEQ • WAQ (0.9) CAT • CAQ • CEQ • WAQ (0.9)		FAM • CAQ • CEQ • WAQ (1.6) CAT • CAQ • CEQ • WAQ (1.4)		
Max SS%/df		94.9% / 24		92.1% / 16		87.1% / 18		91.0% / 18		84.9% / 17		
Min Res MS/df		1307 / 39		135,940 / 47		1482 / 45		198,970 / 45		0.105 / 46		

for the remaining six design factors should reveal the distinction between possible 3FI and 4FI.

With the exclusion of U2 = CAM, the design matrix (table 10) represents a full factorial for the remaining six design factors. The corresponding ANOVAs produced ME (6df), 2FI (15df), 3FI (20df), 4FI (15df) and residual effects (7df) that were used to infer which of the 3FI and 4FI were significant. The resulting inferences are shown at the bottom of tables 18 and 19, as probable 3FI and probable 4FI.

The two bottom lines of tables 18 and 19 show the maximum SS percent that is explained when the probable 3FI and 4FI are included as explanatory variables, and the residual mean squares that would remain after inclusion of all significant ME, 2FI, 3FI, and 4FI. When these residual mean squares were compared with replicate mean squares, however, significant F ratios still existed for X1 = SLMP, X4 = AIR, Y1 = fr7AVE, and Y5 = Ec28AVE.

In summary, the ANOVAs for X1 through X4 and Y1 through Y5 show which terms (one-factor through four-factor) are the most likely explanatory variables for the respective dependent variables. The relative explanatory value of each independent variable is indicated by its SS percent in tables 18 and 19. Variables whose SS percent exceed 10 percent of the total variation are as follows:

- For X1 = SLMP: U4 = AEQ (38%) and U7 = WAQ (22%) for 60% total.
- For X2 = UNWT: U4 = AEQ (37%) and U6 = CEQ (23%) for 60% total.
- For X3 = YLD: U3 = FAM (10%), U5 = CEQ(14%) and U6 = WAQ(30%), for 54% total.
- For X4 = AIR: U4 = AEQ (50%), and U6 = CEQ (10%) for 60% total.
- For Y1 = fr7AVE: U6 = CEQ (62%).
- For Y2 = fpc7AVE: U6 = CEQ (66%).
- For Y3 = ft7AVE: U6 = CEQ (51%).
- For Y4 = fpc28AVE: U6 = CEQ (62%).
- For Y5 = Ec28AVE: U4 = AEQ (14%), and U6 = CEQ (36%), for 50% total.

Thus, the main effects of U4 = AEQ and U6 = CEQ are of foremost importance in predicting virtually all of the dependent variables. The SS percent for two-factor variables are generally less than 5 percent, and SS percent for higher-order interactions are generally 1 percent or less.

Covariance Analysis

The design factor functions, V1 = FAQ, V2 = WCR, and V3 = ACR were treated as covariables in the COVAR columns of tables 18 and 19. The purpose of these analyses was to determine whether the Vi had additional explanatory value beyond the SS percent produced by the seven design

factors. The covariance routines can be run to first show the Vi effects, then show the Ui effects after adjustments have been made for the Vi effects. Alternatively, the routines can be run to first show the Ui effects then show any additional effects that are provided by the Vi. Both alternatives were run for each Xi and Yi, but only the results of the second alternative are shown in the COVAR columns of tables 18 and 19. It was learned that, under the first alternative, the Vi effects are significant and relatively large, but that the Vi effects plus the adjusted Ui effects give lower total SS percent than those for the Ui main effects only. For the second alternative, as shown in tables 18 and 19, the Vi effects are seldom significant after the Ui effects were taken into account. Furthermore, the second approach often reduces the SS percent that was explained by 2FI in the ANOVA.

As can be seen in the total SS percent row of tables 18 and 19, the inclusion of Vi produces a somewhat smaller total SS percent and somewhat larger residual mean square for almost every dependent variable. It was concluded that since each Vi is completely determined by specific Ui (see equations 26, 27 and 28), it is more appropriate to regard the Vi as alternative variables for the Ui, rather than to regard the Vi as additional explanatory variables.

Finally, each ANOVA produced a table of means that show main effects and two-factor effects in units of the dependent variables. The tables of means are given in table 20 for all significant effects of the Ui on each Xi and Yi.

The first row of table 20 shows, for example, that the effect of coarse aggregate type (U1 = CAT) is significant for only X2 = UNWT of the mix variables. For the Y variables, CAT has a significant effect for all but Y5 = Ec28AVE, and the difference between aggregate types ranges from 28 psi (2.0 kg/cm²) for Y1 = fr7AVE to 520 psi (37 kg/cm²) for Y4 = fpc28AVE. The sixth row shows that U5 = CEQ has a significant effect on all Xi and Yi, and that the two CAEQ levels produce, for example, a difference of 197 psi (13.9 kg/cm²) for Y1 = fr7AVE and a difference of 1832 psi (129 kg/cm²) for Y2 = fpc7AVE.

Significant two-factor interactions between two independent variables arise when the dependent variable difference for one variable is not the same at both levels of the other variable. Examples are the interaction between U1 = CAT and U2 = FAM for the two compressive strengths (Y2 and Y4), as shown in the first line of two-factor interactions in table 20. For both Y2 and Y4, strength differences between the two levels of FAM are over 500 psi (35 kg/cm²) for the crushed stone aggregate, but are of the order of only 100 psi (7.0 kg/cm²) for the gravel aggregate. The calculated differences between the crushed stone and river gravel aggregates are also considerably less at FAM = 2.10 than at FAM = 2.84.

Table 20 contains only the two-factor interactions that were significant (in tables 18 and 19) for at least one Xi or Yi. As noted the bottom of table 20, four two-factor effects were not significant for any Xi or Yi.

Table 20. Effects of design factors on mix properties and hardened PCC properties.

DESIGN FACTORS AND LEVELS		MIX PROPERTIES				HARDENED PCC PROPERTIES				
		X1 SLMP (in)	X2 UNWT (pcf)	X3 YLD (cy)	X4 AIR (%)	Y1 fr 7 (psi)	Y2 fc 7 (psi)	Y3 II.7 (psi)	Y4 fc 28 (psi)	Y5 Ec 28
Main Effects and (Significance Levels)										
CAT Course Aggregate Type	Stone Gravel	2.21 1.84	139.0 140.9	27.7 27.8	5.4 5.8	549 521	3765 3365	395 368	4637 4117	4.33 4.32
	Dif & (SL)	-0.37 (NS)	1.9 (00)	0.1 (NS)	0.4 (NS)	-28 (06)	-400 (00)	-27 (08)	-520 (00)	.01 (NS)
CAM Coarse Aggregate Maximum Size	0.75 in. 1.50 in.	1.86 2.20	139.9 140.0	27.7 27.8	5.7 5.5	550 520	3630 3500	379 383	4509 4246	4.34 4.31
	Dif & (SL)	0.34 (NS)	0.1 (NS)	-0.1 (NS)	-0.2 (NS)	-30 (01)	-130 (NS)	4 (NS)	-263 (08)	-.03 (NS)
FAM Fine Aggregate Modules	2.10 2.84	1.64 2.41	138.8 141.1	28.0 27.5	6.1 5.1	516 555	3415 3716	368 394	4204 4551	4.23 4.43
	Dif & (SL)	0.77 (03)	2.3 (00)	-0.5 (00)	-1.0 (00)	39 (01)	301 (01)	26 (08)	347 (02)	0.20 (01)
AEO Air Entrapment Agent Quant.	0 oz. 6.5 oz.	0.74 3.31	143.0 136.9	27.8 27.7	3.6 7.6	560 511	3829 3302	398 364	4666 4089	4.59 4.06
	Dif & (SL)	2.57 (00)	-6.1 (00)	-0.1 (NS)	4.0 (00)	-49 (00)	-527 (00)	-34 (03)	-577 (00)	-.53 (00)
CAO Coarse Aggregate Quant.	1542 lb. 1850 lb.	1.45 2.60	138.8 141.2	28.0 27.5	6.2 5.0	509 562	3388 3742	364 398	4163 4592	4.16 4.49
	Dif & (SL)	1.15 (00)	2.4 (00)	-0.5 (00)	-1.2 (00)	53 (00)	354 (00)	34 (03)	429 (00)	0.33 (00)
CEO Cement Quantity	426 lb. 584 lb.	2.48 1.57	137.5 142.4	28.1 27.3	6.5 4.7	437 634	2649 4481	317 445	3395 5360	3.90 4.75
	Dif & (SL)	-0.91 (01)	4.9 (00)	-0.8 (00)	-1.8 (00)	197 (00)	1832 (00)	128 (00)	1965 (00)	0.85 (00)
WAO Water Quantity	236 lb. 270 lb.	0.88 3.17	140.9 139.0	27.8 27.7	5.5 5.7	553 518	3626 3305	399 363	4603 4152	4.46 4.19
	Dif & (SL)	2.29 (00)	-1.9 (00)	-0.1 (NS)	0.2 (NS)	-65 (01)	-521 (00)	-36 (02)	-451 (00)	-.27 (01)
Two Factor Interactions Significant at (10)										
CAT = Stone	FAM = 2.10 FAM = 2.84		137.5 140.6	28.1 27.4			3513 4016		4358 4917	
	Dif	(NS)	3.1	-0.7	(NS)	(NS)	503	(NS)	559	(NS)
CAT = Gravel	FAM = 2.10 FAM = 2.84		140.2 141.7	27.9 27.6			3316 3415		4050 4185	
	Dif	(NS)	1.5	-0.3	(NS)	(NS)	99	(NS)	135	(NS)
CAT = Stone	CAO = 1542 lbs. CAO = 1850 lbs.	1.34 3.08	138.4 139.7	27.9 27.5	5.7 5.1	542 557		390 399		4.30 4.37
	Dif	1.74	1.3	-0.4	-0.6	15	(NS)	9	(NS)	0.07
CAT = Gravel	CAO = 1542 lbs. CAO = 1850 lbs.	1.56 2.13	139.2 142.7	28.1 27.4	6.8 4.9	475 567		338 397		4.03 4.61
	Dif	0.57	3.5	-0.7	-1.9	92	(NS)	59	(NS)	0.58
CAT = Stone	CEO = 426 lbs. CEO = 584 lbs.		137.0 141.0							4.04 4.63
	Dif	(NS)	4.0	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	0.59
CAT = Gravel	CEO = 426 lbs. CEO = 584 lbs.		138.0 143.8							3.76 4.88
	Dif	(NS)	5.8	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	1.12
CAT = Stone	WAO = 236 lbs. WAO = 270 lbs.				4.9 5.8	579 520				
	Dif	(NS)	(NS)	(NS)	0.9	59	(NS)	(NS)	(NS)	(NS)
CAT = Gravel	WAO = 236 lbs. WAO = 270 lbs.				6.0 5.6	526 516				
	Dif	(NS)	(NS)	(NS)	-0.4	-10	(NS)	(NS)	(NS)	(NS)
CAM = 0.75 in.	AEO = 0 oz AEO = 6.5 oz							406 353	4917 4101	4.68 4.00
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	53	816	-0.68
CAM = 1.50 in.	AEO = 0 oz AEO = 6.5 oz							390 376	4415 4076	4.50 4.13
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	-14	-339	-.37
CAM = 0.75 in.	CAO = 1542 lbs CAO = 1850 lbs		139.2 140.6	28.0 27.6						
	Dif	(NS)	1.4	-0.4	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)
CAM = 1.50 in.	CAO = 1542 lbs CAO = 1850 lbs		138.3 141.7	28.1 27.3						
	Dif	(NS)	3.4	-0.8	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)
CAM = 0.75 in	CEO = 426 lbs CEO = 584 lbs									4.41 6.59
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	2.18
CAM = 1.50 in	CEO = 426 lbs CEO = 584 lbs									4.32 6.08
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	176
CAM = 0.75 in	WAO = 236 lbs WAO = 270 lbs									
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)
CAM = 1.50 in	WAO = 236 lbs WAO = 270 lbs									
	Dif	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)	(NS)

Table 20. Effects of design factors on mix properties and hardened PCC properties (continued).

DESIGN FACTORS AND LEVELS		MIX PROPERTIES				HARDENED PCC PROPERTIES				
		Y1 SLMP (in)	Y2 UNWT (pcf)	Y3 YLD (cy)	Y4 AIR (%)	Y1 f' 7 (psi)	Y2 fpc 7 (psi)	Y3 f' 7 (psi)	Y4 fpc 28 (psi)	Y5 Ec 28
FAM = 2.10	AEO = 0 oz AEO = 6.5 oz DfI					522 510				
FAM = 2.84	AEO = 0 oz AEO = 6.5 oz DfI	(NS)	(NS)	(NS)	(NS)	-12	(NS)	(NS)	(NS)	(NS)
FAM = 2.10	CAO = 1542 lbs CAO = 1850 lbs DfI					598 512				
FAM = 2.84	CAO = 1542 lbs CAO = 1850 lbs DfI	(NS)	(NS)	(NS)	(NS)	-86	(NS)	(NS)	(NS)	(NS)
FAM = 2.10	CEO = 426 lbs CEO = 584 lbs DfI	1.75 1.53				474 558				3.98 4.48
FAM = 2.84	CEO = 426 lbs CEO = 584 lbs DfI	-22	(NS)	(NS)	(NS)	84	(NS)	(NS)	(NS)	0.50
FAM = 2.10	WAO = 236 lbs WAO = 270 lbs DfI	3.21 1.62				544 566				4.36 4.50
FAM = 2.84	WAO = 236 lbs WAO = 270 lbs DfI	-1.59	(NS)	(NS)	(NS)	22	(NS)	(NS)	(NS)	0.15
FAM = 2.10	CAO = 1542 lbs CAO = 1850 lbs DfI									
FAM = 2.84	CAO = 1542 lbs CAO = 1850 lbs DfI	(NS)	(NS)	(NS)	(NS)					
FAM = 2.10	WAO = 236 lbs WAO = 270 lbs DfI							375 360		4.27 4.18
FAM = 2.84	WAO = 236 lbs WAO = 270 lbs DfI	(NS)	(NS)	(NS)	(NS)			-15	(NS)	-0.09
AEO = 0 oz	CEO = 426 lbs CEO = 584 lbs DfI	0.78 0.70	141.3 144.7	28.0 27.5	3.9 3.4			422 367		4.65 4.20
AEO = 6.5 oz	CEO = 426 lbs CEO = 584 lbs DfI	-0.08	3.4	-0.5	-0.5			-56	(NS)	-45
AEO = 0 oz	WAO = 236 lbs WAO = 270 lbs DfI	4.18 2.45	133.7 140.1	28.3 27.2	9.1 6.0					
AEO = 6.5 oz	WAO = 236 lbs WAO = 270 lbs DfI	-1.73	6.4	-1.1	-3.1					
AEO = 0 oz	CAO = 1542 lbs CAO = 1850 lbs DfI	0.12 1.36	142.9 143.1	28.0 27.5	4.1 3.1	519 600		549 521	4337 4996	
AEO = 6.5 oz	CAO = 1542 lbs CAO = 1850 lbs DfI	1.24	0.2	-0.5	-1.0	81	(NS)	(NS)	559	(NS)
AEO = 0 oz	CEO = 426 lbs CEO = 584 lbs DfI	1.64 4.98	138.9 134.9	27.5 27.9	6.8 8.3	498 523			3990 4182	
AEO = 6.5 oz	CEO = 426 lbs CEO = 584 lbs DfI	-0.34	-4.0	0.4	1.5	25	(NS)	(NS)	192	(NS)
CAO = 1542 lbs	WAO = 236 lbs WAO = 270 lbs DfI									
CAO = 1850 lbs	WAO = 236 lbs WAO = 270 lbs DfI	(NS)	(NS)	(NS)	(NS)					
CEO = 426 lbs	WAO = 236 lbs WAO = 270 lbs DfI									
CEO = 584 lbs	WAO = 236 lbs WAO = 270 lbs DfI	(NS)	(NS)	(NS)	(NS)					
						566 554				4.60 4.58
						-12	(NS)	(NS)	(NS)	-0.02
						540 481				4.32 3.81
						-69	(NS)	(NS)	(NS)	-51
						507 511	3553 3223	371 357	4284 4042	
						-4	-330	-14	-242	(NS)
						595 529	4098 3386	426 370	4922 4261	
						-66	-712	-56	-661	(NS)
							2806 2493		3517 3273	
						(NS)	-313	(NS)	-244	(NS)
							4846 4116		5690 5030	
						(NS)	-730	(NS)	-660	(NS)

NOTE: Four Two-factor interactions were not significant for any of the nine properties and are not included in Table 21. These are CATxCAM, CATxAEO, CAMxFAM and CAOxCEO.

Multiple Regression Analyses for PCC Properties

This section presents results from five sets of multiple regression analyses that produce equations for the prediction of mix properties (X) and hardened PCC properties (Y) from other study variables.

The first set is for regressions of Y_i on X_i , the second for regressions of X_i on U_i and V_i , the third for regressions of Y_i on U_i and V_i , and the fourth for regressions of Y_i on U_i , V_i , and X_i . The final set of regressions is for selected pairs of Y_i whose associations were not fully developed in previous sections and that have special utility in the development of PCC PRS systems.

Regressions of PCC Properties (Y) on Mix Properties (X)

Results for the regression of each Y_i on all four mix properties (X_i) and cross-products of mix factors ($X_i X_j$) are given in table 21. The full set of $N = 64$ mixes in data base A was used in each regression. The stepwise regression procedure was used in connection with a 10 percent significance level. In this procedure, the independent variable that has the highest simple correlation with the dependent variable is entered in step 1. The remaining independent variables (nine in table 21) are then searched in step 2 for the variable that will attain the highest level of significance in conjunction with the step 1 variable. The procedure ends when none of the unentered variables will attain the 10 percent level of significance in conjunction with the previously entered variables.

In table 21, it can be seen that $X_2 = \text{UNWT}$ and/or $X_3 = \text{YLD}$ are the only two predictors that attain the 10 percent level of significance for any Y . In addition to these variables, the step procedure selected two interaction effects, $\text{UNWT} \times \text{YLD}$ for the prediction of Y_2 and $\text{SLMP} \times \text{UNWT}$ for the prediction of Y_3 . By coincidence, each of the five regression equations in table 21 for predicting Y_1 through Y_5 contains three terms, two involving the X_i and one for the constant term of the equation.

The summary lines for each regression show total sum of squares, regression sum of squares, R-squares (i.e., regression SS divided by total SS) ranging from 57 to 74 percent, residual sum of squares, residual mean square, and the residual root-mean-square or standard error of estimate (SEE). Replicate mean squares are those shown for each X_i in table 17, and the error mean square (EMS) ratio is the quotient of the residual and replicate mean squares. All five EMS are significant at the 5 percent level or less. It is therefore inferred that all of the derived equations show lack-of-fit, and that variation in the Y variables cannot be adequately explained by only the X variables. Somewhat better fits might be attained through the use of higher-order X interactions, or perhaps through non-linear regression models, but it did not appear that mix variables are sufficient by themselves to give good predictions for the hardened PCC properties.

Maximum and minimum residuals are shown following the EMS ratios, and mix sequence numbers are shown for residuals that exceed 2.8 times SEE in absolute value. The two extreme residuals are shown to be for mixes 49 and

Table 21. Regressions of hardened PCC properties (Y) on mix properties (X).

Depend. Var. Indep. Var.	Y1 = fr 7 AVE	Y2 = fpc 7 AVE	Y3 = ft 7 AVE	Y4 = fpc 28 AVE	Y5 = Ec 28 AVE
	COEFF + (SL) N = 64				
X1 = SLMP (in)	(33)	(29)	(14)	(48)	(41)
X2 = UNWT (pcf)	10.4 (00)	214 (00)	(19)	106 (00)	.093 (00)
X3 = YLD (cy)	-73.4 (00)	(89)	-83.0 (00)	-663 (00)	-.255 (01)
X4 = AIR (%)	(99)	(38)	(41)	(60)	(73)
SLMP·UNWT	(29)	(25)	-.077 (00)	(42)	(34)
SLMP·YLD	(37)	(32)	(10)	(52)	(46)
SLMP·AIR	(99)	(96)	(21)	(90)	(67)
UNWT·YLD	(60)	-3.9 (01)	(20)	(94)	(17)
UNWT·AIR	(96)	(37)	(41)	(59)	(70)
YLD·AIR	(99)	(39)	(42)	(61)	(80)
CONSTANT TERM	1121 (20)	-11265 (00)	2704 (00)	7916 (39)	-1.444 (72)
Total SS/df	991,514 / 63	81,082,943 / 63	518,053 / 63	99,375,898 / 63	31.829 / 63
Regr. SS/df	609,270 / 2	46,432,353 / 2	302,044 / 2	56,506,068 / 2	23.654 / 2
R - Square	61.4%	57.3%	58.3%	56.9%	74.3%
Resid. SS/df	382,244 / 61	34,650,590 / 61	216,008 / 61	42,869,831 / 61	8.175 / 61
Resid MS	6266	568,042	3541	702,784	0.134
Resid. RMS = SEE	79.2 psi	754 psi	59.5 psi	838 psi	0.366
Rep. MS/df	236.2 / 8	61,197 / 8	1008 / 8	122,090 / 8	.0267 / 8
EMS Ratio (SL)†	22.5 (00)	9.28 (00)	3.51 (05)	5.76 (01)	5.02 / (01)
Max Y Res / Max Z Res††	186 / 2.36	1675 / 2.22	177 / 2.97	1983 / 2.37	1.121 / 3.06
Min Y Res / Min Z Res	-178 / -2.25	-1566 / -2.08	-149 / -2.50	-1753 / -2.09	-1.028 / -2.81
Mix / Yobs / Y pred			64 / 665 / 488		49 / 1.80 / 2.83

Resid. Exceeding 28 SEE

† EMS Ratio = $\left(\frac{SEE}{Rep\ SD}\right)^2$ †† Z Res = $\frac{Y\ Res}{SEE}$

64, i.e., for two of the four extreme values that were identified previously.

For each of Y1 through Y5, figures 13 through 17 are plots of the 64 regression residuals (Y observed versus Y predicted) for the five regression equations that are shown in table 21. For each Y, the corresponding Y observed versus Y predicted figure gives (at bottom) the table 21 regression equation for predicting Y, and includes the R-square and SEE that are also given in table 21. In each figure, the value of any particular residual is either the horizontal or vertical distance of the residual point from the line of equality. Residuals exceeding 2.8 SEE are identified by mix number, as was done at the bottom of table 21.

The results given in table 21 are of considerable importance in the construction control of variables that relate to PCC strength and will be discussed further in the next section.

Regressions of Mix Properties (X) on Significant Design Factors (U) and Design Factor Functions (V)

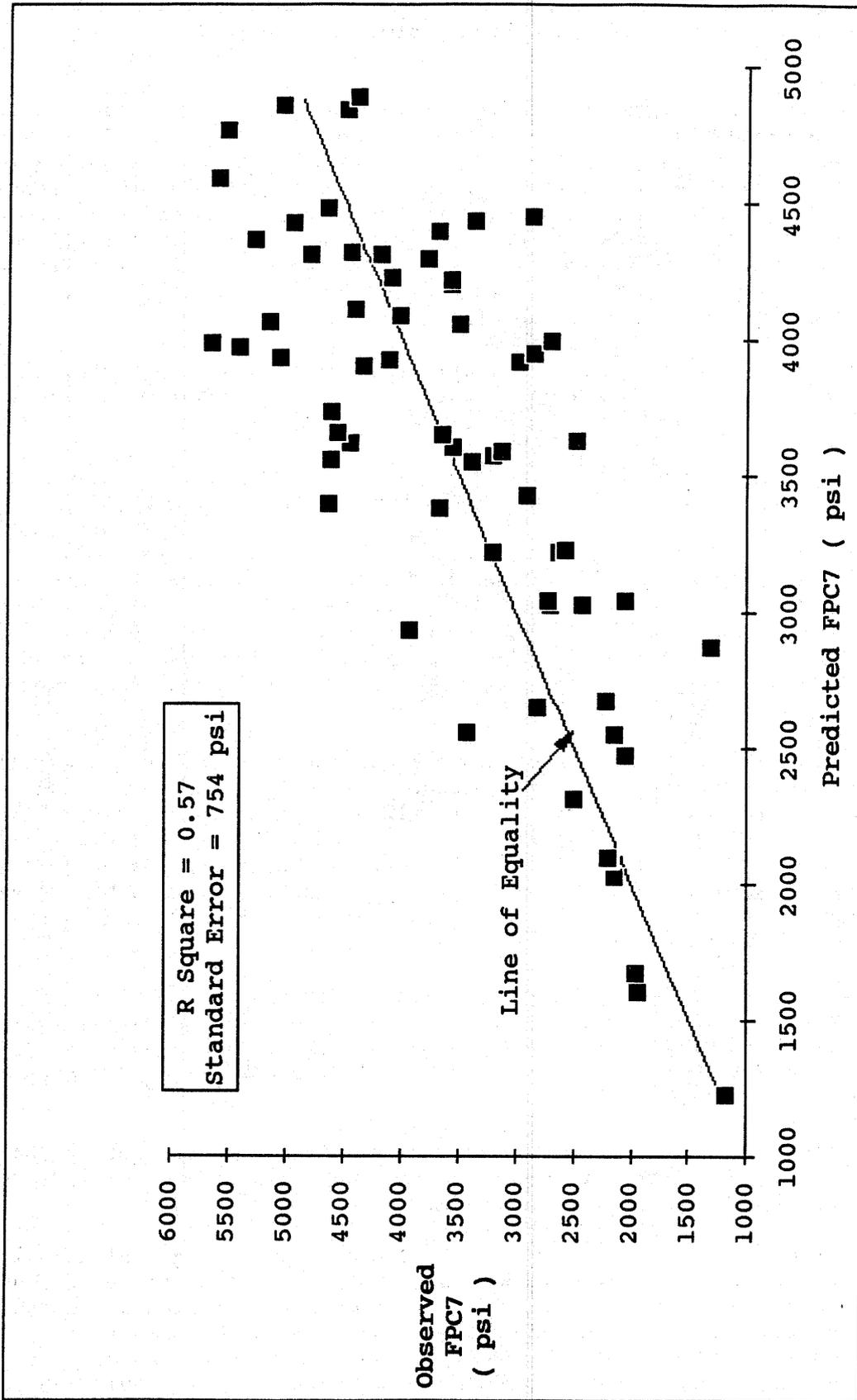
The second set of regression analyses were run to determine how and how well PCC mix properties (X_i) can be predicted from experimental design factors (U_j) and functions of the design factors (V_j). The regression results are shown in table 22. Independent variables used in the regressions are shown at the left of the table and include only those U_j and V_j and cross-products thereof that were significant for at least one of the four X_i in the ANOVAs of table 18.

To reduce intercorrelations among independent variables, especially those induced by the use of cross-products, each U_j and V_j was first transformed by subtraction of its mean value as given in the diagonal of table 15. For example, table 15 shows that the mean values (across all 64 mixes) were 2.47 for $U_3 = \text{FAM}$ and 0.51 for $V_2 = \text{WCR}$. The corresponding independent variables for the table 22 regressions are thus $V_{MD} = (V_M - 2.47)$ and $W_{CRD} = (W_{CR} - 0.51)$, where the letter D is used to denote the deviation of the original variable from its mean value.

Without these transformations, the correlation between U_i and $U_i \times U_j$, for example, might be around 0.90, whereas the correlation between $(U_i - U_{iAVE})$ and $(U_i - U_{iAVE})(U_j - U_{jAVE})$ might be nearly zero. As a result, the regression coefficients for U_{iD} and $U_{iD} \times U_{jD}$ are not confounded because of high intercorrelations.

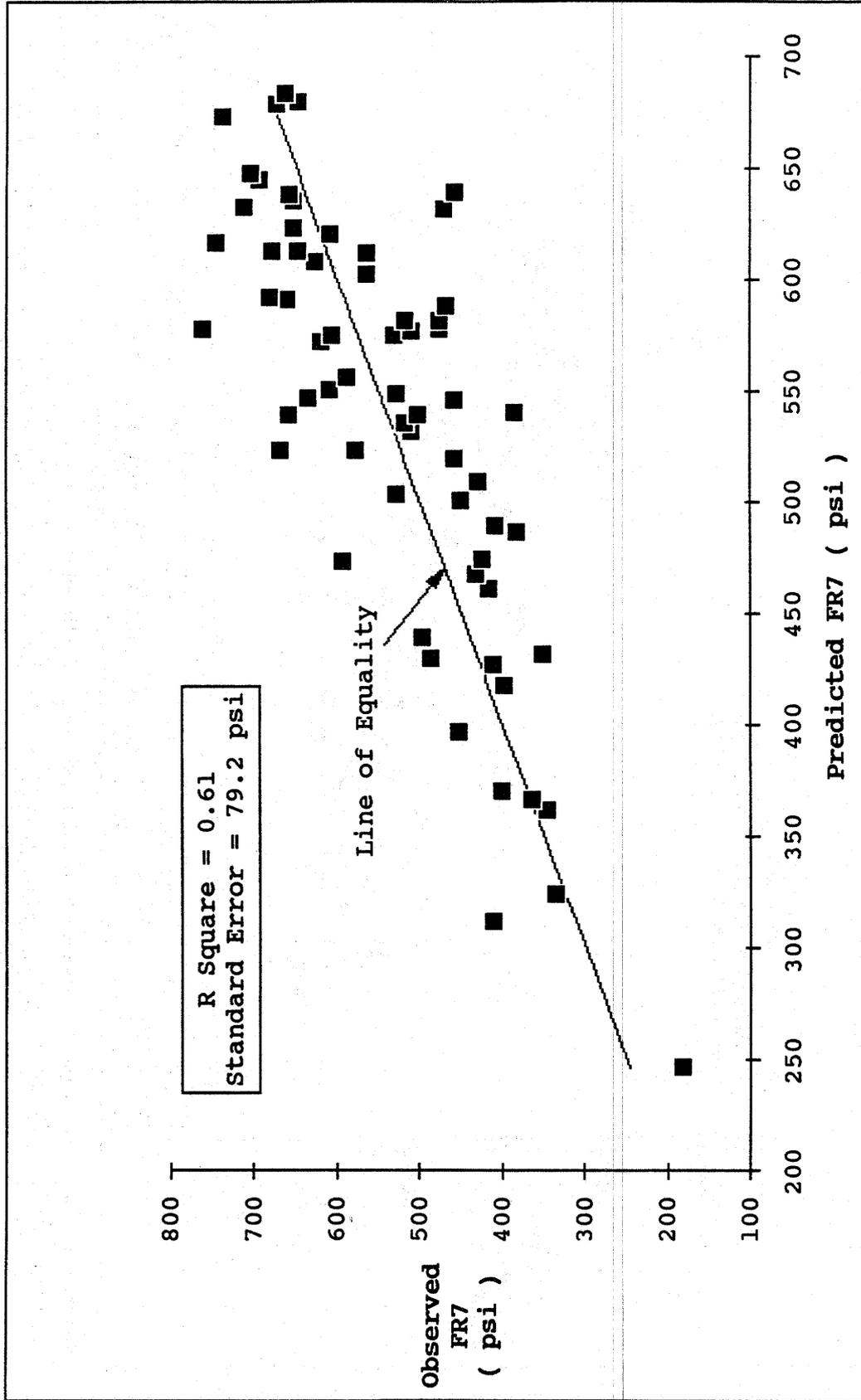
As shown at the top of table 22, regressions were run both using data from all mixes ($N = 64$) and using data for the 60 mixes that exclude the 4 mixes noted at the bottom of the table.

The table 22 results represent $4 \times 2 = 8$ stepwise regression analyses. Except for the constant term in each regression equation, table 22 shows regression coefficients for only those independent variables whose effects were significant at the 10 percent level ($SL = 10$). The number of independent variables in each regression equation is shown as df for regression SS in the regression summary and varies (for $N = 60$) from 7 variables for the prediction of $X_4 = \text{AIR}$ to 10 variables for the prediction



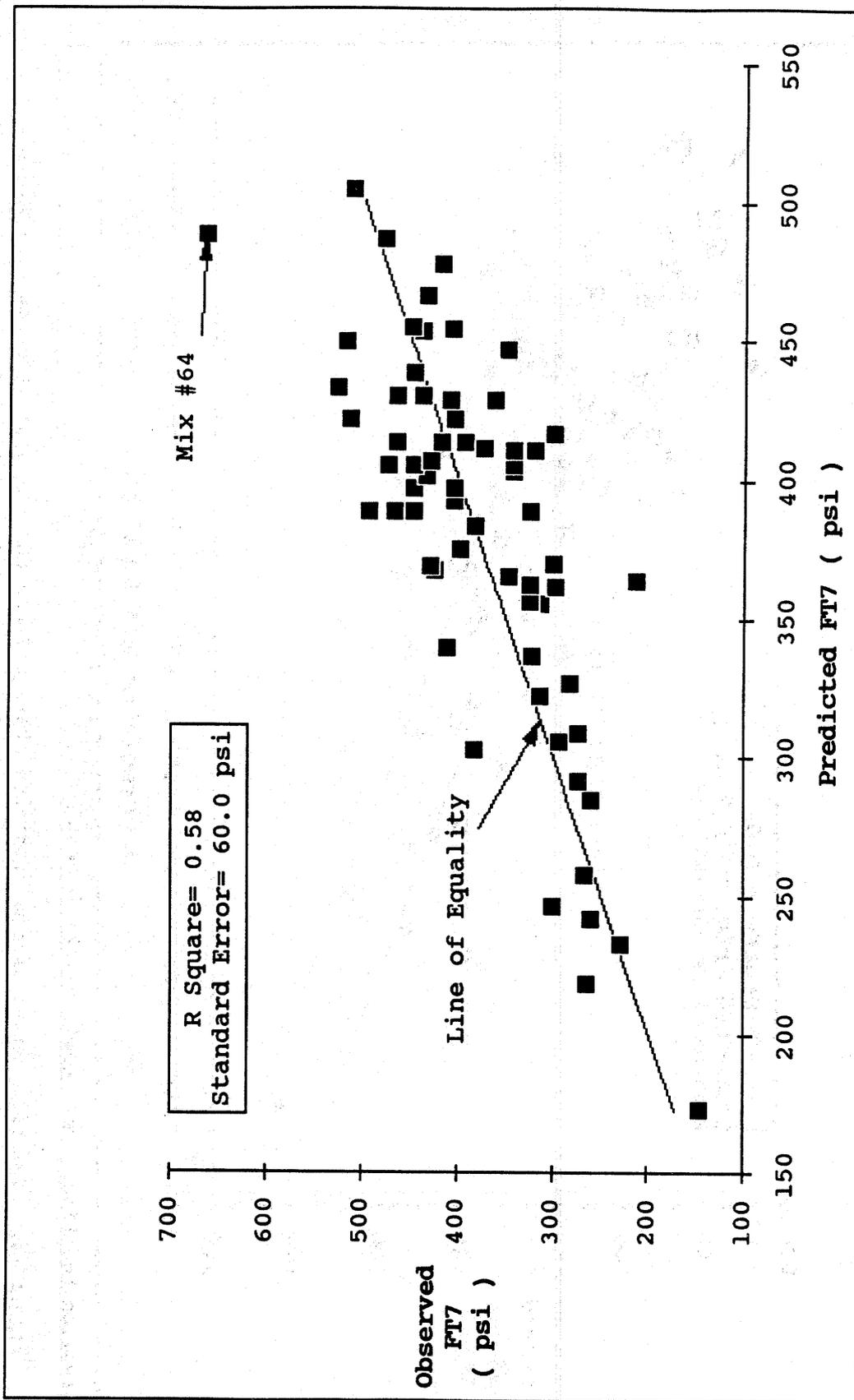
Regression Equation: $FPC7 (psi) = -11265 + 214UNWT - 3.9UNWTxYLD$

Figure 13. Plot of observed versus predicted average 7-day PCC compressive strength ($Y_2 = FPC7_{AVE}$), where only mix properties (X_i) were independent variables.



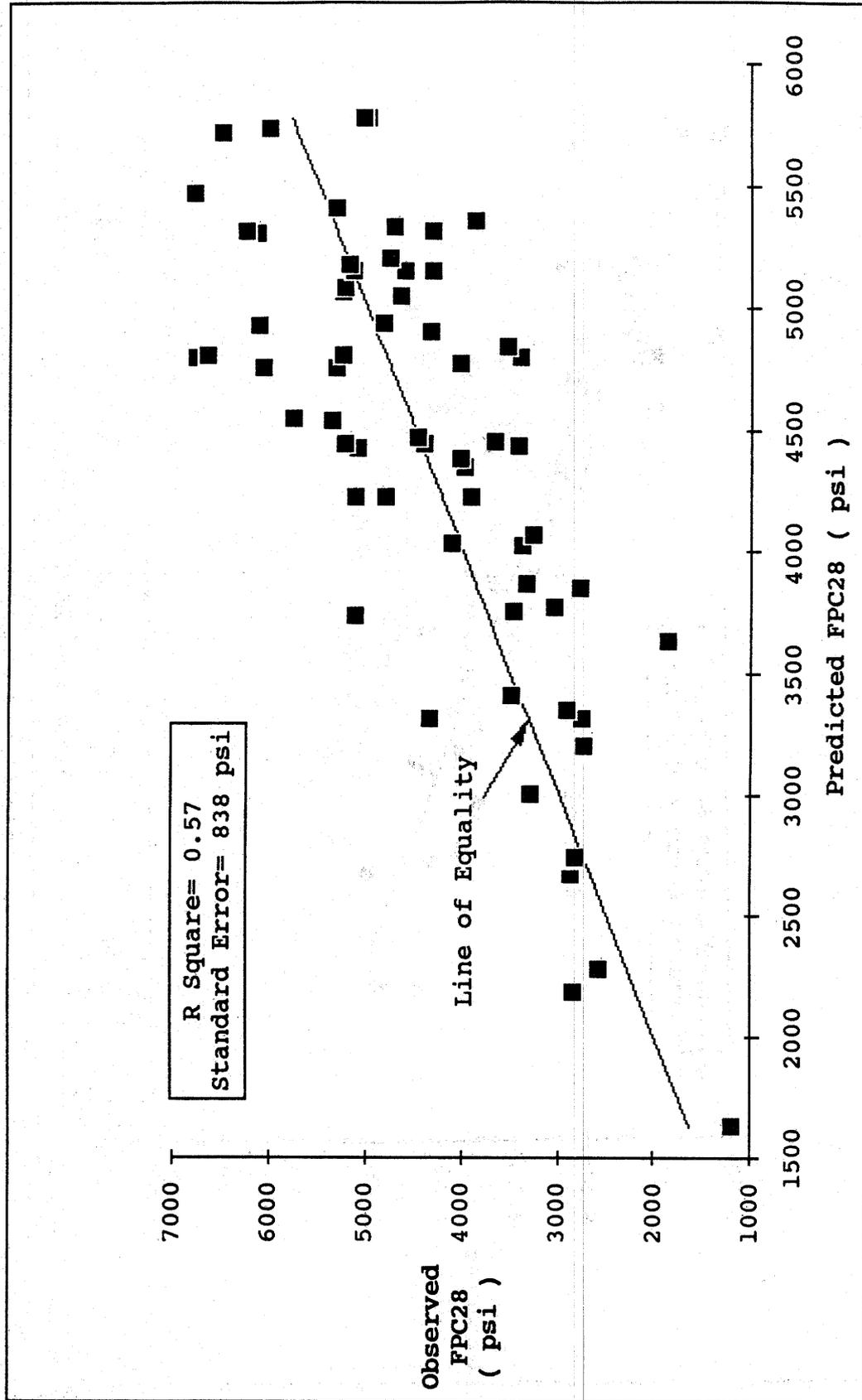
Regression Equation: $FR7 \text{ (psi)} = 1121 + 10.4UNWT - 73.4YLD$

Figure 14. Plot of observed versus predicted average 7-day PCC flexural strength ($Y1=FR7/AVE$), where only mix properties (Xi) were independent variables.



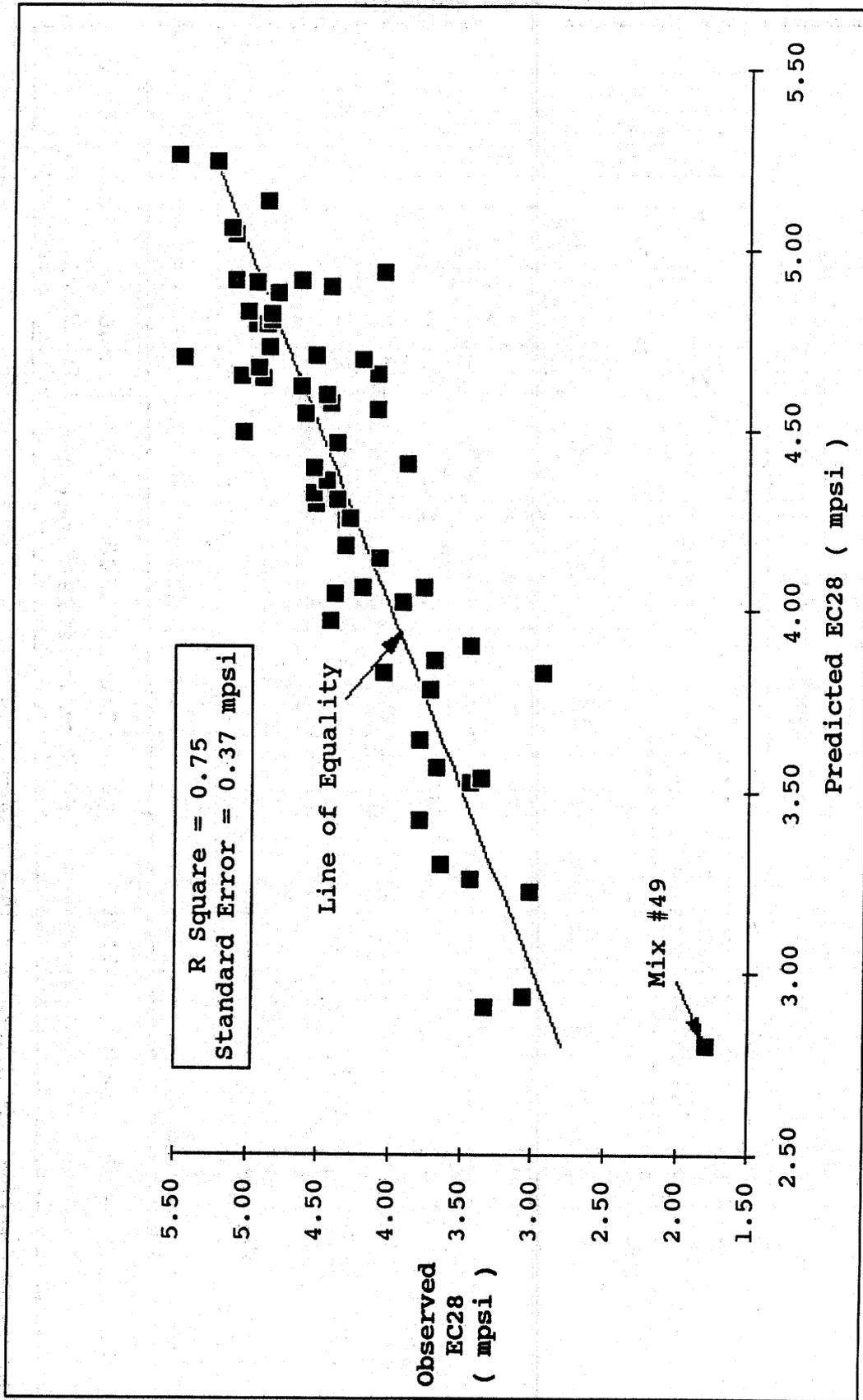
Regression Equation: $FT7 \text{ (psi)} = 2704 - 83.0YLD - 0.077SLMPxUNWT$

Figure 15. Plot of observed versus predicted average 7-day PCC tensile strength ($Y3=FT7AVE$), where only mix properties (Xi) were independent variables.



Regression Equation: $FPC28 (psi) = 7916 + 106UNWTI - 663YLD$

Figure 16. Plot of observed versus predicted average 28-day PCC compressive strength ($Y_4 = FPC28AVE$), where only mix properties (X_i) were independent variables.



Regression Equation: EC28 (mpsi) = 1.444+0.093UNWT-0.255YLD

Figure 17. Plot of observed versus predicted average PCC elastic modulus (Y5=EC28AVE), where only mix properties (Xi) were independent variables.

Table 22. Multiple regressions for mix properties (X) on significant design factors (U) and factor functions (V).

Depend. Var.		X1 = SLMP (in)		X2 = UNWT (pcf)		X3 = YLD (cy)		X4 = AIR (%)	
		COEFFICIENT & (SL)		COEFFICIENT & (SL)		COEFFICIENT & (SL)		COEFFICIENT & (SL)	
		N = 64	N = 60*	N = 64	N = 60	N = 64	N = 60	N = 64	N = 60
Design Factors (U)	U1 CATD=(CAT-0.5)	(28)	(37)	1.81 (00)	1.54 (00)	(67)	(28)	(16)	(15)
	U2 CAMD=(CAM-1.125)	(56)	(67)	(63)	(19)	(13)	-.26 (01)	(57)	(72)
	U3 FAMD=(FAM-2.47)	(17)	(26)	2.88 (00)	2.64 (00)	-.53 (00)	-.49 (00)	-1.05 (02)	(11)
	U4 AEQD=(AEQ-3.25)	.42 (00)	.42 (00)	-.98 (00)	-.98 (00)	(96)	(97)	.62 (00)	.64 (00)
	U5 CAPD=(CAP-38.59)	.11 (03)	.09 (07)	.34 (00)	.33 (00)	-.07 (00)	-.07 (00)	-.13 (00)	-.13 (00)
	U6 CEPD=(CEP-9.29)	(95)	(83)	1.55 (00)	1.55 (00)	-.23 (00)	-.23 (00)	-.56 (00)	(40)
	U7 WAPD=WAP-14.64)	.82 (00)	.77 (00)	-.82 (00)	-.89 (00)	(27)	(52)	(80)	(38)
	V2 WCRD=(WCR-0.51)	8.59 (00)	8.94 (00)	(80)	(75)	(27)	(49)	(78)	10.02 (00)
Dsgn Fctrs (U,V)	CATD • CAPD	-.17 (07)	-.20 (05)	.27 (01)	.22 (04)	-.04 (05)	-.03 (09)	(36)	(58)
	CATD • CEPD	(74)	(58)	(29)	(25)	(31)	(25)	(31)	(13)
	CATD • WAPD	(57)	(41)	(15)	(48)	(13)	(45)	-.70 (03)	(69)
Two-Factor Products	CAMD • CAPD	(46)	(48)	.27 (06)	(25)	-.05 (06)	(25)	-.21 (06)	-.16 (08)
	FAMD • CEPD	-.66 (04)	-.60 (08)	(58)	(37)	(67)	(32)	(77)	(29)
	AEQD • CEPD	-.14 (00)	-.14 (00)	.19 (00)	.21 (00)	-.04 (00)	-.04 (00)	-.14 (00)	-.14 (00)
	AEQD • WAPD	.16 (01)	.16 (01)	-.29 (00)	-.22 (00)	.06 (00)	.04 (00)	.15 (00)	.12 (01)
Three-Factor Products	CATD•FAMD•AEQD	(27)	(21)	(19)	-.08 (03)	(30)	.01 (03)	(17)	.05 (02)
	AEQD•CAPD•WAPD	-.03 (01)	-.03 (03)	.04 (01)	(19)	-.873 (00)	(25)	(41)	(82)
	FAMD•AEQD•CEPD	(27)	(35)	(28)	(25)	(42)	(47)	(54)	(38)
Constant Term		2.05 (00)	2.09 (00)	140.0 (00)	140.1 (00)	27.74 (00)	27.72 (00)	5.60 (00)	5.37 (00)
Regression Summary	Total SS	376.1	362.7	1625.5	1477.9	33.6	24.1	505.7	439.6
	Reg. SS / df	261.6/9	252.4/9	1484.7/11	1361.9/10	27.8/8	19.9/8	416.3/8	385.6/7
	R - Square	.696	.696	.913	.921	.829	.827	.823	.877
	Res. SS / df	114.4/54	110.3/50	140.7/52	116.1/49	5.8	4.2/51	89.4	54.0
	SEE	1.46 in	1.49 in	1.65 pcf	1.54 in	.32 cy	.29 cy	1.28%	1.02%
	Rep.SD / df	.484 in/8		1.35 pcf/8		.369 cy/8		.524%/8	
	EMS Ratio (SL)†	9.10 (00)	9.48 (00)	1.49 (NS)	1.30 (NS)	.75 (NS)	.62 (NS)	5.58 (01)	3.54 (05)
Max Y Res / Max Z Res††		3.8/2.6	3.47/2.34	3.3/2.0	3.9/2.5	.8/2.6	.6/2.2	3.6/2.9	2.4/2.4
Min Y Res / Min Z Res		-3.1/-2.2	-3.4/-2.3	-4.7/-2.8	-4.1/-2.7	-.7/-2.0	-.8/-2.7	-2.9/-2.3	-2.3/-2.3
Residuals Over 2.8 SEE	a. Mix No / Res.			15/4.7				34/3.6	
	b. " "								

* - N = 60 omits Mix Numbers 34,37,49, and 64.
 (NS) - Ratio not significant at the 10% level.

$$\dagger \text{ EMS Ratio} = \left(\frac{\text{SEE}}{\text{Rep SD}} \right)^2$$

$$\dagger\dagger \text{ Z Res} = \frac{\text{Y Res}}{\text{SEE}}$$

of $X_2 = UNWT$. Each equation includes at least three two-factor products and one three-factor product of the independent variables.

R-squares range from about 70 percent for slump to 92 percent for unit weight. The standard errors of estimate (SEE) are significantly greater than the corresponding replicate standard deviations for slump (X_1) and air (X_4), but not for unit weight (X_2) or yield (X_3). It was concluded that the prediction equation for X_2 and X_3 provide good fits to the data, but that there are significant lack-of-fits for both the slump and air prediction equations.

Residuals for the $N = 60$ regression equations are plotted in figures 18 through 21. As shown at the bottom of table 22 and in figures 19 and 21, extreme residuals were identified with mix 15 for unit weight and with mix 34 for air content.

Regressions of PCC Properties (Y) on Significant Design Factors (U,V)

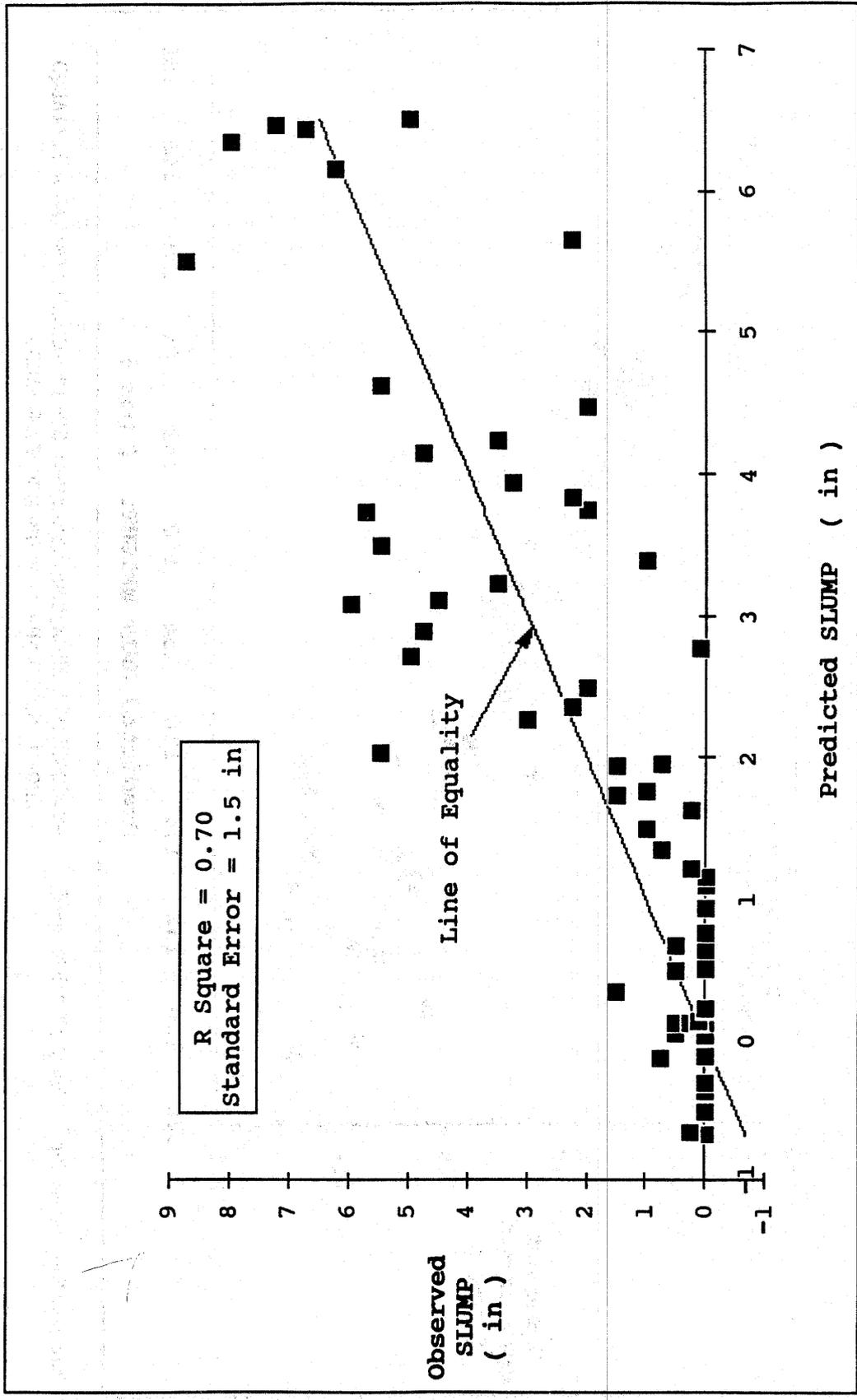
The third set of regression results are shown in table 23 and give equations for predicting PCC strength (Y_1 through Y_4) and modulus (Y_5) from the experimental design factors (U and V).

Regression terms represent all seven experimental design factors (U) and two functions (V) of design factors (FAP and WCR). As was done in the table 22 regressions for X, all U and V are included as deviations from their respective means. Cross-product terms for two-factors, three-factors, and four-factors were included in the regression models in accordance with the ANOVA results that were given in table 19. Stepwise regression was used to narrow down the final regression equations to those terms that attained the 10 percent significance level.

Regressions for each dependent variable were run with both the full data set ($N = 64$) and the reduced data set ($N = 60$). Since R-squares are generally somewhat larger and SEEs are somewhat smaller for the reduced data set, the remaining discussion will relate only to those regressions for which $N = 60$. For the $N = 60$ case, R-squares range from about 83 percent for $Y_3 = ft_7AVE$ to about 97 percent for fpc_7AVE .

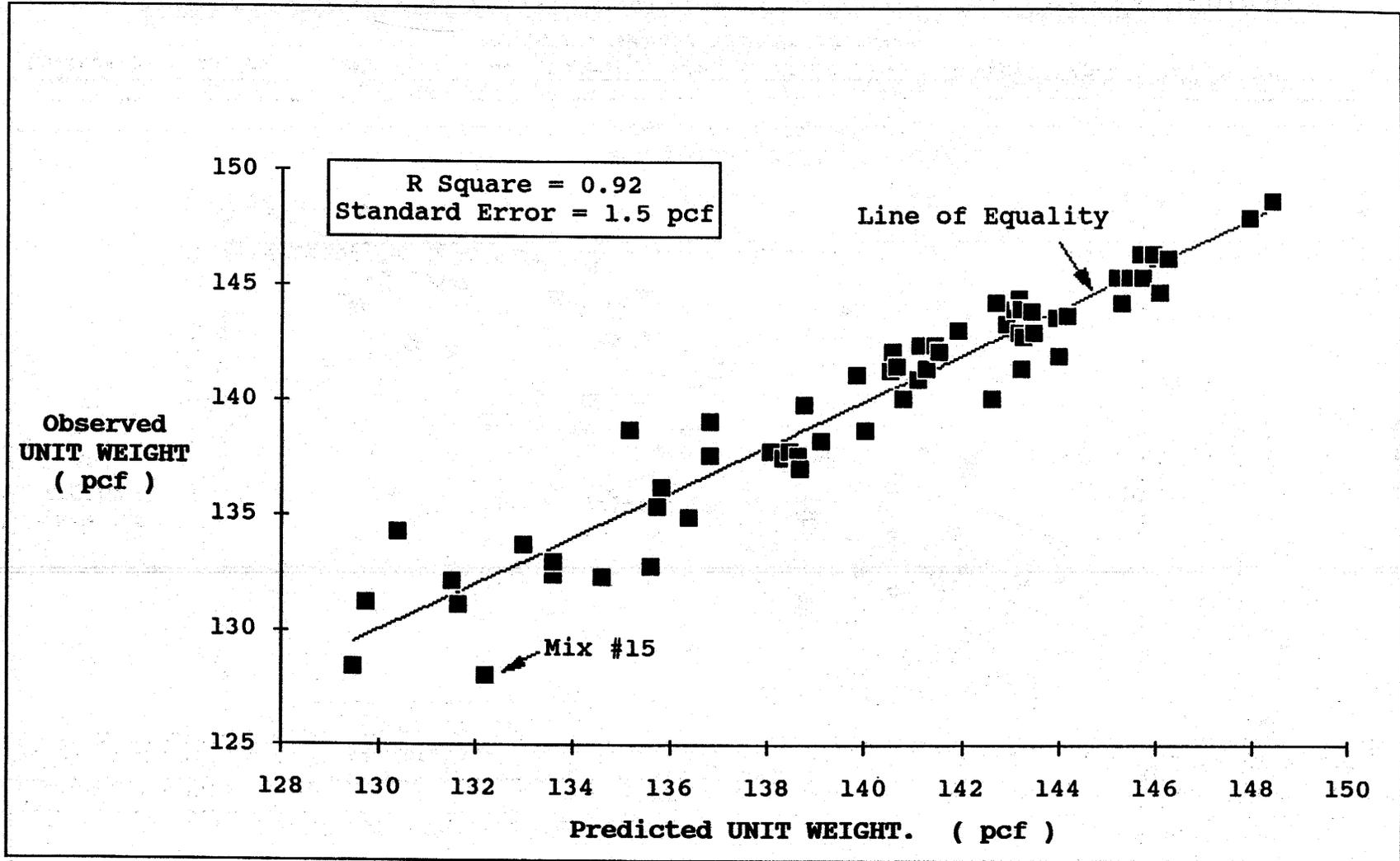
Except for flexural strength ($Y_1 = fr_7$), all SEE are within chance variation of the corresponding replicate standard deviations, and therefore give error mean square (EMS) ratio that are not statistically significant. As observed earlier, the replicate standard deviation for fr_7AVE is quite quite low [15.4 psi (1.08 kg/cm²)] and represents a coefficient of variation of only about 3 percent. If the standard deviation of fr_7AVE between replicate mixes were around 30 psi (2.1 kg/cm²), the coefficient of variation would be 30/535 or about 5.6 percent, and the error mean square ratio for fr_7 would be around 2.0 and would not be significant. It was believed that the assumption of a replicate standard deviation of 30 psi (2.1 kg/cm²) for fr_7AVE was not unreasonable.

With the foregoing assumption, it was concluded that none of the regression equations in table 23 show serious lack-of-fit and that all



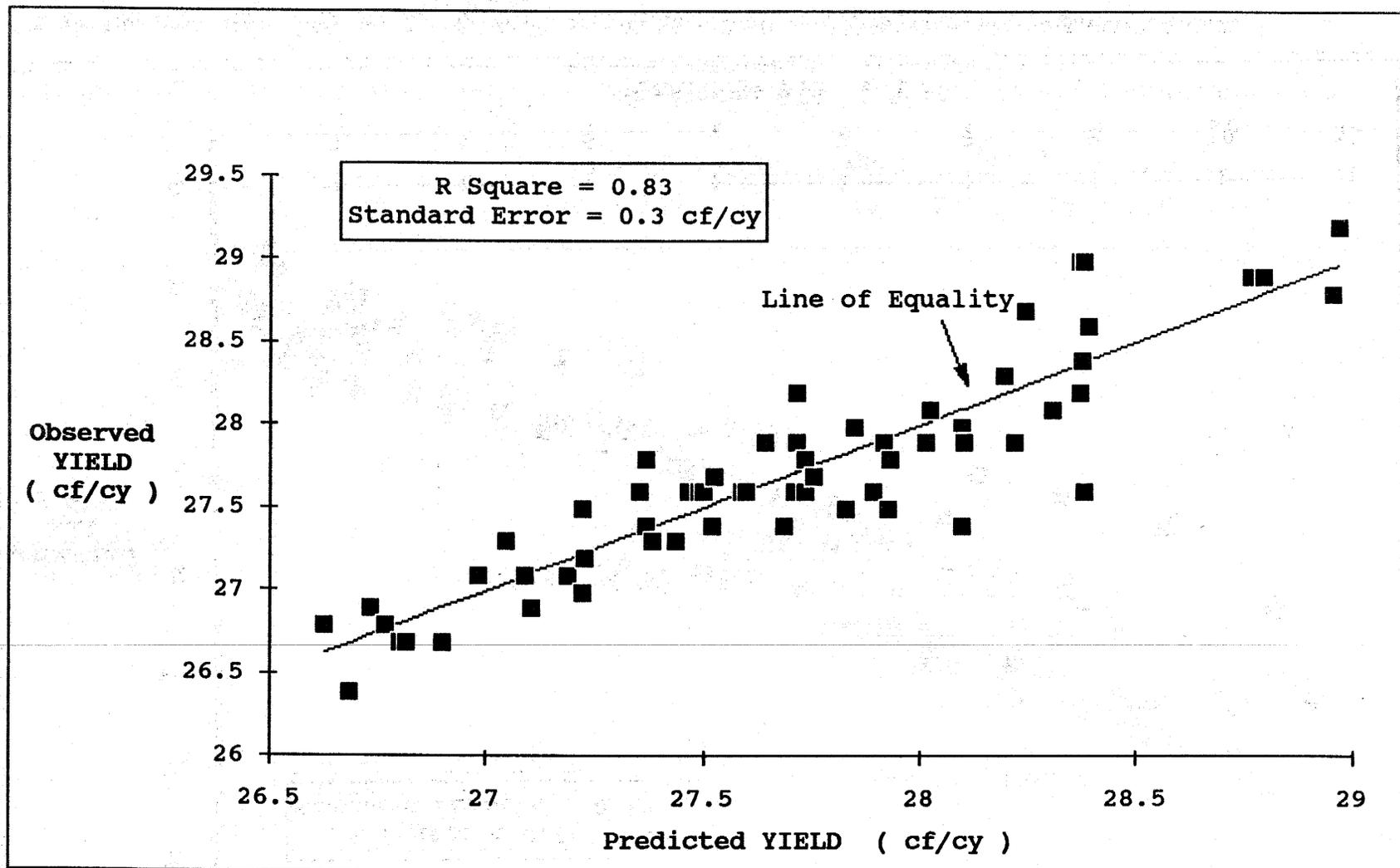
Regression equation: $SLUMP (in) = +0.77WAPD + 8.94WCRD - 0.20CATD * CAPD - 0.60FAMD * CEPD - 0.14AEQD * CEPD + 0.16AEQD * WAPD - 0.03AEQD * CAPD * WAPD$

Figure 18. Plot of observed versus predicted PCC slump (Xi=SLMP), where only significant design factors (Ui) and design factor functions (Vi) were independent variables.



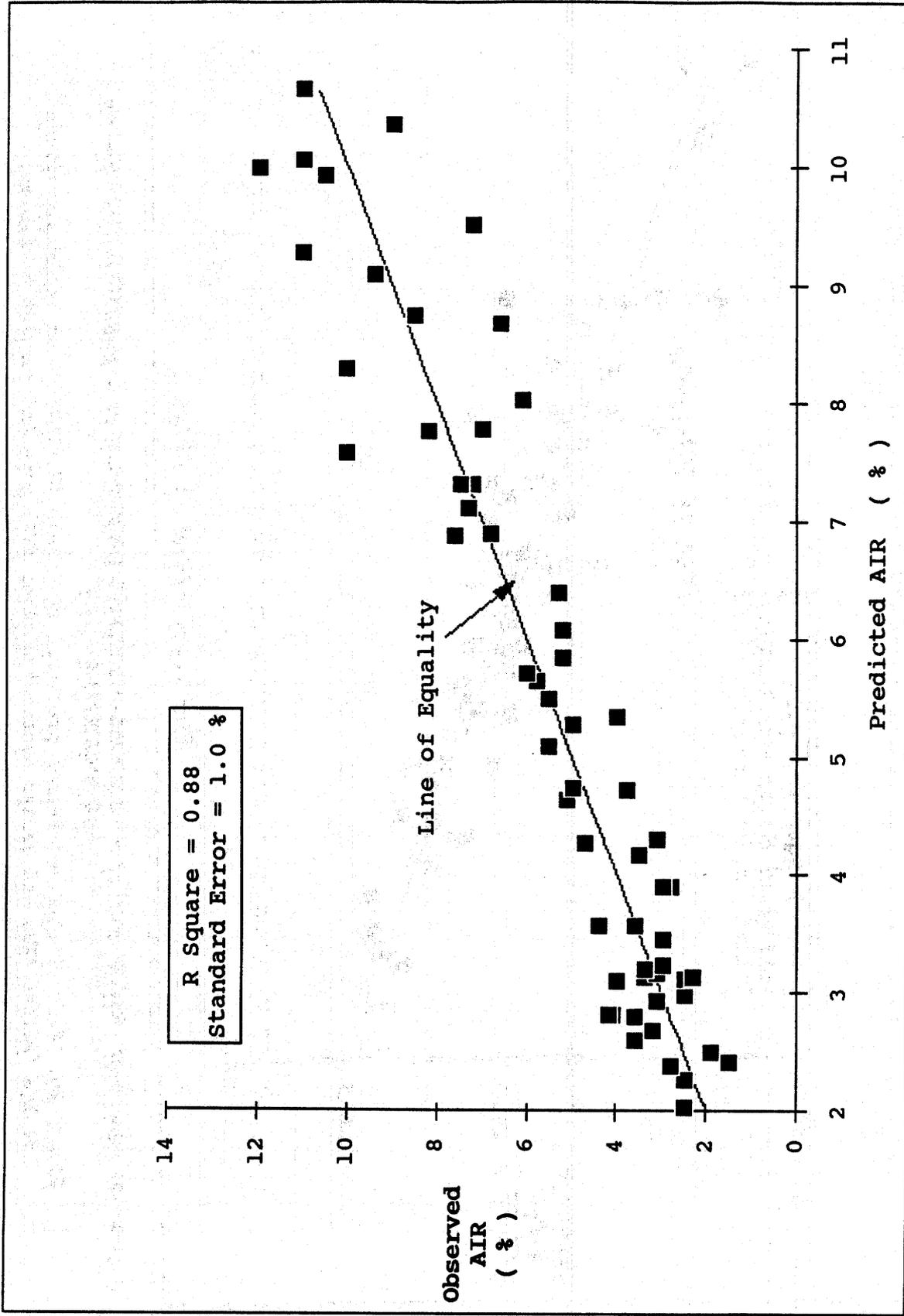
Regression equation: $\text{UNIT WEIGHT (pcf)} = -0.98\text{AEQD} + 0.33\text{CAPD} + 1.55\text{CEPD} - 0.89\text{WAPD} + 0.22\text{CATD} * \text{CAPD} + 0.21\text{AEQD} * \text{CEPD} - 0.22\text{AEQD} * \text{WAPD} - 0.08\text{CATD} * \text{FAMD} * \text{AEQD}$

Figure 19. Plot of observed versus predicted PCC unit weight (X2=UNWT), where only significant design factors (Ui) and design factor functions (Vi) were independent variables.



Regression equation: $\text{YIELD (cf/cy)} = -0.07\text{CAPD} - 0.23\text{CEPD} - 0.03\text{CATD} * \text{CAPD} - 0.04\text{AEQD} * \text{CEPD} + 0.04\text{AEQD} * \text{WAPD} + 0.01\text{CATD} * \text{FAMD} * \text{AEQD}$

Figure 20. Plot of observed versus predicted PCC yield (X3=YLD), where only significant design factors (Ui) and design factor functions (Vi) were independent variables.



Regression equation: $AIR (\%) = +10.02WCRD - 0.16CAMD * CAPD - 0.14AEQD * CEPD + 0.12AEQD * WAPD + 0.05CATD * FAMD * AEQD$

Figure 21. Plot of observed versus predicted PCC air content ($X_4=AIR$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.

Table 23. Multiple regressions for hardened properties (Y) on significant design factors (U) and factor functions (V).

Depend. Var.		Y1 = fr 7 AVE		Y2 = fpc 7 AVE		Y3 = ft 7 AVE		Y4 = fpc 28 AVE		Y5 = Ec 28 AVE	
		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)	
		N = 64	N = 60*	N = 64	N = 60	N = 64	N = 60	N = 64	N = 60	N = 64	N = 60
Design Factors (U,V)	CATD=(CAT-0.5)	-54.4(00)	-37.1(00)	-581.3(00)	-649.3(00)	-41.5(00)	-24.9(00)	734.5(00)	-661.9(00)	(46)	-.207(00)
	CAMD=(CAM-1.125)	-41.0(00)	-49.8(00)	(21)	-193.2(01)	(69)	(71)	326.2(01)	-320.7(00)	(64)	(39)
	FAMD=(FAM-2.47)	(99)	(92)	(38)	(41)	(65)	(54)	(71)	(53)	(65)	(17)
	AEQD=(AEQ-3.25)	5.2(04)	(86)	(76)	46.2(01)	(11)	-7.4(00)	(32)	(29)	(27)	.025(07)
	CAPD=(CAP-38.59)	22.9(00)	16.0(00)	161.8(00)	201.5(00)	14.1(00)	(42)	196.1(00)	164.9(00)	.151(00)	.169(00)
	CEPD=(CEP-9.29)	67.2(00)	(89)	621.1(00)	515.7(00)	43.2(00)	(71)	654.7(00)	474.6(00)	.290(00)	.280(00)
	WAPD=(WAP-14.64)	(59)	32.7(00)	-128.3(00)	(95)	(44)	(54)	(29)	(62)	(94)	(41)
	FAPD=(FAP-31.55)	22.3(00)	14.6(00)	141.1(00)	203.4(00)	12.4(00)	(71)	175.4(00)	149.1(00)	.144(00)	.175(00)
WCRD=(WCR-0.51)	(52)	1179.8(00)	(70)	1864.1(00)	(40)	-520.7(00)	(28)	-2859(00)	(92)	(37)	
Mix Prp (X)	SLMPD=(SLMP-2.03)										
	UNWTD=(UNWT-140.0)										
	YLDD=(YLD-27.74)										
	AIRD=(AIR-5.59)										
Two Variable Interactions	CATD • CAMD	(23)	(50)	(68)	(83)	(43)	(68)	(89)	(83)	(18)	.279(05)
	CATD • FAMD	69.0(02)	(40)	(32)	(17)	(78)	-78.5(00)	(65)	-334.4(10)	(80)	(50)
	CATD • CAPD	(12)	(14)	37.0(08)	(41)	(18)	(37)	(67)	(66)	.035(03)	(32)
	CATD • CEPD	(90)	(92)	-195.5(00)	-235.1(00)	(20)	-16.4(01)	204.8(00)	-262.9(00)	0.72(07)	(31)
	CAMD • AEQD	(72)	(52)	(17)	(76)	(17)	(91)	71.2(07)	(35)	(29)	(87)
	CAMD • CEPD	(15)	-21.0(02)	(43)	(80)	(21)	(85)	(75)	(80)	(54)	(39)
	FAMD • AEQD	-12.8(00)	-10.9(01)	(38)	-56.5(02)	(66)	-7.2(06)	(17)	-81.0(01)	(44)	(63)
	FAMD • CAPD	-7.0(06)	-6.7(07)	-59.3(03)	-38.0(06)	(46)	(33)	(19)	(72)	-0.47(03)	(34)
	FAMD • WAPD	(56)	(58)	(85)	(51)	(21)	(94)	(56)	(98)	-1.73(04)	(41)
	AEQD • CAPD	-1.1(02)	-1.0(02)	-7.8(01)	-7.9(00)	(68)	(89)	-12.6(00)	-10.8(00)	-0.004(07)	(30)
	AEQD • WAPD	(68)	(55)	(20)	(36)	(61)	(97)	28.1(06)	(18)	(68)	(96)
	CAPD • WAPD	-3.0(02)	(13)	-28.5(00)	-18.4(01)	-2.7(05)	(59)	-37.3(00)	-18.2(04)	-0.019(01)	(20)
CEPD • WAPD	7.7(03)	(52)	(77)	(50)	(91)	(91)	(83)	(51)	(95)	(46)	
SLMPD • UNWTD											
Three & Four Varia. Interactions	CATD • FAMD • CAPD	(72)	(53)	(28)	(16)	(85)	(71)	(20)	(37)	.016(00)	(22)
	CATD • CEPD • WAPD	(55)	(44)	(39)	(98)	(12)	(42)	(68)	(23)	(91)	.059(09)
	FAMD • CAPD • CEPD	(22)	(35)	(75)	(42)	(12)	-0.7(00)	(83)	(21)	(38)	(81)
	CAPD • WAPD • AIRD										
CATD • CAPD • WAPD • AIRD											
Constant Term		536.3(00)	540.7(00)	3588.6	3579.8	383.1	381.1	4403.9	4410.0	4.330	4.340
Regression Summary	Total SS	883024	805441	81.1E6	65.1E6	518053	347969	99.4E6	76.8E6	29.68	19.83
	Reg. SS / df	877875/12	732920/10	76.0E6/10	63.0E6/13	395963/5	258064/7	91.8/6/10	73.3E6/11	26.57/10	17.63/7
	R - Square	.912	.910	.937	.968	.764	.828	.924	.954	.895	.889
	Res. SS / df	85148/51	72521/49	5.1E6/53	2.1E6/46	122090/58	59895/52	7.66/53	3.6E6/48	3.12/53	2.20/52
	SEE	40.9	38.5	310.0	212.5	45.9	33.9	377.5	272.1	.242	.206
	Rep. SD / df	15.4 psi / 8		247 psi / 8		31.8 psi / 8		349 psi / 8		0.163 psi / 8	
	EMS Ratio (SL)†	7.1(00)	6.2(01)	1.58(NS)	.74(NS)	2.08(NS)	1.14(NS)	1.17(NS)	.61(NS)	2.20(NS)	1.60(NS)
Max Y Res / Max Z Res††	91/2.2	78/2.0	923/3.0	350/1.6	190/4.1	80/2.4	1076/2.9	441/1.6	0.5M/2.0	.4M/2.2	
Min Y Res / Min Z Res	-94.9/-2.3	-88/-2.3	-990/-32	-491/-2.3	-79/-1.7	-79/-2.3	-1071/-2.8	-620/-2.3	-7M/-2.8	-4M/-1.9	
Residuals Over 2.8 SEE	a. Mix No. / Z			34/3.0		64/4.1		34/2.9			
	b. " " "			37/-3.2							

* - N = 60 omits Mix Numbers 34,37,49, and 64.
(NS) - Ratio not significant at the 10% level.

$$\dagger \text{ EMS Ratio} = \left(\frac{\text{SEE}}{\text{Rep SD}} \right)^2$$

$$\dagger\dagger \text{ Z Res} = \frac{\text{Y Res}}{\text{SEE}}$$

provide adequate relationships for prediction hardened PCC properties from the experimental design factors.

The number of independent variables in the table 23 regression equations range from 7 (for ft7 and Ec28) to 13 for fpc7. These variables include individual factors and two-factor products within each equation, and two equations (for ft7 and Ec28) contain three-factor cross-products.

It was found that all nine factors ($7U_i + 2V_i$) are involved in the regression equations for flexural and compressive strengths, but that CAM, WAP and FAP did not enter the equations for tensile strength, nor did FAM and WCR enter the equations for PCC modulus.

Graphs for the residuals between observed and predicted Y values are shown in figures 22 through 26, respectively. Each graph includes the regression equation from which residuals were calculated, and the R-Square and SEE that indicate the closeness of fit.

Regressions of PCC Properties (Y) on Significant Design Factors (U,V) and Mix Properties (X)

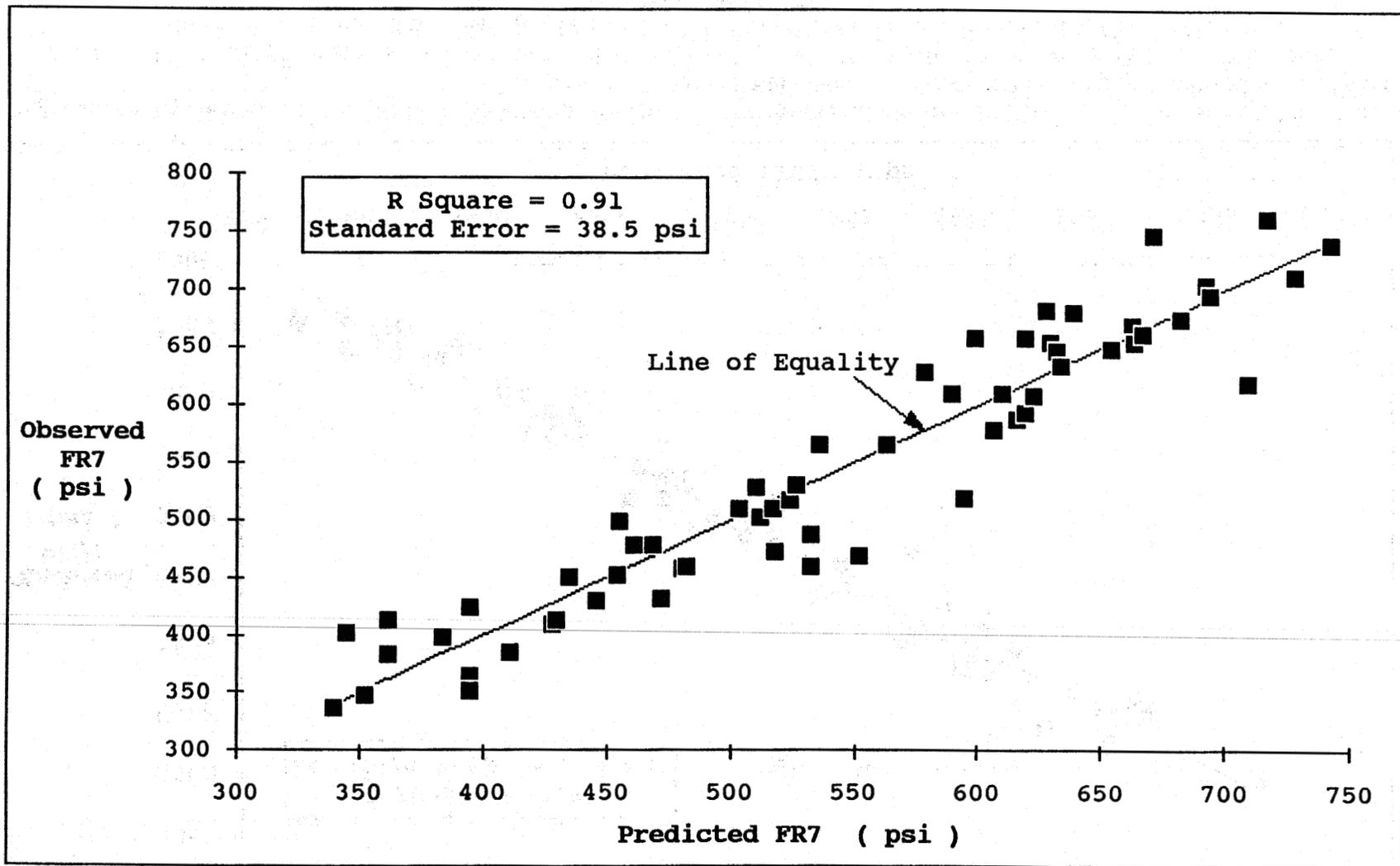
The fourth set of regression analyses produce equations for predicting the hardened PCC properties (Y) from not only the experimental design factors (U and V), but also from mix properties (X). The analytical results for these equations are shown in table 24. The U and V variables of table 23 are now augmented by the four X variables and certain cross products of U, V and X variables that were implied by significant effects in the ANOVAs for Y (table 19) and the regressions of Y on X (table 21).

As for the table 23 analyses, the independent variables were all transformed to deviations from their respective means, and regressions were run for both data sets ($N = 64$ and $N = 60$). The following discussion is for the $N = 60$ regression results.

In each of the five equations, the total number of terms remains nearly equal for the table 23 and table 24 regressions. Each table 24 equation, however, contains terms involving $X_2 = UNWT$ and $X_4 = AIR$. In addition, the prediction equation for $Y_5 = Ec28AVE$ contains $X_1 = SLMP$ as an independent variable.

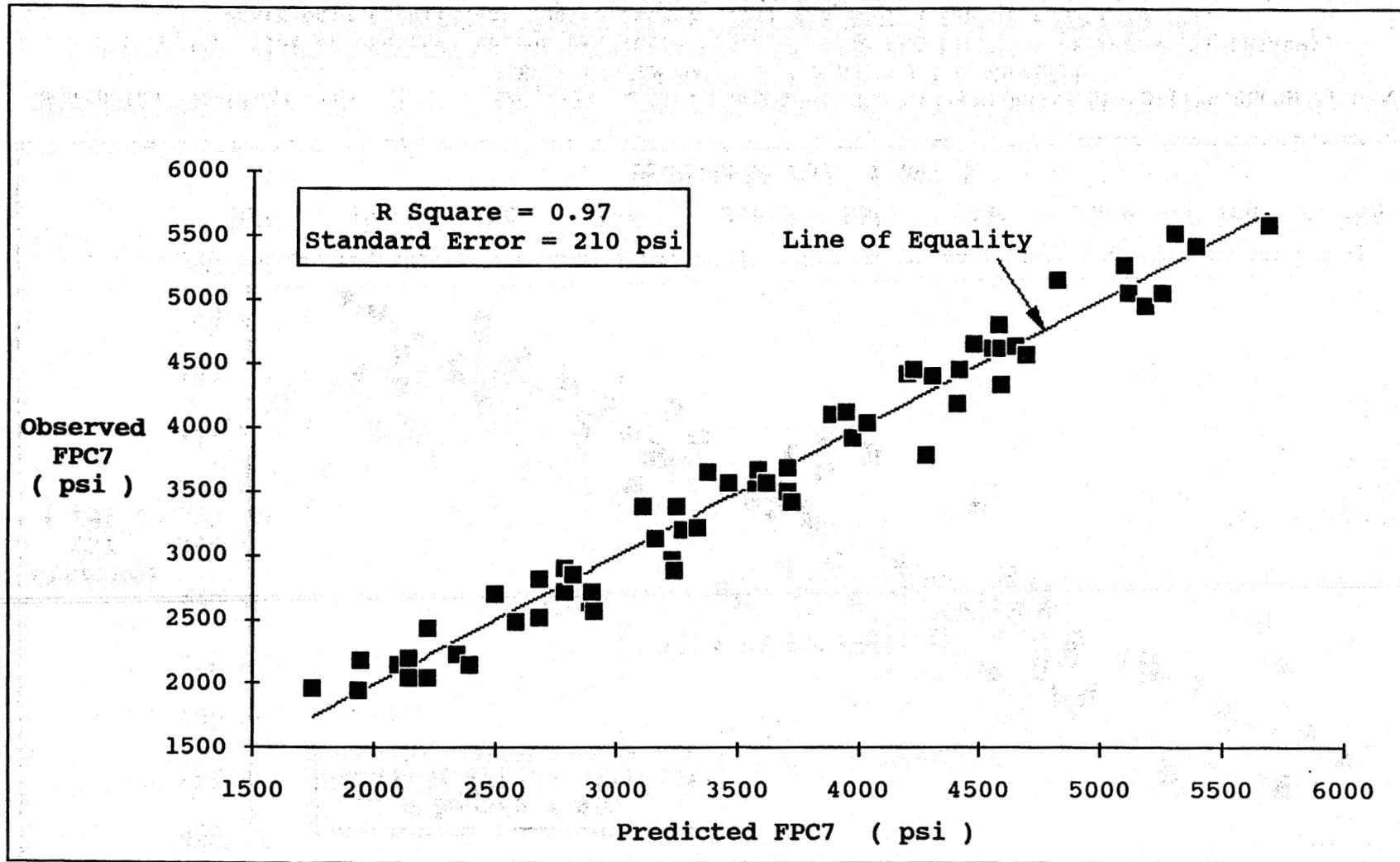
Comparison of the R-squares, SSE, and EMS ratio lines of tables 23 and 24 show that all the table 24 regression equations provide somewhat better fits to the data than were provided by the table 23 equations. R-squares in table 24 range from about 86 percent for $Y_3 = ft7$ to 98 percent for fpc7, and are about 2 percent higher than for the table 23 regressions. The SEE in table 24 are at least 10 percent less than the corresponding SEE in table 23. None of the EMS ratios in table 24 are significant, provided that 30 psi (2.1 kg/cm^2) is used as the replicate standard deviation for fr7.

It was concluded that the regression equations in table 24 are quite satisfactory secondary relationships for the prediction of hardened PCC properties from the mix design properties (U and V) and the plastic mix properties (X). Figures 27 through 31 show residuals for table 24.



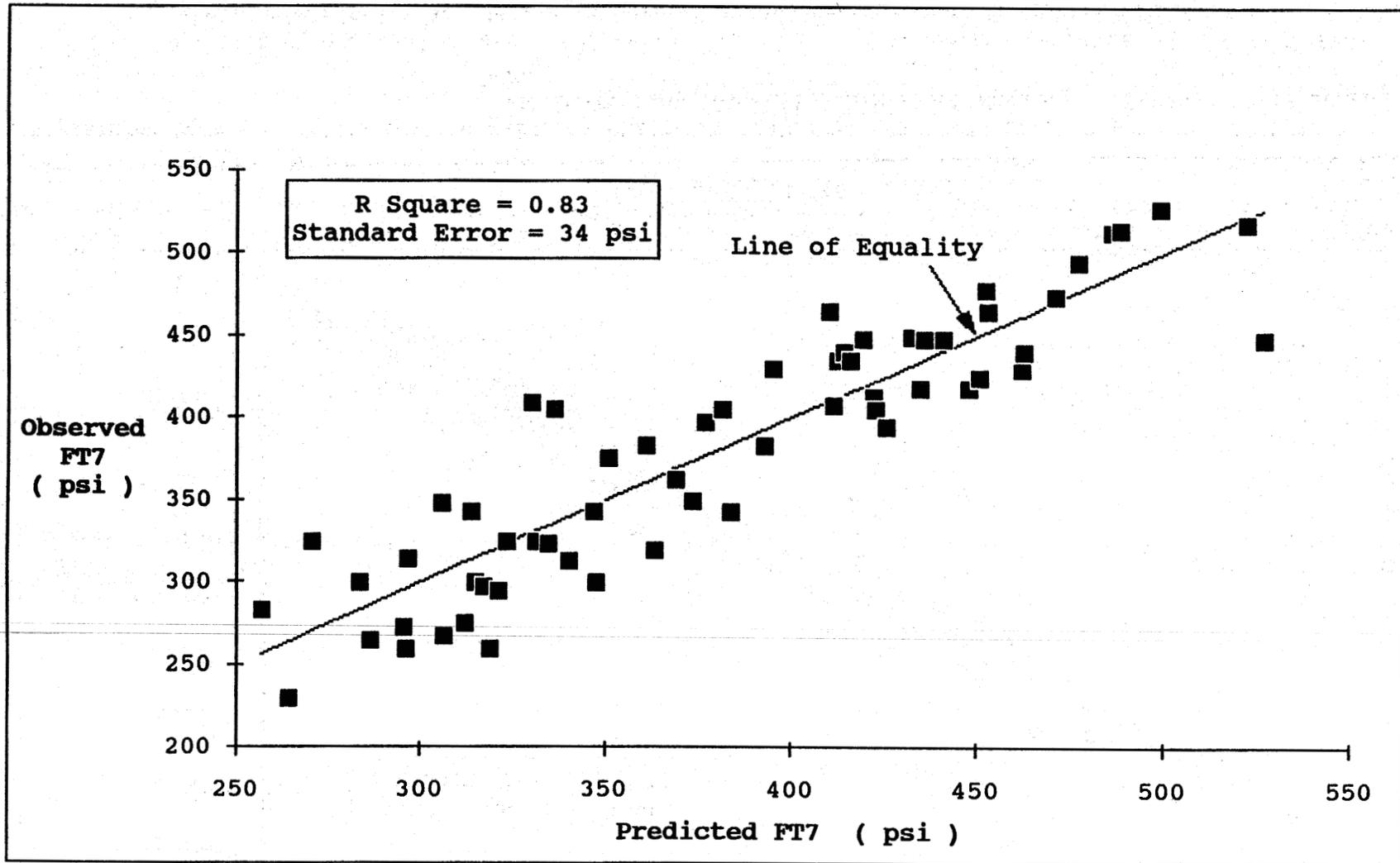
Regression Equation: $FR7 \text{ (psi)} = 540.7 - 37.1CATD - 49.8CAMD + 16.0CAPD + 32.7WAPD + 14.6FAPD + 1179.8WCRD - 21.0CAMD * CEPD - 10.9FAMD * AEQD - 6.7FAMD * CAPD - 1.0AEQD * CAPD$

Figure 22. Plot of observed versus predicted average 7-day PCC flexural strength ($Y_1=FR7AVE$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.



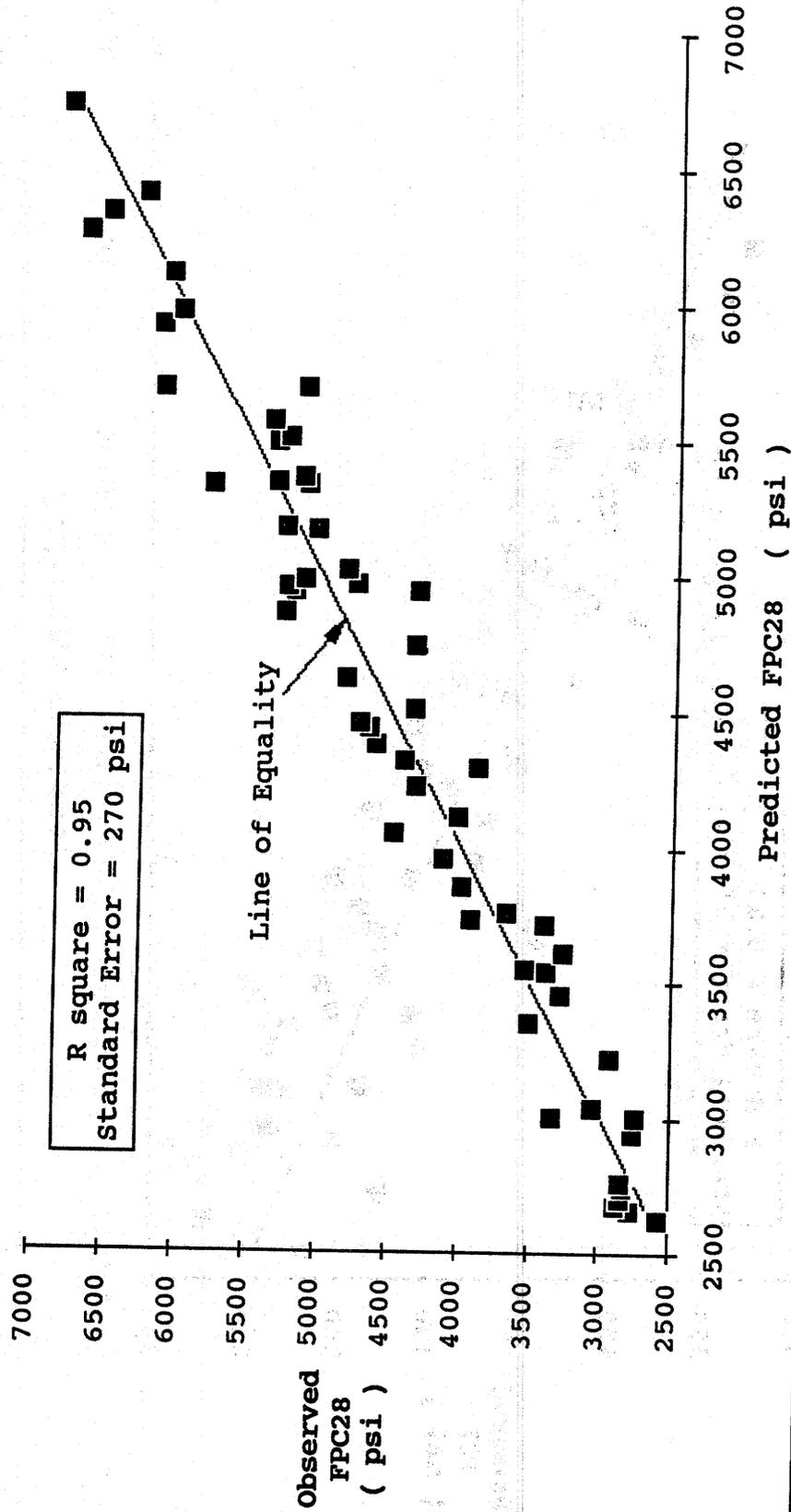
Regression Equation: $FPC7 \text{ (psi)} = 3580 - 649.3CATD - 193.2CAMD + 46.2AEQD + 201.5CAPD + 515.7CEPD + 203.4FAPD - 1864.1WCRD - 235.1CATD*CEPD - 56.5FAMD*AEQD - 38.0FAMD*CAPD - 7.9AEQD*CAPD - 18.4CAPD*WAPD$

Figure 23. Plot of observed versus predicted average 7-day PCC compressive strength ($Y_2 = FPC7AVE$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.



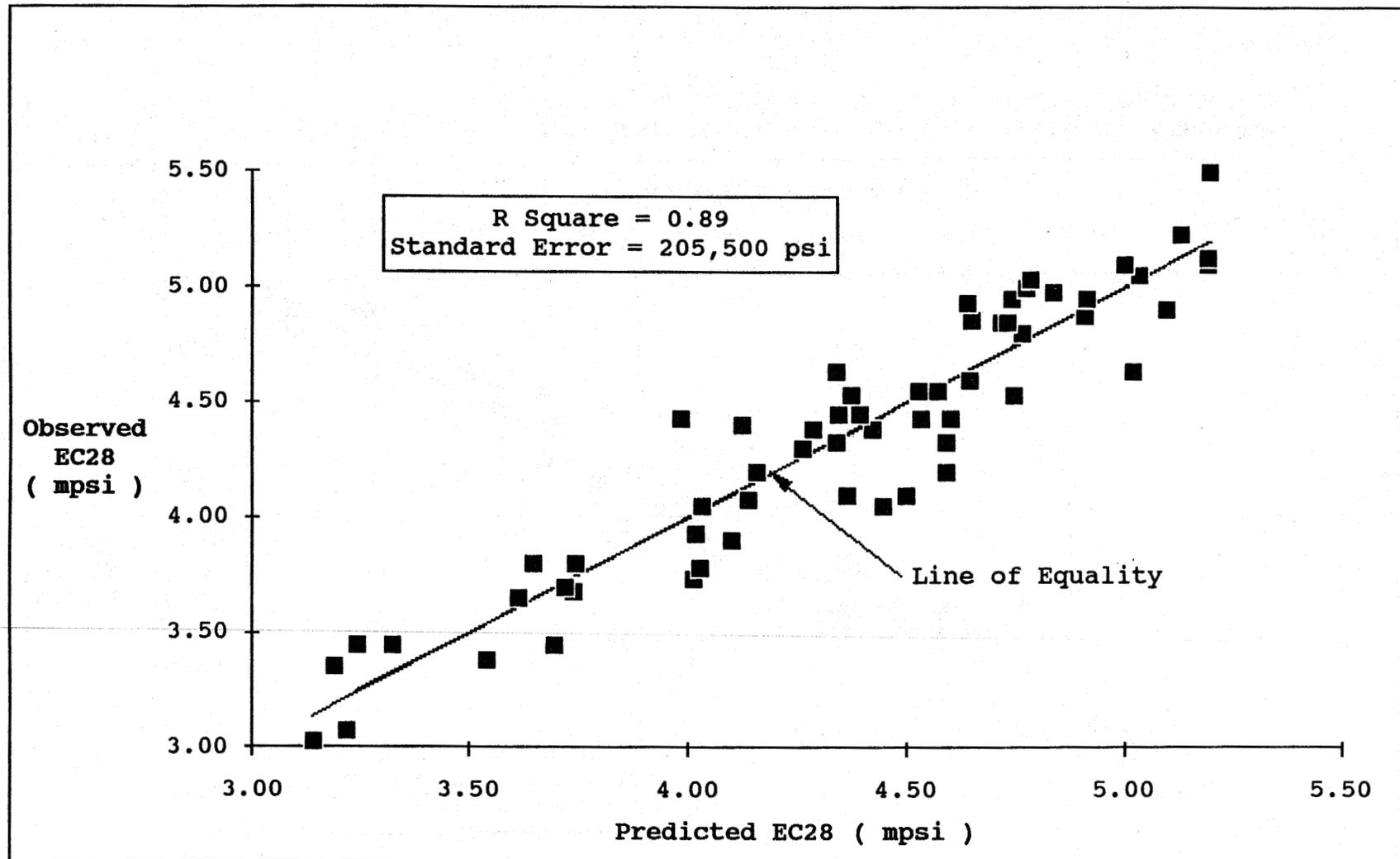
Regression Equation: $FT7 \text{ (psi)} = 381.1 - 24.9CATD - 7.4AEQD - 520.7WCRD - 78.5CATD * FAMD - 16.4CATD * CEPD - 7.2FAMD * AEQD - 0.7FAMD * CAPD * CEPD$

Figure 24. Plot of observed versus predicted average 7-day PCC tensile strength ($Y3=FT7AVE$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.



Regression Equation: $FPC28 (psi) = 4410 - 661.9CATD - 320.7CAMD + 164.9CAPD + 474.8CEPD + 149.1FAPD - 2859WCRD - 334.4CATD * FAMD - 262.9CATD * CEPD - 81.0FAMD * AEQD - 10.8AEQD * CAPD - 18.2CAPD * WAPD$

Figure 25. Plot of observed versus predicted average 28-day PCC compressive strength ($Y_4 = FPC28_{AVE}$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.



Regression Equation: $EC28 \text{ (mpsi)} = 4.340 - 0.207CATD + 0.025AEQD + 0.169CAPD + 0.280CEPD + 0.175FAPD + 0.279CATD * CAMD + 0.059CATD * CEPD * WAPD$

Figure 26. Plot of observed versus predicted average PCC elastic modulus ($Y_5=EC28AVE$), where only significant design factors (U_i) and design factor functions (V_i) were independent variables.

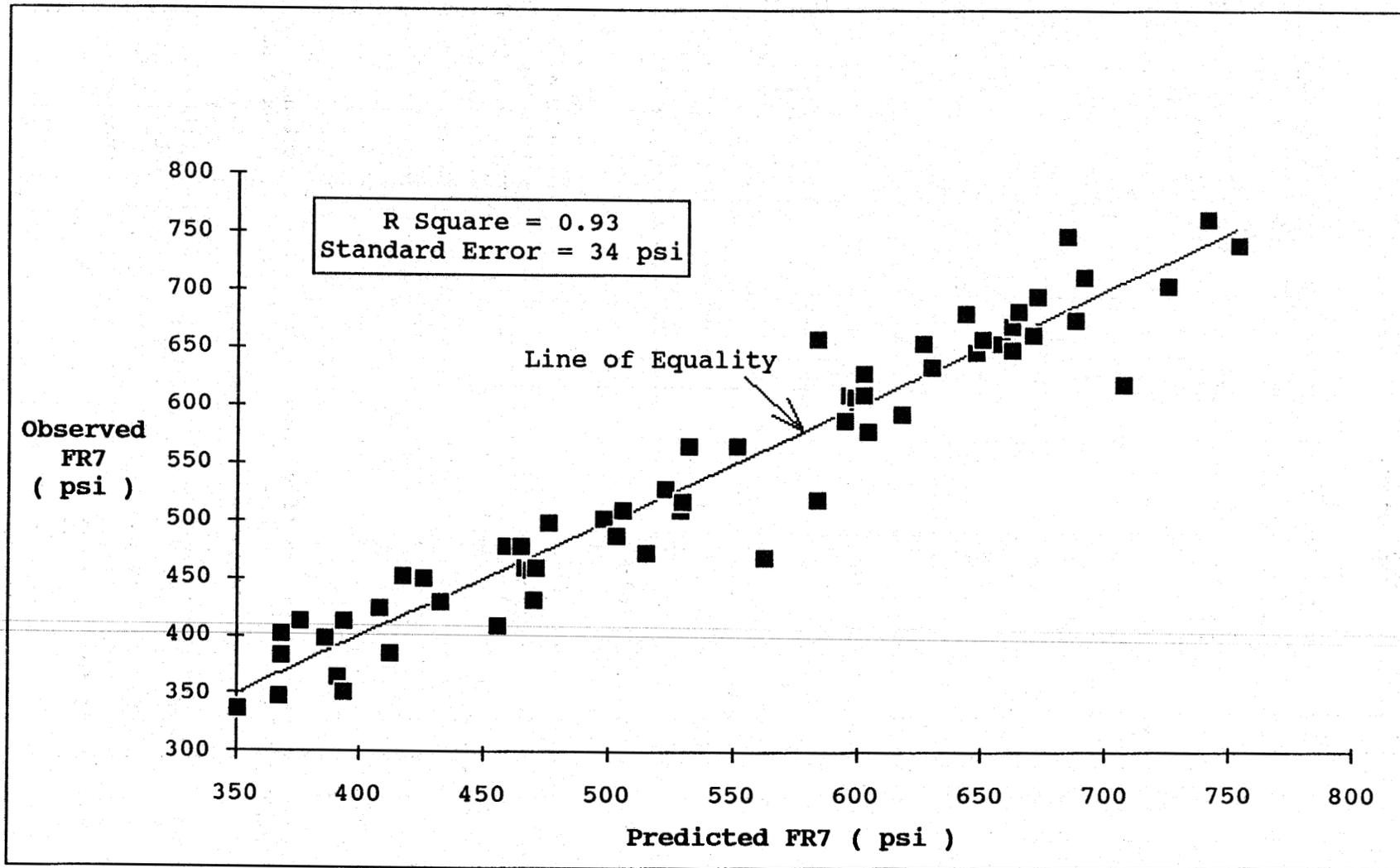
Table 24. Multiple regressions for hardened properties (Y) on significant design factors (U,V) and mix properties (X).

Depend. Var.		Y1 = fr 7 AVE		Y2 = fpc 7 AVE		Y3 = ft 7 AVE		Y4 = fpc 28 AVE		Y5 = Ec 28 AVE	
		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)		COEFF. & (SL)	
		N = 64	N = 60*	N = 64	N = 60	N = 64	N = 60	N = 64	N = 60	N = 64	N = 60
Design Factors (U,V)	CATD=(CAT-0.5)	-59.8(00)	-45.4(00)	-603.9(00)	-662.5(00)	-48.0(00)	-41.6(00)	-702.7(00)	-711.8(00)	(40)	-.25(00)
	CAMD=(CAM-1.125)	-40.4(00)	-51.0(00)	(37)	-210.5(00)	(74)	(86)	-283.1(02)	-350.1(00)	(54)	(29)
	FAMD=(FAM-2.47)	(83)	(66)	(58)	(40)	(78)	(46)	(53)	(59)	(88)	(69)
	AEQD=(AEQ-3.25)	9.3(00)	(44)	(76)	(23)	5.5(04)	(93)	(83)	(48)	(17)	(97)
	CAPD=(CAP-38.59)	(31)	(24)	(30)	(32)	(72)	(87)	(95)	17.66(05)	(43)	(77)
	CEPD=(CEP-9.29)	35.4(00)	(30)	440(00)	158.4(00)	22.3(00)	(31)	171.9(03)	226.4(00)	(91)	(86)
	WAPD=(WAP-14.64)	(56)	18.4(00)	-219.4(00)	(74)	(21)	(20)	(78)	(54)	(94)	(79)
	FAPD=(FAP-31.55)	(27)	(23)	(28)	(32)	(82)	(63)	-25.4(06)	(90)	(43)	-.01(08)
WCRD=(WCR-0.51)	(46)	-832.2(00)	(56)	-4472.3(00)	(19)	-456.2(00)	-4761.2(00)	-4029.7(00)	-2.0(00)	-1.18(00)	
Mix Prop (X)	SLMPD=(SLMP-2.03)	-5.1(06)	(47)	(32)	(15)	(31)	(50)	(92)	(47)	(85)	-.06(00)
	UNWTD=(UNWT-140.0)	15.7(00)	10.1(00)	101.0(00)	120.8(00)	11.4(00)	6.9(00)	110.6(00)	91.4(00)	.09(00)	.13(00)
	YLDD=(YLD-27.74)	(77)	(58)	(99)	(30)	(16)	(62)	(71)	(39)	(22)	(97)
	AIRD=(AIR-5.59)	(25)	(19)	(12)	55.1(05)	(88)	(75)	(31)	(35)	(27)	.09(00)
Two Variable Interactions	CATD • CAMD	(36)	(64)	(75)	(44)	(45)	(88)	(98)	(47)	(18)	(44)
	CATD • FAMD	77.8(00)	(45)	-376.9(06)	-425.3(00)	(89)	-60.1(01)	(21)	-425.4(02)	(80)	(35)
	CATD • CAPD	(24)	(12)	(36)	(53)	(25)	(65)	(86)	(69)	(12)	(59)
	CATD • CEPD	(68)	(80)	-195.1(00)	-220.0(00)	(12)	-18.7(00)	219.2(00)	-251.5(00)	.08(05)	(17)
	CAMD • AEQD	(52)	(54)	(16)	(76)	(20)	(92)	66.9(08)	(44)	(33)	(99)
	CAMD • CEPD	-13.4(10)	-24.0(00)	-152.7(02)	(28)	(25)	(96)	(23)	(95)	(14)	(26)
	FAMD • AEQD	-10.8(01)	-10.7(01)	(76)	-56.6(00)	(89)	(15)	(40)	-83.3(00)	(77)	(70)
	FAMD • CAPD	-7.2(03)	-5.3(10)	(29)	-27.3(10)	(44)	(23)	(33)	(92)	-.04(06)	(15)
	FAMD • WAPD	(69)	(18)	(21)	147.8(02)	(28)	(97)	(91)	(34)	(14)	(70)
	AEQD • CAPD	-0.9(02)	-1.0(01)	-6.7(02)	-7.7(00)	(65)	(63)	-10.9(00)	-12.0(00)	(14)	(41)
	AEQD • WCRD	49.2(01)	30.1(07)	(37)	175.9(04)	(29)	(33)	(37)	(59)	(98)	(75)
	CAPD • WAPD	-2.6(03)	(73)	(17)	(72)	-2.3(08)	(88)	-25.9(03)	(36)	-.02(05)	(88)
	CEPD • WCRD	7.8(02)	(11)	(84)	(92)	(70)	(34)	(68)	(65)	(64)	(52)
SLMPD • UNWTD	(37)	(22)	-8.1(01)	(37)	(61)	(1.0)	(24)	(69)	(49)	(85)	
Three & Four Varia. Interactions	CATD • FAMD • CAPD	(57)	(58)	(48)	(14)	(85)	(57)	(36)	(26)	.01(00)	(26)
	CATD • CEPD • WAPD	(53)	(24)	(42)	(88)	(17)	(47)	(90)	(45)	(77)	.05(08)
	FAMD • CAPD • CEPD	(20)	(24)	(61)	(49)	(27)	(12)	(66)	(38)	(38)	(44)
	CAPD • WAPD • AIRD	(39)	1.1(02)	(82)	9.2(00)	(90)	(42)	(52)	12.0(00)	(43)	(52)
CATD • CAPD • WAPD • AIRD	(42)	-2.4(01)	-28.5(00)	-18.5(00)	(25)	-1.8(02)	-21.0(01)	-16.1(01)	-.02(00)	(67)	
Constant Term		536.0(00)	543.4(00)	3543.3(00)	3607(00)	382.8(00)	382.6(00)	4423.5(00)	4425.9(00)	4.34(00)	4.36(00)
Regression Summary	Total SS	96E4	81E4	81E6	65E6	52E4	35E4	99E6	77E6	29.68	19.83
	Reg. SS / df	89E4/14	75E4/12	77E6/10	64E6/15	40E4/5	30E4/6	93E6/11	74E6/12	26.37/7	18.16/7
	R - Square	.929	.932	.948	.980	.770	.857	.932	.964	.888	.916
	Res. SS / df	6.9E4/49	5.5E4/47	4.1E6/53	1.3E6/44	11.9E4/58	5.0E4/53	6.7E6/52	2.8E6/47	3.31/56	1.66/52
	SEE	37.4 psi	34.3 psi	281.1 psi	172.5 psi	45.3 psi	30.6 psi	359.4/ psi	242.1 psi	.243	.179
	Rep.Std Dev. / df	15.4 psi / 8		247 psi / 8		31.8 psi / 8		349 psi / 8		0.163 psi / 8	
	EMS Ratio (SL)†	5.91(01)	4.93(03)	1.30(NS)	0.49(NS)	2.03(NS)	0.93(NS)	1.06(NS)	0.48(NS)	2.22(NS)	1.21(NS)
Max Y Res / Max Z Res††		57/1.5	75/2.7	833/3.0	294/1.7	179/3.9	63/2.1	1051/2.9	461/1.9	.66/2.7	.46/2.6
Min Y Res / Min Z Res		-104/-2.8	-93/-2.7	-790/-2.8	-506/-2.9	-96.3/-2.1	-68.9/-2.3	-1057/-2.9	-661/-2.7	-.51/-2.1	-.41/-2.31
Resid. Over 2.8 SEE	a. Mix No. / Z Res	34/-2.8		34/3.0	9/-2.9	64/3.9		34/2.9			
	b. " " "			37/-2.8				37/-2.9			

* - N = 60 omits Mix Numbers 34,37,49, and 64.
(NS) - Ratio not significant at the 10% level.

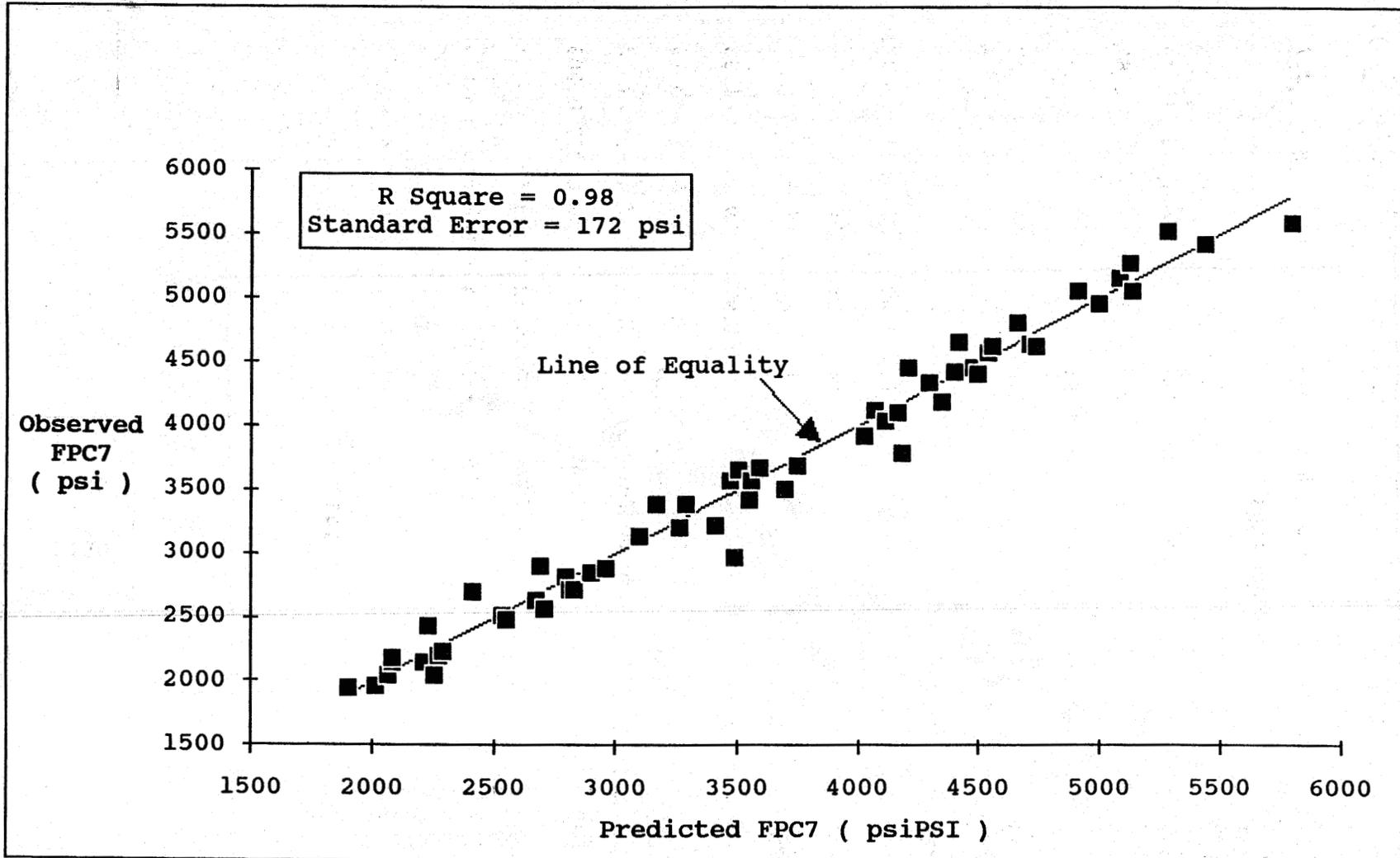
† EMS Ratio = $\left(\frac{SEE}{Rep SD}\right)^2$

†† Z Res = $\frac{Y Res}{SEE}$



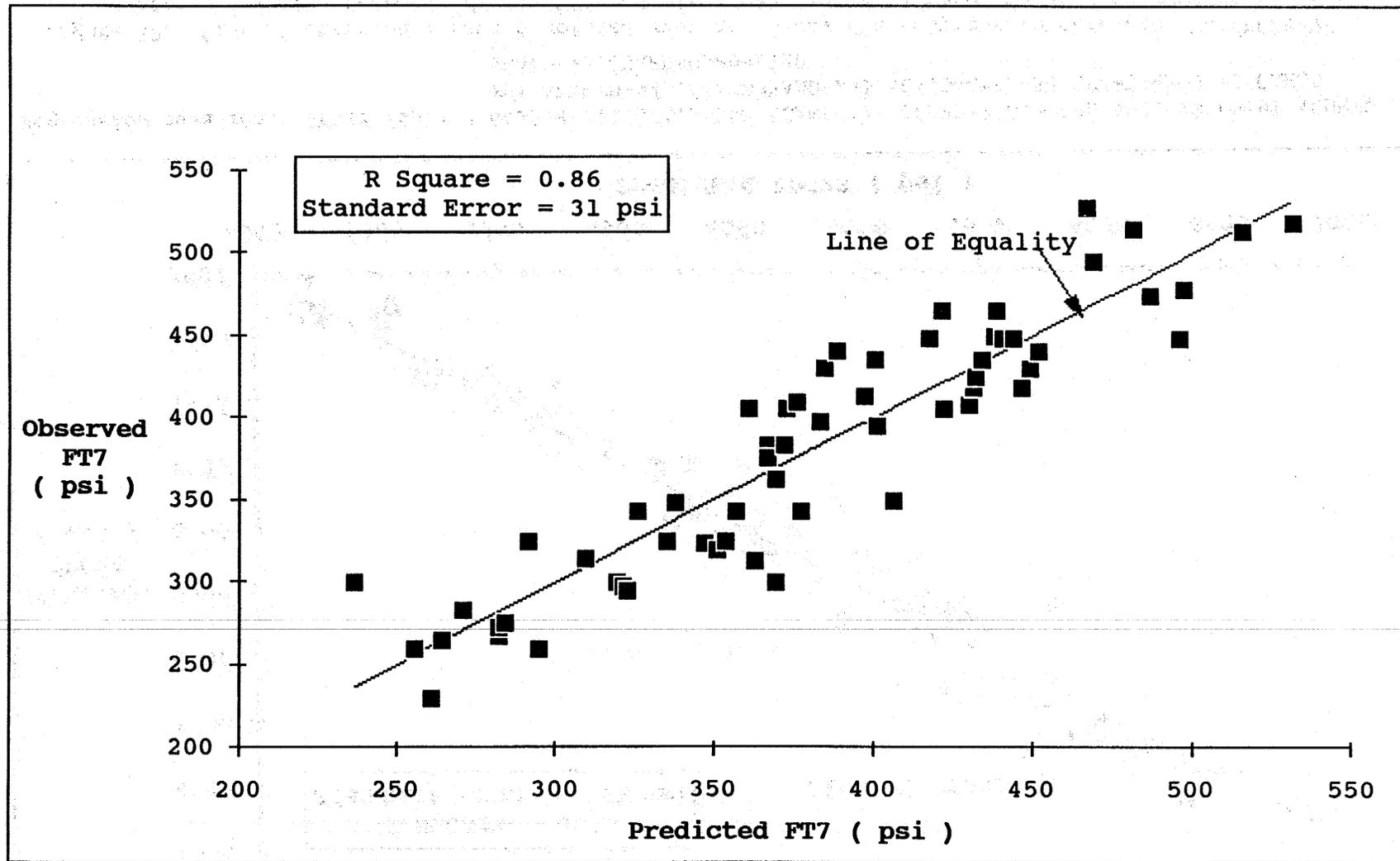
Regression equation: $FR7 \text{ (psi)} = 543.4 - 45.4CATD - 51.0CAMD + 18.4WAPD - 832.2WCRD + 01.1UNWTD - 24.0CAMD*CEPD - 10.7FAMD*AEQD - 5.3FAMD*CAPD - 1.0AEQD\&CAPD + 30.1AEQD*WCRD + 1.1CAPD *WAPD*AIRD$

Figure 27. Plot of observed versus predicted average 7-day PCC flexural strength ($Y1=FR7AVE$), where all design factors (U_i and V_i) and mix properties (X_i) were independent variables.



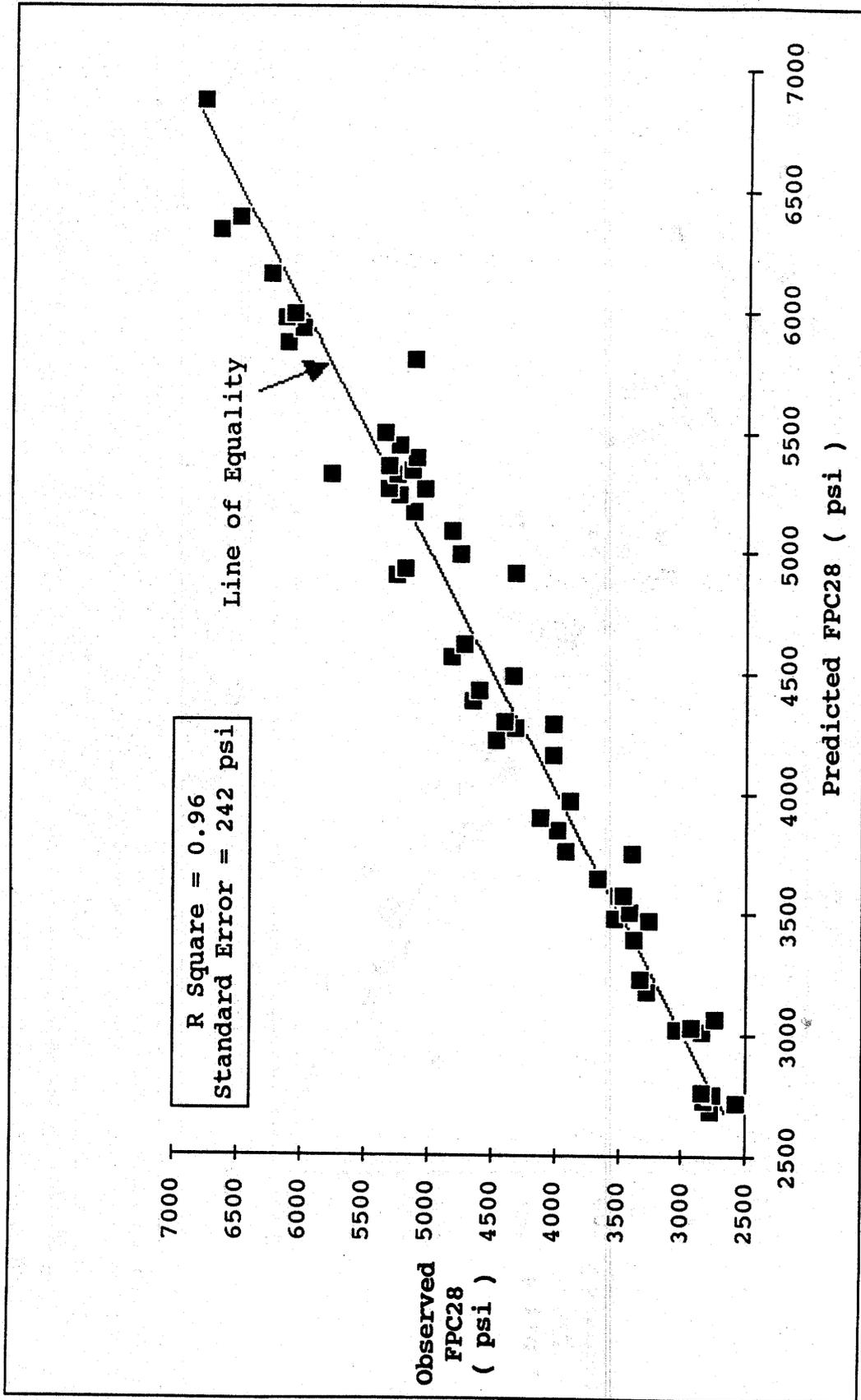
Regression equation: $FPC7 \text{ (psi)} = 3607 - 662.5CATD - 210.5CAMD + 158.4CAPD - 4472.3WCRD + 120.8UNWTD + 55.1AIRD - 425.3CATD * FAMD - 220.0CATD * CEPD - 56.6FAMD * AEQD - 27.3FAMD * CAPD + 147.8FAMD * WAPD - 7.7AEQD * CAPD + 175.9AEQD * WCRD + 9.2CAPD * WAPD * AIRD - 18.5CATD * CAPD * WAPD * AIRD$

Figure 28. Plot of observed versus predicted average 7-day PCC compressive strength ($Y_2 = FPC7AVE$), where all design factors (U_i and V_i) and mix properties (X_i) were independent variables.



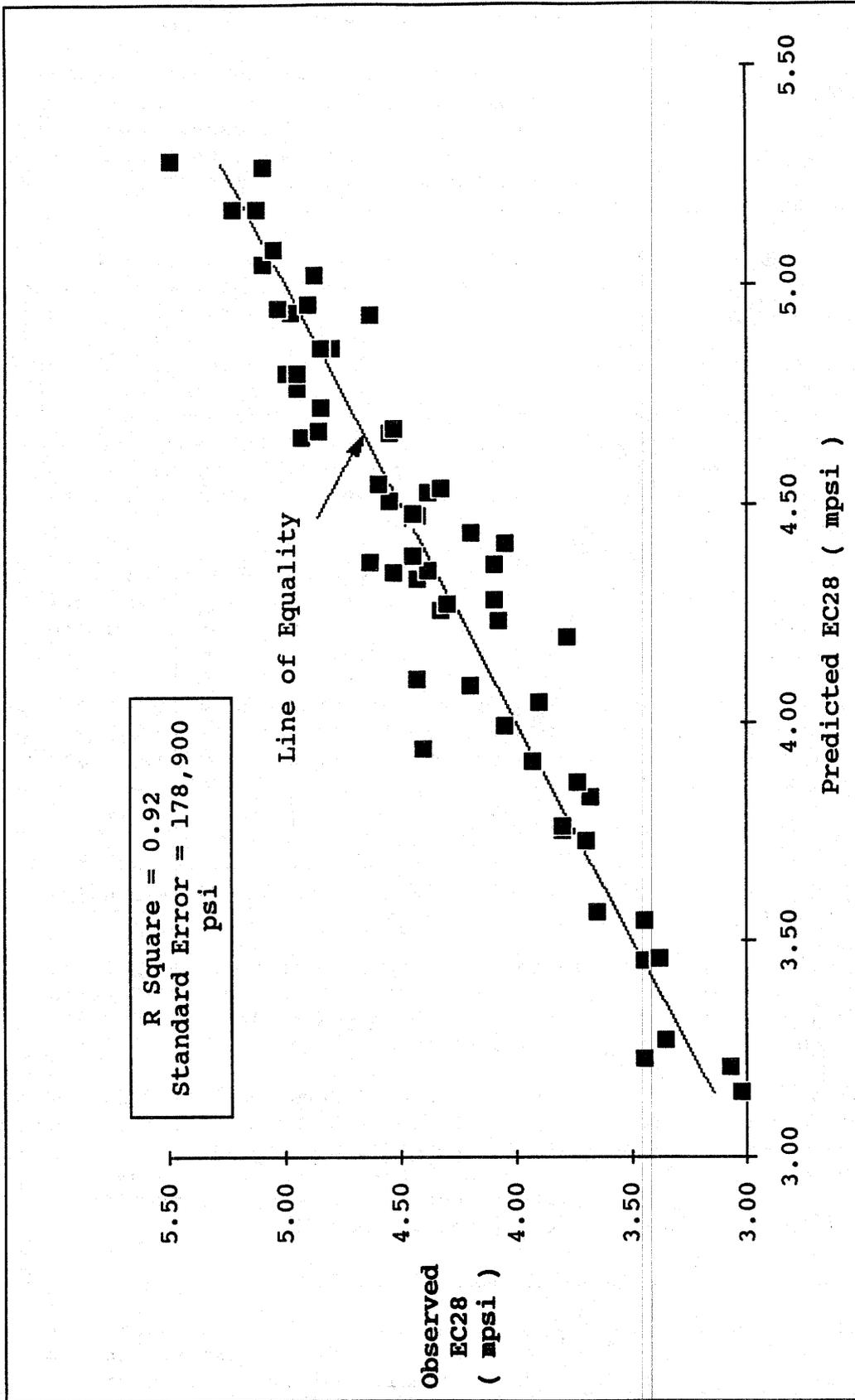
Regression equation: $FT7 \text{ (psi)} = 382.6 - 41.6CATD - 456.2INCRD + 6.9UNWTD - 60.1CATD * FAMD - 18.7CATD * CEPD - 1.8CATD * CAPD * WAPD * AIRD$

Figure 29. Plot of observed versus predicted average 7-day PCC tensile strength ($Y_3=FT7AVE$), where all design factors (U_i and V_i) and mix properties (X_i) were independent variables.



Regression equation: $FPC28 \text{ (psi)} = 4425.9 - 711.8CATD - 350.1CAMD + 17.66CAPD + 226.4CEPD - 4029.7WCED + 91.4UNWTD + 91.4UNWTD - 425.4CATD * FAMD - 251.5CATD * CEPD - 83.3FAMD * AEQD - 12.0AEQD * CAPD + 12.0CAPD * WAPD * AIRD$

Figure 30. Plot of observed versus predicted average 28-day PCC compressive strength (Y4=FPC28AVE), where all design factors (Ui and Vi) and mix properties (Xi) were independent variables.



Regression equation: $EC28 \text{ (mpsi)} = 4.36 - 0.25CATD - 0.01FAPD - 1.18WCRD - 0.06SLMPD + 0.13UNWTD + 0.09AIRD + 0.05CATD * CEPD * WAPD$

Figure 31. Plot of observed versus predicted average PCC elastic modulus ($Y5=EC28AVE$), where all design factors (U_i and V_i) and mix properties (X_i) were independent variables.

Simple Regressions for Selected Pairs of PCC Properties

The final set of regression equations that were derived are for the prediction of one PCC property (Y_i) from values of another PCC property (Y_j). Although each two-way ($Y_i Y_j$) graph (previously discussed) was accompanied by a simple regression equation, all were based on the complete data set of $N = 64$ mixes. Regression results in this section are given in table 25 and are based on the reduced data set ($N = 60$).

The first three regressions in table 25 are for the prediction of $Y_1 = fr_7$ from $Y_2 = fpc_7$. Three different models were used, first with Y_1 versus Y_2 , then with Y_1 versus $\sqrt{Y_2}$, and finally with $\log Y_1$ versus $\log Y_2$. The square root transformation was used to provide comparisons with secondary relationships A and E in appendix C, both of which express modulus of rupture as a constant (7.5 or 9.5) times the square root of compressive strength. Table 25 shows that the study data gave the constant 9.2 as a multiplier of $\sqrt{fpc_7}$. Table 25 values for R-square, SEE, and extreme residuals show that the log transformations for Y_1 and Y_2 give the best fit to the study data, and that the untransformed Y_1 and Y_2 produce a somewhat better fit than the square root transformation.

The fourth and fifth regressions for Y_1 show how well fr_7 can be predicted from $Y_3 = ft_7$ and from $Y_4 = fpc_{28}$. The results show that fpc_{28} is a much better predictor of fr_7 than is ft_7 . It can also be seen that fpc_7 is a somewhat better predictor of fr_7 than is fpc_{28} .

The final Y_1 equation in table 25 is for the prediction of $Y_1^* = fr_{28}$ from $Y_1 = fr_7$. Since the laboratory study did not include 28-day tests of flexural strength, the Y_1^* versus Y_1 relationship has been assumed to be the same as for converting fpc_7 to fpc_{28} .

The next equation is for predicting $Y_4 = fpc_{28}$ from $Y_2 = fpc_7$. This relationship is very close ($R^2 = 0.942$), and shows that 28-day compressive strength at 28 days is about 122 percent of the corresponding 7-day compressive strength.

The next pair of equations are for predicting tensile strength from either flexural strength (Y_3 versus Y_1) or compressive strength (Y_3 versus Y_2). It is apparent that compressive strength is considerably better than flexural strength for predicting tensile strength.

The final set of regressions are for the prediction of $Y_5 = Ec_{28}$ from $Y_1 = fr_7$, from $Y_2 = fpc_7$, from $Y_3 = ft_7$, from $Y_4 = fpc_{28}$, and finally from the combination of fpc_{28} and $X_2 = UNWT$. In the last case, log transformations were used for all three variables. The results show that fr_7 is the best single predictor of Ec_{28} , but that much better predictions for Ec_{28} are given by the combination of fpc_{28} and $UNWT$.

Sensitivity Analyses for Selected Dependent Variables

Forty-six regression equations have been derived from the laboratory data and appear in tables 21 through 23. Three of these equations have been selected from table 22 for analysis of the sensitivity of mix properties (X) to the design factors (U, V), and three equations have been selected from

table 24 for analysis of the sensitivity of PCC properties (Y) to design factors (U, V) and mix properties (X). The six selected equations are reproduced in the right-hand columns of table 26. All six equations are based on N = 60 mixes.

The three selected mix properties are X1 = SLMP, X2 = UNWT, and X4 = AIR. Slump is an important indicator of concrete workability and air content is a valid predictor of PCC durability. Both are included as PCC specifications variables by 9 of the 12 States represented in table 4. Regression equations in table 24 show that unit weight is an important predictor of PCC strength; it may be the simplest and best strength indicator that can be evaluated from the plastic PCC mix, but only 3 of 12 States include unit weight in their current specifications.

The three selected PCC properties in table 26 are Y1 = fr7, Y2 = fpc7, and Y5 = Ec28. Flexural strength (fr) is a primary predictor of PCC performance, not only in the AASHTO rigid pavement design equation, but also in all mechanistic models that contain the PCC stress/strength ratio. The PCC modulus (Ec) is also a primary element of stress and performance predictions. Compressive strength (fpc) is not only highly correlated with flexural strength and PCC modulus, but is relatively easy to evaluate from samples taken during construction. Table 4 shows that compressive strength is a specifications variable in 10 of 12 States, and that flexural strength specifications are used in the other 2 States.

For the foregoing reasons, it is believed that the six dependent variables that have been selected for sensitivity analysis in table 26 are of foremost importance in the development of M&C acceptance plans and performance-related specifications for rigid pavements.

All independent variables for each regression equation in table 26 are expressed as deviations from their respective means, as shown in the second column of the table. Thus, the mean value for each deviation variable is zero. Minimum and maximum values for the deviation variables are shown in the third and fourth columns. Since predictions for any dependent variable are calculated by summing the products of regression coefficients times the independent variable deviations, the max/min values provide general indications of the amount of change that the independent variables (and their cross-products) can produce in the dependent variable predictions.

More specific sensitivity analyses are shown in table 27 for the SLMP, UNWT, and AIR prediction equations, and in table 28 for the fr7, fpc7, and Ec28 prediction equations. The remainder of this section describes the methods used to produce the tables and the sensitivity results that are contained in the tables.

Each of the seven experimental design factors (U1 = CAT through U7 = WAP) was run at two levels in the laboratory study. For example, levels for U1 = CAT were gravel and stone, levels for U2 = CAM were 0.75 in (19.05 mm) and 1.50 in (38.1 mm), levels for U6 = CEP were 7.7 percent and 10.9 percent, and levels for U7 = WAP were 13.6 percent and 15.6 percent. The four combinations of CEP and WAP levels produce four different levels for U2 = WCR, namely, 0.40, 0.46, 0.55, and 0.63. Based on the ANOVA means for PCC strength (fr7, fpc7, and fpc28 in table 20), one level of each U factor was

Table 26. Inputs for sensitivity analysis of selected dependent variables.

REGRESSION VARIABLES				TABLE N EQUATIONS			TABLE P EQUATIONS		
CODE	DEFINITIONS	MINIMUM VALUE	MAXIMUM VALUE	X1 = SLMP	X2 = UNWT	X4 = AIR	Y1 = fr7	Y2 = fpc7	Y5 = Ec28
U1	CATD = (CAT - 0.5)	-0.50	+0.50		1.54		-45.4	-662.5	-250
U2	CAMD = (CAM - 1.125)	-0.38	+0.38				-51.0	-210.5	
U3	FAMD = (FAM - 2.47)	-0.37	+0.37		2.64				
U4	AEQD = (AEQ - 3.25)	-3.25	+3.25	.42	-.98	.64			
U5	CAPD = (CAP - 38.6)	-3.8	+3.8	.09	.33	-.13			
U6	CEPD = (CEP - 9.3)	-1.6	+1.6		1.55			158.4	
U7	WAPD = (WAP - 14.6)	-1.0	+1.0	.77	-.89		18.4		
V2	WCRD = (WCR - 0.51)	-0.11	+0.12	8.94		10.02	-832.2	-4472	-1.18
X1	SLMPD = (SLMP - 2.03)	-2.0	+6.7				10.1	120.8	-0.06
X2	UNWTD = (UNWT - 140)	-12.0	+8.8					55.1	0.13
X4	AIRD = (AIR - 5.6)	-3.9	+6.6						0.09
U1 U3	CATD-FAMD	-0.19	+0.19					-425.3	
U1 U5	CATD-CAPD	-0.19	+0.19	-0.20	.22				
U1 U6	CATD-CEPD	-0.8	+0.8					-220.0	
U2 U5	CAMD-CAPD	-1.4	+1.4			-.16			
U2 U6	CAMD-CEPD	-0.6	+0.6				-24.0		
U3 U4	FAMD-AEQD	-1.2	+1.2				-10.7	-56.6	
U3 U5	FAMD-CAPD	-1.4	+1.4				-5.3	-27.3	
U3 U6	FAMD-CEPD	-0.6	+0.6	-0.60					
U3 U7	FAMD-WAPD	-0.4	+0.4					147.8	
U4 U5	AEQD-CAPD	-12.4	+12.4				-1.0	-7.7	
U4 U6	AEQD-CEPD	-5.2	+5.2	-0.14	.21	-.14			
U4 U7	AEQD-WAPD	-3.2	+3.2	.16	-.22	.12			
U4 V2	AEQD-WCRD	-0.39	+0.39				30.1	175.9	
U1 U3 U4	CATD-FAMD-AEQD	-0.60	+0.60		-0.08	.05			
U4 U5 U7	AEQD-CAPD-WAPD	-12.4	+12.4	-0.03					
U5 U7 X4	CAPD-WAPD-AIRD	-24.3	+24.3				1.1	9.2	
U1 U5 U7 X4	CATD-CAPD-WAPD-AIRD	-12.5	+12.5				-2.4	-18.5	
MEAN VALUE = EQUATION CONSTANT				2.09	140	5.37	543.4	3607	4.36
R - Square				.70	.92	.88	.93	.98	.92
Standard Error of Estimate				1.5 in.	1.5 pcf	1.0%	34 psi	172 psi	.18 mpsi
Error Mean Square Ratio				9.5	1.3	3.5	4.9	0.5	1.2
Significance Level of EMS Ratio				(00)	(NS)	(05)	(03)	(NS)	(NS)

Table 27. Sensitivity of mix properties to changes in design factors.

EXPERIMENT DESIGN FACTOR LEVELS AND CHANGES	X1. SLMP (in)		X2. UNWT (pcf)		X4. AIR (%)	
	SEE = 1.5 in Value	Change	SEE = 1.5 pcf Value	Change	SEE = 1.0 % Value	Change
ALL U, V AT LOW-STR LEVELS	6.6 in		130 pcf		10.5%	
U1. CAT Gravel to Stone	5.8	-.8	129	-1	10.5	0
U2. CAM 1.50" to 0.75"	6.6	0	130	0	10.0	-.5
U3. FAM 2.10 to 2.84	7.3	+.7	132	+2*	10.5	0
U4. AEQ 6.5 oz to 0 oz	0.6	-6.0**	140	+10***	4.1	-6.4***
U5. CAP 34.8% to 42.4%	5.8	-.8	133	+3**	9.0	-1.5*
U6. CEP 7.7% to 10.9% (& WCR .63 to .46)	4.3	-2.3*	137	+7***	7.3	-3.2***
U7. WAP 15.6% to 13.6% (& WCR .63 to .55)	2.6	-4.0**	133	+3**	8.9	-1.6*
V2. WCR .63 to .40 (& CEP & WAP changes)	0.5	-6.1**	140	+10***	5.9	-4.6***
ALL U, V AT MID-STR LEVELS	2.1 in	-4.5	140 pcf	+10	5.4%	-5.1%
U1. CAT mid to Gravel	2.1	0	141	+1	5.4	0
CAT mid to Stone	2.1	0	139	-1	5.4	0
U2. CAM 1.125" to 1.50"	2.1	0	140	0	5.4	0
CAM 1.125" to 0.75"	2.1	0	140	0	5.4	0
U3. FAM 2.47 to 2.10	2.1	0	139	-1	5.4	0
FAM 2.47 to 2.84	2.1	0	141	+1	5.4	0
U4. AEQ 3.25 oz to 6.5 oz	3.5	+1.4	137	-3**	7.5	2.1**
AEQ 3.25 oz to 0 oz	0.7	-1.4	143	+3**	3.3	-2.1**
U5. CAP 38.6% to 34.8%	1.7	-.4	139	-1	5.9	+.5
CAP 38.6% to 42.4%	2.4	+.3	141	+1	4.9	-.5
U6. CEP 9.3% to 7.7% (& WCR .51 to .60)	2.9	+.8	138	-2*	6.3	+.9
CEP 9.3% to 10.9% (& WCR .51 to .42)	1.3	-.8	142	+2*	4.5	-.9
U7. WAP 14.6% to 15.6% (& WCR .51 to .53)	3.0	+.9	139	-1	5.6	+.2
WAP 14.6% to 13.6% (& WCR .51 to .46)	0.9	-1.2	141	+1	4.9	-.5
V2. WCR .51 to .63 (& CEP & WAP changes)	3.9	+1.8*	138	-2*	6.6	+1.2*
WCR .51 to .40 (& CEP & WAP changes)	0.3	-1.8*	142	+2*	4.3	-1.1*
ALL U, V AT HI-STR LEVELS	0.2 in	-1.9	146 pcf	+6	3.1%	-2.3%
U1. CAT Stone to Gravel	0	-.2	148	+2	3.0	-.1
U2. CAM .75" to 1.5"	0.2	0	146	0	2.6	-.5
U3. FAM 2.84 to 2.10	0.9	+.7	144	-2*	3.0	-.1
U4. AEQ 0 oz to 6.5 oz	0.5	+.3	143	-3**	4.9	+1.8*
U5. CAP 42.4% to 34.8%	0	-.2	138	-8***	3.6	+.5
U6. CEP 10.9% to 7.7% (& WCR .40 to .55)	0.9	+.7	143	-3**	3.2	+.1
U7. WAP 13.6% to 15.6% (& WCR .40 to .46)	2.0	+1.8*	145	-1	2.9	-.2
V2. WCR .40 to .63 (& CEP & WAP changes)	2.8	+2.6*	143	-3**	3.1	0

* Change exceeds 1 SEE, ** Change exceeds 2 SEE, *** Change exceeds 3 SEE

Table 28. Sensitivity of PCC properties to changes in design factors.

EXPERIMENT DESIGN FACTOR LEVELS AND CHANGES	Y1. fr7 SEE = 34 psi		Y2. fpc7 SEE = 172 psi		Y5. Ec28 SEE = .18 mpsi	
	Value	Change	Value	Change	Value	Change
ALL U, V AT LOW-STR LEVELS	354		1790		2.89	
U1. CAT Gravel to Stone	346	-8	1504	-286*	3.17	+.28*
U2. CAM 1.50" to 0.75"	363	+9	1923	+133	2.85	-.04
U3. FAM 2.10 to 2.84	380	+26	2167	+377**	3.10	+.21*
U4. AEQ 6.5oz to 0 oz	395	+41*	2267	+477**	3.97	+1.08***
U5. CAP 34.8% to 42.4%	364	+10	1818	+28	3.25	+.36**
U6. CEP 7.7% to 10.9% (& WCR .63 to .46)	522	+168***	3303	+1513***	3.96	+1.07***
U7. WAP 15.6% to 13.6% (& WCR .63 to .55)	405	+51*	2808	+1018***	3.58	+.69***
V2. WCR .63 to .40 (& CEP & WAP changes)	560	+206***	3957	+2166***	4.47	*1.58***
ALL U, V AT MID-STR LEVELS	543	+189	3607	+1817	4.36	+1.47
U1. CAT mid to Gravel	521	-23	3276	-331*	4.34	-.02
CAT mid to Stone	566	+23	3938	+331*	4.37	+.01
U2. CAM 1.125" to 1.50"	524	-19	3528	-79	4.36	0
CAM 1.125" to 0.75"	562	+19	3686	+79	4.36	0
U3. FAM 2.47 to 2.10	534	0	3607	0	4.21	-.15
FAM 2.47 to 2.84	543	0	3607	0	4.47	+.11
U4. AEQ 3.25 oz to 6.5 oz	543	0	3607	0	4.05	-.31*
AEQ 3.25 oz to 0 oz	543	0	3607	0	4.62	+.26*
U5. CAP 38.6% to 34.8%	543	0	3607	0	4.28	-.09
CAP 38.6% to 42.4%	543	0	3607	0	4.40	+.04
U6. CEP 9.3% to 7.7% (& WCR .51 to .60)	448	-95**	2748	-859***	4.09	-.27*
CEP 9.3% to 10.9% (& WCR .51 to .42)	638	+95**	4444	+837***	4.67	+.31*
U7. WAP 14.6% to 15.6% (& WCR .51 to .53)	535	-8	3397	-210*	4.15	-.21*
WAP 14.6% to 13.6% (& WCR .51 to .46)	578	+35*	3913	+306*	4.55	+.19*
V2. WCR .51 to .63 (& CEP & WAP changes)	442	-101**	2630	-977***	3.93	-.43**
WCR .51 to .40 (& CEP & WAP changes)	637	+94**	4522	+915***	4.73	+.37**
ALL U, V AT HI-STR LEVELS	772	+229	5830	+2223	5.28	+.92
U1. CAT Stone to Gravel	774	+2	4921	-909***	5.28	0
U2. CAM .75" to 1.5"	709	-63*	5679	-151	5.24	-.04
U3. FAM 2.84 to 2.10	754	-18	5600	-230*	5.00	-.28*
U4. AEQ 0 oz to 6.5 oz	675	-97**	5129	-701***	5.09	-.19*
U5. CAP 42.4% to 34.8%	642	-130***	4753	-1257***	4.30	-.98***
U6. CEP 10.9% to 7.7% (& WCR .40 to .55)	566	-206***	3949	-1881***	4.63	-.65***
U7. WAP 13.6% to 15.6% (& WCR .40 to .46)	704	-68**	5224	-606***	4.97	-.31*
V2. WCR .40 to .63 (& CEP & WAP changes)	492	-280***	3201	-2629***	4.46	-.82***

* Change exceeds 1 SEE, ** Change exceeds 2 SEE, *** Change exceeds 3 SEE

designated as its "low-strength" level and one as its "high-strength" level, depending upon which level generally produced lower or higher PCC strength. Because Ec28 is highly correlated with strength, the low and high strength levels for each U also produce low and high Ec28 levels. The water/cement ratio, $V2 = WCR$, is completely determined by CEP and WAP, and has its low-strength level (0.63) when $U6 = CEP$ is at its low-strength level of 7.7 percent, and $U7 = WAP$ is at its low-strength level of 15.6 percent. The high-strength level for WCR (0.40) occurs when CEP and WAP are at their high-strength levels of 10.9 percent and 13.6 percent, respectively. It should be noted that the high-strength level for any U is not necessarily its higher numerical value. For example, the low-strength level for AEQ is 6.5 oz (192 mL) and the high-strength level is 0 oz (0 mL).

As shown in the top lines of tables 27 and 28, substitution of the low strength levels for all U and V in the regression equations produces 6.6 in (167.64 mm) for slump, 130 pcf (2080 kg/m³) for unit weight, 10.5 percent for air, 354 psi (24.9 kg/cm²) for fr7, 1790 psi (126 kg/cm²) for fpc7, and 2.89 mpsi (203,000 kg/cm²) for Ec28.

As shown near the bottom of tables 27 and 28, substitutions of the high-strength levels for all U, V produces 0.2 in (5.08 mm) for slump, 146 pcf (2336 kg/m³) for unit weight, 3.1 percent for air, 772 psi (54.3 kg/cm²) for fr7, 5830 psi (410 kg/cm²) for fpc7, and 5.28 mpsi (371,000 kg/m³) for Ec28. Comparisons of the foregoing calculations with the observed data in data base A (appendix F) shows close agreement between the calculated and observed highs and lows for the dependent variables.

Tables 27 and 28 also show calculated values for the dependent variables when all U, V are at mid-strength levels. It is noted, however, that $U1 = CAT$ has only the qualitative levels of gravel (low-strength) and stone (high-strength), whose coded values are 1 for gravel and 0 for stone. Thus, the coded mid-value for CAT is 0.5, but has no physical interpretation.

The purposes of the sensitivity analyses are to show how each dependent variable changes as:

1. Each U, V changes from its low-strength level to its high-strength level when all remaining U, V are at low-strength levels.
2. Each U, V changes from its mid-strength level to its low-strength level, and from its mid-strength level to its high-strength level when all remaining U, V are at mid-strength levels.
3. Each U, V changes from its high-strength level to its low-strength level when all remaining U, V are at high-strength levels.

In general terms, it can be said that all-low levels produce "weak" PCC, all-mid levels produce "medium" strength PCC, and all-high levels produce "strong" PCC. Thus, purpose (1) is to see by how much weak PCC can be improved by a gross change in only one factor. At the other extreme, purpose (3) is to see by how much strong PCC can be denigrated by a gross change in only one factor. Purpose (2) is to see by how much medium-strength PCC will be changed through partial changes in each factor.

All of the foregoing changes for U, V are shown at the left in both tables 27 and 28. It must be noted that any change in CEP when WAP is fixed will produce a change in WCR, and that any change in WAP for fixed CEP will also produce a change in WCR. It is also true that a change in WCR must generally be accompanied by changes in both CEP and WCR. The left-hand columns of tables 27 and 28 show how WCR changes when CEP or WAP is changed, and how CEP and WAP must change when WCR is changed.

Standard errors of estimate (SEE) for the regression equations were given at the bottom of table 26 and are repeated in the top rows of tables 27 and 28. For each dependent variable, the corresponding column in table 27 or 28 shows the amount of dependent variable change that is associated with the row-by-row changes in U, V. Dependent variable changes are marked with *, **, or *** if the dependent variable changes by 1, 2, or 3 times its SEE, respectively. Unmarked changes are less than one SEE and have no statistical significance.

At low-strength levels, table 27 shows that the largest changes in X1 = SLMP, X2 = UNWT, and X4 = AIR are produced by changing U4 = AEQ, U6 = CEP, and V2 = WCR from low to high levels. Much lesser changes are produced when all U, V are at high levels. No great changes in X1, X2, or X4 are produced by U, V changes from mid levels to low levels or high levels.

None of X1, X2, or X4 is changed appreciably by CAT, CAM, or FAM changes at any level. An increase or decrease in coarse aggregate quantity (CAP) has a sizeable effect only upon unit weight, as might be expected. Major changes occur in X1, X2, and X4 as CEP, WAP, and WCR change from low to high levels when all other U, V are at low levels. Corresponding changes do not occur, however, when all U, V are at midlevels or high levels.

Table 28 shows that the greatest effect of changing CAT from stone to gravel occurs in fpc7 when all remaining U, V are at high levels. Changes in coarse aggregate maximum size (CAM) have almost no effect on any of Y1, Y2, or Y5. The only pronounced effect of an increase in FAM is on fpc7 when all remaining U, V are at low levels.

Reduction of AEQ from 6.5 oz (192.2 mL) to 0 oz (0 mL) produces sizeable increases in fpc7 and Ec28 when other U, V are at low levels. The reverse is true when AEQ is increased from 0 oz (0 mL) to 6.5 oz (192.2 mL) at high levels for the remaining U, V.

The effect of CAP reduction is quite large for all of Y1, Y2, and Y5 when other U, V are at high levels, but CAP changes have small effects when the other factors are at mid or low levels.

By far, the greatest sensitivities are those of Y1, Y2, and Y5 to changes in CEP, WAP, and WCR. These results are consistent with past research and underscore the fact that PCC strengths (and modulus) are largely determined by the PCC cement quantity in conjunction with water quantities that lead to relatively low water/cement ratios.

A graphical representation of table 28 is given in figure 32. In general, it can be seen that the PCC properties are least affected by aggregate type and gradation (CAT, CAM, and FAM). Of greater importance are

the amounts of air entrainment agent (AEQ) and coarse aggregate (CAP) in the PCC mix. Finally, the greatest changes in Y1, Y2, and Y5 are produced by changes in cement quantity (CEP), water quantity (WAP), and the concomitant changes in water/cement ratio (WCR).

SUMMARY RESULTS OF THE LABORATORY STUDY

As evidenced by the experimental design (table 10) and the resulting data bases (appendix F), the laboratory study has produced a wide range of PCC mix factor combinations and measured properties of both the plastic mixes and the hardened PCC specimens. The study inputs and outputs provided an adequate basis for the many analyses that were performed.

Much has been learned about the intercorrelations (table 15) among the experimental variables and about their batch-to-batch and specimen-to-specimen variability (tables 16 and 17) under controlled laboratory conditions.

Extensive analyses of variance (tables 18, 19 and 20) have shown that every experimental design factor (U, V) has a statistically significant effect upon one or more of the measured PCC properties (X, Y), and that many of the factors have interactive effects that may involve two-, three-, or even four-factors.

More than 20 different types of regression equations were derived (tables 21, 22, 23, 24, 25) and represent secondary relationships among PCC properties and their M&C determinants. Equations derived for prediction of flexural, compressive, and tensile strength, and for prediction of PCC elastic modulus have R-squares greater than 0.90 and standard errors equivalent to the variation between replicate mixes.

The regression equations compare favorably with counterpart relationships that have been derived in previous studies and include new relationships that have not heretofore been published. The equations should be quite useful for the development of M&C acceptance plans, specifications for rigid pavement construction, and perhaps even for better PCC mix design procedures.

More than 10 of the derived equations are for the pairwise association between two different PCC properties (table 25). These relationships provide a rational basis for choosing between alternative field tests on the basis of time and cost. An example might be the use of 7-day compressive strength as a surrogate for the PCC modulus at 28 days.

Rather complete sensitivity analyses were developed from the equations that predict slump, unit weight, air content, flexural strength, compressive strength, and PCC modulus. The analyses showed that strength and modulus are relatively insensitive to changes in aggregate type and gradation, and moderately sensitive to changes in quantities of air entrainment and coarse aggregates. The analyses verified previous knowledge that strength and modulus are highly sensitive to changes in cement content, water content, and to the concomitant changes in water/cement ratio.

It is concluded that the laboratory study was quite effective in producing data and analytical results that are needed for the development of a wide range of performance-related specifications for PCC materials and construction.

CHAPTER 6

PROCEDURES AND ALGORITHMS FOR DERIVATION OF PERFORMANCE-RELATED SPECIFICATIONS

This chapter describes procedures and algorithms that can be used to derive PRS for the construction of PCC pavements. Thus, the contents of this chapter represent a detailed expansion of box H in figure 1 of chapter 2. For convenience of presentation, the procedures are grouped in seven modules as indicated in the center column of figure 33.

Procedures for each module, including inputs and outputs, will be discussed in the seven sections that follow. There are generally a number of alternative procedures that might be used within each module. For the most part, the alternatives relate either to (1) a methodology that has been developed in NCHRP Project 10-26A for asphaltic concrete pavements, or (2) a methodology that has been developed by the New Jersey DOT for rigid pavement construction, or (3) methods that have been conceptualized as part of this study (see references 88.1, 80.1, 82.2, 82.3, and 82.4).

Details shown in figure 33 for the first five modules represent a mixture of the three methodologies. Details shown for modules 6 and 7 represent only the NCHRP 10-26A methods based on economic life and that will be used for the demonstration procedures described in chapter 7.

PAVEMENT DESIGN FOR THE INITIAL PERFORMANCE PERIOD

The derivation of PRS begins with a specific pavement design that is assumed to have been developed by the State highway agency (SHA) for the initial performance period. It will also be assumed that the revised AASHTO design guide for rigid pavements is used to determine design levels for all variables that have direct bearing on the pavement structure and its expected life.^{86.4} In particular, it is assumed that the pavement design has been derived through use of the AASHTO design equation for rigid pavements, as represented by relationship N in appendix B.

As shown at the left in figure 33, design inputs include design criteria, environmental assumptions, and roadbed assumptions. Design criteria include design years for the initial performance period, rehabilitation criteria that indicate the pavement's condition at the end of the initial performance period, expected ESALs during the performance period, and reliability criteria that govern the probability that the constructed pavement will indeed survive the design period years.

For the AASHTO design equation, the rehabilitation criterion is simply a terminal level for PSI, e.g., $PSI = 2.5$. The design equation provides many alternative pavement structures that will meet this criterion along with other criteria and assumptions in boxes A, B, and C. From these alternatives, the SHA presumably selects an optimum structure relative to the economics of local materials and construction options.

Outputs from the design procedures are design levels for both PCC and non-PCC variables, as shown at the right in figure 33. The non-PCC variables in box D include design period ESALs, base layer variables, and

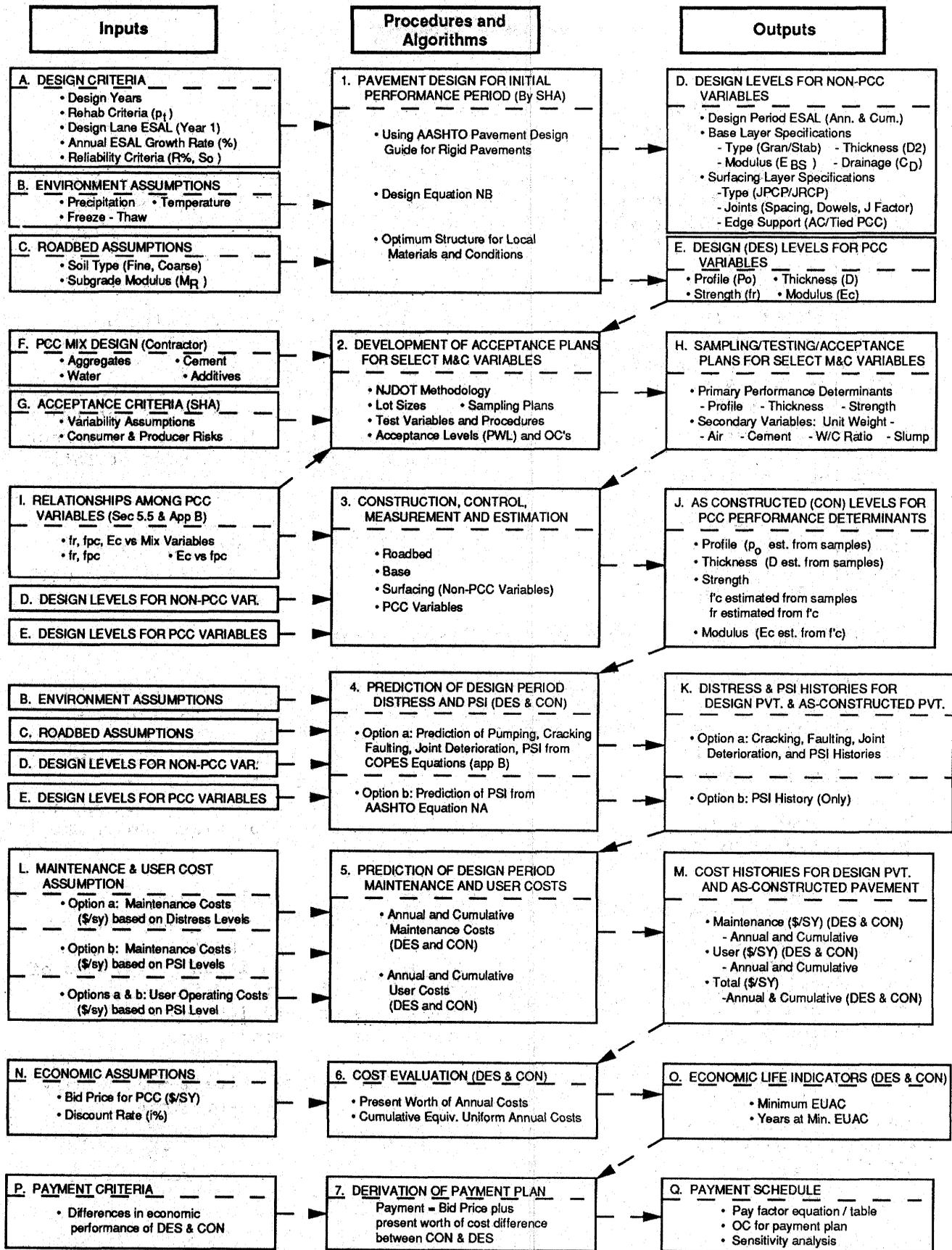


Figure 33. Derivation of performance-related M&C specifications.

surfacing layer variables that are not directly related to the PCC itself, e.g., variables relating to reinforcement, joints, and shoulders.

As indicated in box E, the AASHTO design equation contains only four PCC variables: initial profile (or PSI), PCC thickness, PCC modulus of rupture, and PCC elastic modulus. The design process therefore results in design levels for all four of these variables. It follows that performance-related specifications for construction of the rigid pavement surfacing layer must take all four into account.

Sensitivity analyses of the original AASHTO design equation have been made by the NJDOT and show that changes in the PCC modulus (E_c) from 3 million to 5 million psi (211,000 to 352,000 kg/cm²) have negligible effects on the predicted pavement life.^(82.3) Thus, only initial PSI (P_o), PCC thickness (D), and PCC flexural strength (f_r) need be considered in the PRS system.

Depending upon the choice of cost evaluation alternatives for module 6, it may be necessary for module 1 to include pavement designs for not only the initial performance period, but also for subsequent rehabilitation periods. Both the NCHRP 10-26A and NJDOT methodologies require only initial pavement designs, while the new procedure conceptualized in this study is based on a specified analysis period that extends into at least one rehabilitation period.

ACCEPTANCE PLANS FOR SELECT M&C VARIABLES

The second module in figure 33 contains procedures for the development of acceptance plans for select M&C variables whose levels are to be controlled during pavement construction. For a given M&C variable (e.g., PCC thickness), the acceptance plan is a set of definitions and rules for:

- The construction lot that is to be sampled.
- A sampling plan that specifies how and how many samples (e.g., cores) are to be selected from each lot.
- Test procedures and output values that quantify the sample characteristics.
- Acceptance criteria that determine whether the lot quality is acceptable in view of the test outputs.
- An operating characteristic (OC) that gives the probability that a specific lot quality will result in acceptance via the sampling plan and acceptance criteria.

As indicated in box G of figure 33, acceptance plans and associated operating characteristics are governed by assumptions about the inherent variability of lots with respect to sample characteristics and by consumer and producer risks. The consumer (SHA) risk for a given acceptance plan is the probability that a truly unacceptable lot will nevertheless be accepted. The producer (contractor) risk, on the other hand, is the probability that a truly acceptable lot will nevertheless be rejected. The risks arise, of

course, because only a relatively small portion of any lot has actually been sampled and tested, and, because of inherent variation throughout the lot, the selected samples will sometimes give good results from a bad lot or bad results from a good lot.

Acceptance plans for rigid pavement M&C variables have been worked out in great detail by the NJDOT.^(82.3) It is assumed for purposes of this study that these plans are quite adequate for the development of the new PRS. Salient aspects of the NJDOT acceptance plans are listed below and are recommended for both the demonstration and further development of PCC specifications:

- Lot size should be 1-day's production of the finished PCC pavement layer.
- M&C variables to be evaluated through samples from each lot should include slump, air content, 28-day compressive strength, PCC thickness, and as-constructed PCC profile.
- Samples for evaluation of slump and air content should be taken randomly from the trucks that provide PCC mix for the lot (day's production). Provisions should be specified for retempering and resampling rejected material.
- Material for evaluation of compressive strength should be taken from the same trucks that are sampled for slump and air content and should produce two standard cylinders per sample for 28-day tests.
- Cores for evaluation of PCC thickness should be taken via a stratified random sampling plan over the area covered by the day's production.
- One hundred percent sampling in wheel paths may be used for initial profile evaluation.
- Lot acceptability should be judged in terms of sample statistics from which the lot percent defective can be estimated. Consumer risk may be in the neighborhood of 20 percent (of being accepted when bad) for lots that are 50 percent defective while producer risks may be in the neighborhood of 10 percent (of being rejected when good) for lots that are 10 percent defective.

It may be assumed that slump is included as a construction control for workability of the concrete and that air content is included as a construction control for PCC durability. In the light of the laboratory study results that are reported in chapter 5, it may be that unit weight should also be included as a construction control for PCC strength as shown in box H of figure 33.

Available resources preclude development of specific acceptance plans within the present study, but follow-on studies should produce acceptance plans in detail for all M&C variables that are evaluated in the performance-related construction specifications.

CONSTRUCTION, CONTROL, AND EVALUATION

The third module in the derivation of PRS contains the procedures for pavement construction, sampling, sample evaluation, and lot acceptance or rejection via the acceptance plans that were developed in the second module for both the primary and secondary M&C variables.

It is assumed that M&C quality control is the contractor's responsibility, and that SHA responsibility is for acceptance or rejection of construction lots in terms of observed variables that are set forth in the PRS. If the NJDOT sampling and evaluation plans are used, two performance determinants in the AASHTO design equations are estimated directly, namely, initial profile and PCC thickness. The two remaining determinants (PCC modulus of rupture and PCC elastic modulus) are to be estimated from the 28-day compressive strengths of the sample cylinders. Thus, secondary relationships among PCC variables, as given in chapter 5 and appendix C, must be used to convert observed compressive strength (fpc28) to estimates for flexural strength (fr28) and elastic modulus (Ec28). It is important to note that these estimates are made for each and every construction lot, and that all four estimates have uncertainties relative to the corresponding "true" values for the entire lot, partly because of sampling variability and partly because of uncertainties in the secondary relationships that have been used for the conversions.

It is recommended that serious consideration be given to the use of 7-day compressive strength for the sample cylinders, especially since it was shown in chapter 5 that there is little difference in the statistical uncertainty between estimation of fr28 and Ec28 from fpc28 or fpc7.

The NJDOT procedure uses a square root relationship similar to appendix C, relationships A or E to convert compressive strength to flexural strength. Results of this study, however, provide alternative equations for this conversion (see table 25).

PREDICTION OF DISTRESS AND PERFORMANCE FOR DESIGN PAVEMENT AND CONSTRUCTED PAVEMENT

The fourth module in PRS development contains equations and algorithms for predicting distress and performance for both the pavement as-designed (DES) and as-constructed (CON). Inputs for the as-constructed pavement are assumed to be the same as for the as-designed pavement for all variables in boxes B, C, and D, but contain the as-constructed estimates for PCC variables (box J) instead of the as-designed levels in box E.

As shown in module 4 of figure 33, one option is to use the COPES prediction equations in appendix B for pumping, cracking, faulting, joint deterioration and serviceability loss.^(85.2) Use of these equations produces annual and cumulative distress histories for both the as-designed and as-constructed pavements throughout the initial performance period.

A second option is to predict only the PSI history for the initial performance period using the AASHTO design equation (appendix B, relationship N). If both options are employed, the output distress and performance histories indicated in box K are annual and cumulative vectors

for ESAL, pumping, cracking, faulting, joint deterioration, PSI (COPES), and PSI (AASHTO).

The foregoing procedures are analogous to those used in NCHRP Project 10-26A wherein annual and cumulative vectors are produced for ESAL, AC cracking, AC rut depth, and AC roughness. On the other hand, the NJDOT procedures involve only the time at which the terminal serviceability level (PSI = 2.5) is reached and do not require distress or PSI predictions during the initial performance period years.

PREDICTION OF PERFORMANCE PERIOD COSTS FOR DESIGN PAVEMENT AND CONSTRUCTED PAVEMENT

The fifth procedural module is directed at the transformation of annual and cumulative distress quantities into corresponding unit costs for both the as-designed pavement (DES) and the as-constructed pavement (CON). As shown in box L, input assumptions can be made for dollars per square yard costs (\$/SY), for annual maintenance when distress variables and PSI are at given levels, or more simply, for annual maintenance costs (\$/SY) when PSI is at given levels. If user operating costs are also to be considered, then assumptions must be made for user costs (in \$/SY) at given PSI levels.

From the input assumptions, the algorithms in module 5 produce cost histories, both annual and cumulative, that show maintenance costs, user costs and total costs for both the as-designed pavement (DES) and the as-constructed pavement (CON). As noted in box L of figure 33, one option is to predict both annual maintenance and user costs from the PSI levels that are predicted on a year-by-year basis.

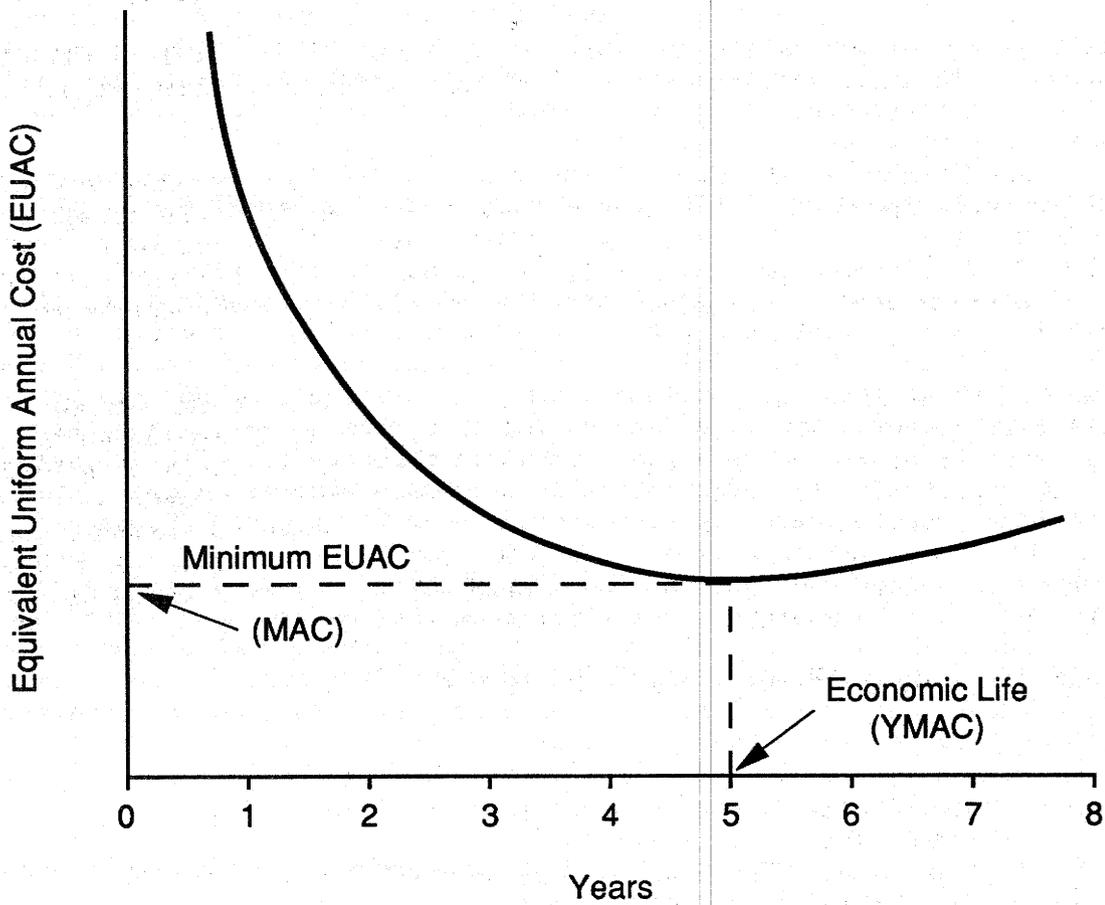
It should be noted that the:

- 1) NCHRP 10-26A methodology employs both maintenance and user operating costs.
- 2) Life cycle cost methodology conceptualized in this study employs only maintenance costs, but could consider user costs, if necessary.
- 3) NJDOT methodology does not depend at all upon maintenance or user costs.

COST EVALUATION FOR DESIGN PAVEMENT AND AS-CONSTRUCTED PAVEMENT

The sixth module contains algorithms for conversion of the cost histories produced in module 5 into summary indicators of the economic performances of the design pavement and the constructed pavement.

The NCHRP 10-26A methodology employs indicators that are based on economic life, as illustrated in figure 34. Economic life is defined as the year (YMAC) within the initial performance period at which the equivalent uniform annual cost (EUAC) has a minimum value (MAC). Thus, in figure 34, MAC and YMAC are coordinates of the minimum point on the EUAC curve. To calculate EUAC, the cumulative present worth of construction, maintenance, and user operating costs is calculated for each year of the performance



EUAC = Cumulative Present Worth of Total Costs Through Year Y
 times $i k / (k-1)$ where i = discount rate and $k = (1+i)^Y$

Figure 34. Economic life indicators.

period, using the cost histories that were produced in module 5. For each year, Y:

$$EUAC = (\text{Cumul. Present Worth of Total Costs})[i(1+i)^Y]/[(1+i)^Y-1] \quad (29)$$

where i is an assumed discount rate. In general, the EUAC values will first decrease, then increase after passing through the minimum point at which $EUAC = MAC$ at year $YMAC$.

If the foregoing calculations are made for both the design pavement and the constructed pavement, the outputs (box 0 of figure 33) are arrays of annual EUAC for both pavements. As shown in figure 35, coordinates for the minimum EUAC in the respective arrays are designated by (DES MAC, DES YMAC) for the design pavement and by (CON MAC, CON YMAC) for the constructed pavement.

The NJDOT methodology for cost evaluation uses the AASHTO design equation to compute the expected life (L) for both the design pavement (LDES) and the constructed pavement (LCON), using inputs from boxes D, E, and J in figure 33. Furthermore, it is assumed that the present unit cost of the first overlay is C_1 , the present unit cost of each subsequent overlay is C_2 , and that every overlay has the same expected life, LOL. Maintenance and user operating costs are not considered, so the present worth of all costs (PWC) for the design pavement is given by:

$$DES \text{ PWC} = C_0 + R^{LDES} [C_1 + C_2 R^{LOL} / (1 - R^{LOL})] \quad (30)$$

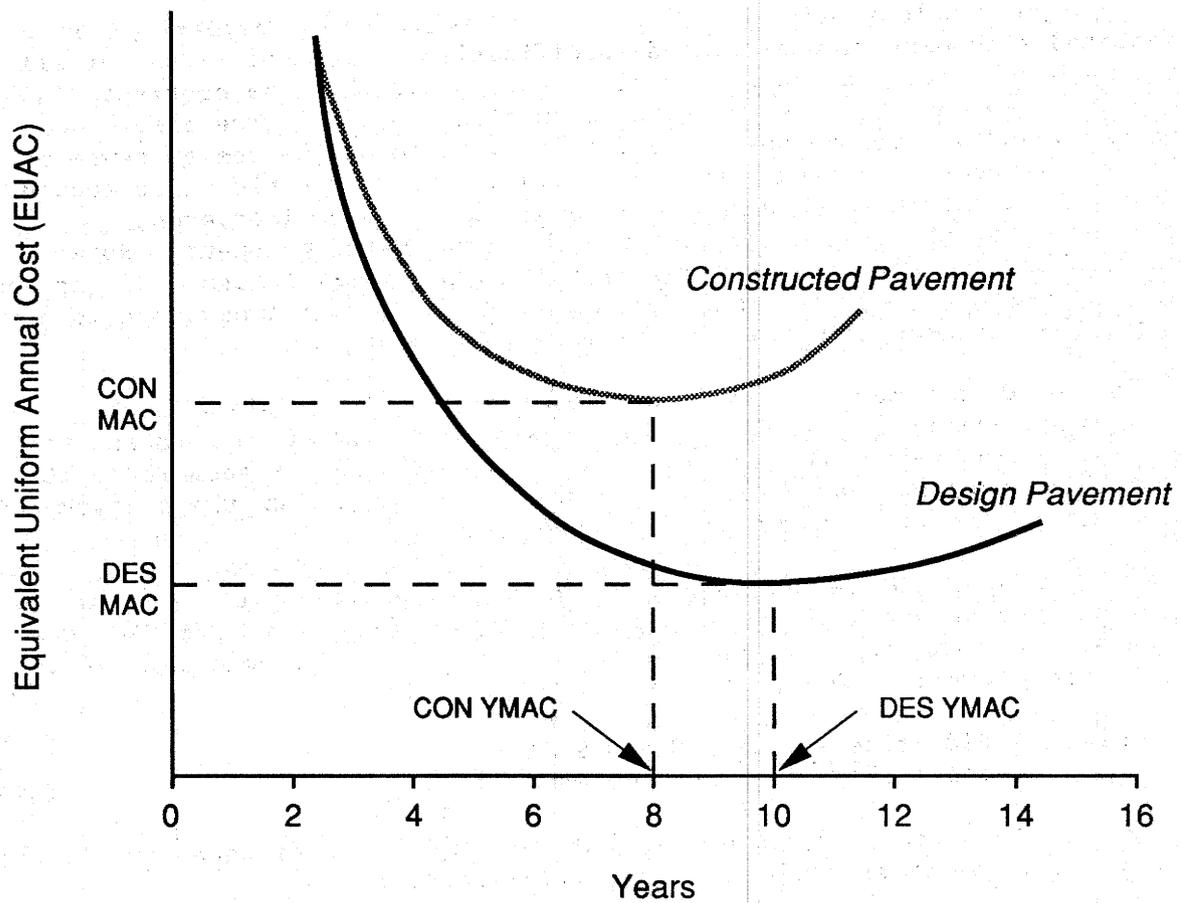
where:

- C_0 = unit cost of initial construction.
- C_1 = unit cost of first overlay.
- C_2 = unit cost of each subsequent overlay.
- LDES = expected life (years) of design pavement.
- LOL = expected life of each overlay (assumed to be constant).
- R = $(1 + \text{inflation rate}) / (1 + \text{interest rate})$.

Corresponding present worth costs for the constructed pavement are given by equation 30 with LDES replaced by LCON, the expected life of the constructed pavement.

The new methodology conceptualized in this study evaluates the present worth of life-cycle costs over a specified analysis period for both the design pavement and the constructed pavement. Cost elements cover initial construction, overlay construction, maintenance and salvage value. The methodology also assumes that different overlay designs will be required if there are differences in the initial performance period for the design and constructed pavements.

Whatever methodology may be used, the outputs of module 6 (box 0 of figure 33) are economic indicators of pavement performance and are assumed to be essential to a true PRS system.



Design Pavement:

DES MAC = Minimum EUAC for design pavement

DES YMAC = Year at minimum EUAC for design pavement

Constructed Pavement:

CON MAC = Minimum EUAC for constructed pavement

CON YMAC = Year at minimum EUAC for constructed pavement

Figure 35. Economic life differentials between design pavement and constructed pavement.

DERIVATION OF PAYMENT PLANS FOR PERFORMANCE-RELATED SPECIFICATIONS

The last module in figure 33 is for the derivation of payment plans associated with performance-related specifications. Payment plans for all three methodologies that have been discussed assume that the contractor's bid price should be paid for every construction lot whose true life expectancy and associated costs are precisely those of the design pavement. All three methods also assume that pay adjustments to the bid price should be based on differences between the economic indicators of pavement performance for the design pavement and the constructed pavement. More specifically, the criterion for any payment plan (box P of figure 33) is the degree to which the economic performance of the constructed pavement is greater than or less than that of the design pavement.

It should be remembered that the acceptance plans (module 2) and construction evaluations (module 3) are based on construction lots and that the payment plan refers to payments that are calculated for each lot. It can thus be expected that overall payment is calculated from the credits and debits associated with individual lot payments.

For the NCHRP 10-26A methodology, the lot payment plan is the bid price minus the present worth of the difference between CON MAC and DES MAC (see figure 35) over the years given by CON YMAC, i.e., the economic life of the constructed pavement. Thus, the payment formula is:

$$\text{Payment} = \text{Bid Price} - (\text{CON MAC} - \text{DES MAC}) * \frac{1}{[(1+i)^{\text{CON YMAC}} - 1] / [i(1+i)^{\text{CON YMAC}}]} \quad (31)$$

If the bid price is factored out from the right side of equation 31, the bid price multiplier is defined to be the pay factor:

$$\text{Pay Factor} = 1 - [(\text{CON MAC} - \text{DES MAC}) / \text{Bid Price}] * \frac{1}{[(1+i)^{\text{CON YMAC}} - 1] / [i(1+i)^{\text{CON YMAC}}]} \quad (32)$$

For example, if $i = 6$ percent, bid price is \$30/sy, DESMAC is \$6/sy, CONMAC is \$7/sy, and CONYMAC = 15 years, then, the pay factor is equal to $1 - [\$1/\$30][2.40-1]/[0.06(2.40)] = 1 - 0.32 = 0.68$ or 68 percent of the bid price. If CON MAC is less than DES MAC, the pay factor will be greater than 1.

In connection with the chapter 7 demonstration of the economic life methodology, a sensitivity analysis was performed to show how the pay factor in equation 32 changes with changes in PCC variables, cost variables, and associated changes in the explicit variables within equation 32.

In the NJDOT methodology, the payment for each lot is given by:

$$\text{Payment} = \text{Bid Price} + (R^{\text{LDES}} - R^{\text{LCON}}) [C_1 + C_2 R^{\text{LOL}} / (1 - R^{\text{LOL}})] \quad (33)$$

where C_1 , C_2 , and LOL are overlay costs and life (as in equation 30). The NJDOT pay factor multiplier for bid price is, therefore:

$$\text{Pay Factor} = 1 + [(R^{\text{LDES}} - R^{\text{LCON}}) / \text{Bid Price}] [C_1 + C_2 R^{\text{LOL}} / (1 - R^{\text{LOL}})] \quad (34)$$

Thus, both the life of the constructed pavement and of the design pavement enter into the pay factor equation. If the former should be zero, for example, the pay factor will be about 0.60 for LDES = 20 years, Bid Price = \$30/sy, $R = 0.96$, $C_1 = \$8/SY$, $C_2 = \$7/SY$, and $LOL = 10$ years.

Since the methodology conceptualized in this study has not been fully developed, no counterparts to equations 32 and 34 are given in this report.

The NJDOT approach makes it possible to write the pay factor equation in terms of the "load ratio", $LCON/LDES$, i.e., the ratio of the constructed life to the design life. Thus, the pay factor can be graphed as a function of the load ratio as shown in figure 36. For ratios between (say) 0.50 and 1.50, the curve for equation 34 can be approximated by a straight line whose equation, for example, might be $F = 0.75 + 0.25*(\text{load ratio})$. For ratios below 0.5, a pay factor of (say) 0.60 can be assumed; for ratios above 1.50, the pay factor might stay constant at 1.12. These substitutions for the pay factor curve provide both minimum and maximum pay factors, and permit simple pay factor calculations between the limiting load ratios. Similar approaches can be developed for both the NCHRP 10-26A methodology and the conceptual approach identified in this study, but such developments are left for further research.

A final important consideration for any specification payment plan is its so-called operating characteristic (OC), i.e., the contractor's expected payment, relative to the acceptance sampling plans that were discussed earlier in this chapter. For each construction lot (e.g., 1 day's pavement production), only a few samples (e.g., five) have been selected and evaluated to estimate the PCC properties of the entire lot. All outputs from module 4 (distress predictions), module 5 (cost predictions), and module 6 (economic indicators of performance) are based on sample characteristics that vary by chance about the true characteristics of the entire lot. These same sampling variations affect the pay factors that are calculated for individual lots, and thus lead to pay factors that are sometimes smaller and sometimes larger than would be produced from 100 percent sampling of each lot. The contractor's expected payment, shown in figure 36 as the OC curve, is derived from probabilities that relate to differences between pay factors derived from sample data and pay factors that would result from full knowledge of the quality characteristics of the entire lot. As has been done by NJDOT, it is useful to derive sampling and acceptance plans whose expected payment OCs give close approximations to the pay factor equations that are part of the PRS. Again, development of expected payment OCs for either the NCHRP 10-26A approach (or the new methodology conceptualized in this study) will require further research, as is discussed in chapter 8.

Additional research will also be required to develop PRS that take into account the uncertainties of equations that are used to predict both physical and economic performance. In both the present study and in the NCHRP 10-26A report, prediction equation errors have been disregarded, but eventually, the OCs for acceptance and payment plans must take these uncertainties into account.

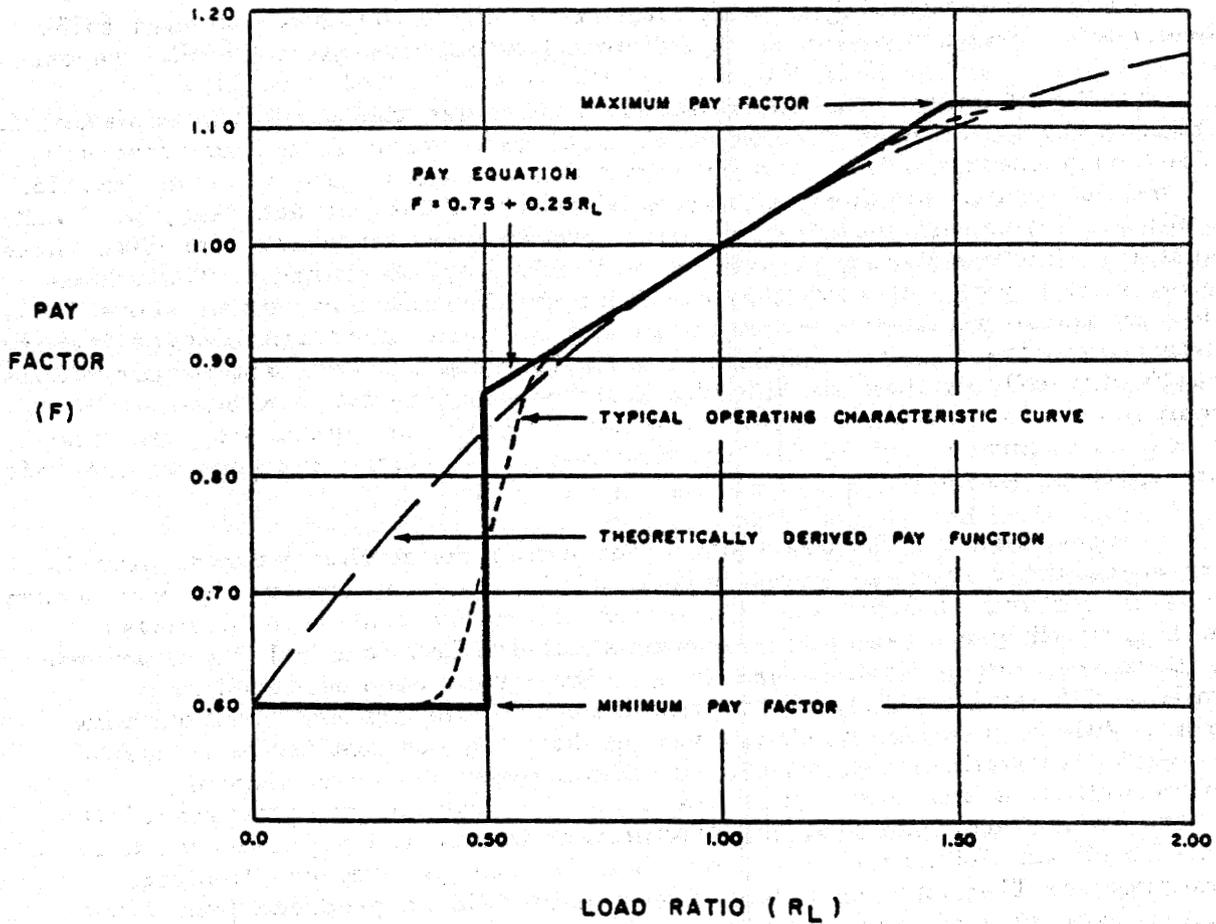


Figure 36. Determination of acceptance plan payment system based on theoretically derived pay function and expected payment operating characteristic curve. (82.3)

CHAPTER 7

DEMONSTRATION DEVELOPMENT OF A PERFORMANCE-RELATED M&C SPECIFICATION FOR PCC CONSTRUCTION

This chapter presents a demonstration of the procedures and algorithms that were discussed in chapter 6. The primary methodology is based on economic life, as has been the basis for AC construction specifications in NCHRP Project 10-26A. The demonstration procedures are computerized through the use of a spreadsheet program (Lotus 1-2-3).

The first section explains and illustrates all input data for the procedure. The second section presents procedures and outputs for the year-by-year histories of illustrative traffic, serviceability, distress, and costs for a specified design pavement and for a corresponding as-constructed pavement. In the third section, the historical data are used to develop indicators of economic performance (e.g., economic life) for both pavements and to produce an illustrative payment plan for the as-constructed pavement.

Sensitivity analyses for the payment plan are given in the fourth section and show how payment factors depend upon determinants of the plan, both for physical determinants (e.g., PCC strength) and for economic determinants (e.g., maintenance costs).

The final section contains comments and conclusions that were reached by the research team with respect to the demonstration procedures and results. It is acknowledged that the demonstration represents only the formative stages of the complete development of a performance-related M&C specification for the construction of PCC pavements.

INPUT DATA FOR THE DEMONSTRATION SPECIFICATION

Inputs for the demonstration specification are listed in table 29 in six categories (A through F). Input requirements are dictated by the use of the AASHTO design equation for rigid pavements (appendix B, relationship N), by the COPEs equations for prediction of various types of distress (appendix B, relationships A, B, H, J, and O), and by the cost evaluation procedures and equations that were presented in chapter 6.

Category A in table 29 is for the primary PCC specifications factors that have been selected for the demonstration. These factors are (1) initial serviceability level, P_o ; (2) PCC slab thickness, D_c ; (3) 28-day compressive strength, F'_c ; (4) 28-day flexural strength, S'_c ; and (5) PCC elastic modulus, E_c . As was shown in box J of figure 33, it is assumed that only P_o , D_c , and F'_c are evaluated through construction sampling and acceptance plans. Since the AASHTO design equation requires inputs for S'_c and E_c , the demonstration assumes that these properties can be estimated adequately from equations that were developed in chapter 5 and presented in table 25. The equation used for estimating flexural strength from compressive strength is:

Table 29. Input data for the demonstration specification.

A. Primary PCC Specifications Factors	Design Pvt. (DES)	Constructed Pvt. (CON)
1. Initial PSI (P_o)	4.3	4.0
2. Slab Thickness (D_c)	9.0 in	8.5 in
3. 28-day Compressive Strength (F'_c)	4000 psi	3500 psi
4. 28-day Flexural Strength (S'_c) (estimated from F'_c by equation 35)	614 psi	557 psi
5. PCC Elastic Modulus (E_c) (estimated from F'_c by equation 36)	4.17 mpsi	3.96 mpsi
B. Non-PCC M&C Factors		<u>DES & CON</u>
1. Load Transfer Coefficient (J)		3.2
2. Drainage Coefficient (C_d)		1.0
3. Subbase Thickness		6.0 in
4. Joint Spacing		20 ft
5. Subbase Type (0 = Gran, 1 = Stab)		1 (Stab)
6. Shoulder Type (0 = AC, 1 = PCC Tied)		0 (AC)
7. Dowel Bar Diameter		1.25 in
8. Reinf. Steel Quantity		0.12 in ² /ftwidth
9. Type of Joint Filler (0 = None, 1 = Unitube)		1 (Unitube)
10. Modulus of Subgrade Reaction (k)		60 pci
C. Traffic Factors		
1. Initial 4-lane ESAL (W_o)		500,000
2. Direction Distribution Factor		50%
3. Lane Distribution Factor		90%
4. Annual Growth Rate (r)		5%
D. Environmental Factors		
1. Freeze Index		625
2. Avg. Monthly Temperature		18°C
3. Max. Annual Temp. Range		33°C
4. Avg Annual Precipitation		25 in
E. Other Distress Factors (for COPES equations)		
1. D-Crack Potential (0 = No, 1 = Yes)		0
2. Reactive Aggregates (0 = No, 1 = Yes)		0
3. Incompressible Potential (0 = No, 1 = Yes)		0
4. Joint Damage Potential (0 = Low, 1 = Med/High)		0
F. Economic and Cost Factors		
1. Interest Rate (i)		6%
2. Cost of PCC Construction (Bid Price)		\$ 30.00/sy
3. Annual Maintenance Costs when PSI = 2.5 (m in equation 37)		\$ 0.28/sy
4. Percent of Vehicle Operating Costs (q in equation 38)		10%

$$S'_c = 1.22 (131.2 + 0.093 F'_c) \quad (35)$$

and the equation for estimating elastic modulus from compressive strength is:

$$E_c = 2,508,000 + 416 F'_c \quad (36)$$

Thus, the demonstration data input begins with entries for P_o , D_o , and F'_c for both the design pavement (DES) and the as-constructed pavement (CON). The spreadsheet program then uses equations 35 and 36 to produce estimates for both S'_c and E_c . All remaining entries in table 29 are for factors whose levels are assumed to be the same for both the design pavement and the as-constructed pavement.

Category B is for M&C factors that are independent of the PCC properties in category A. The first two factors, load transfer (J) and drainage coefficient (C_d) and the tenth factor, modulus of subgrade reaction (k), are values required by the AASHTO design equation. The remaining factors are required by one or another of the COPES equations for predicting rigid pavement distress.

Category C contains traffic factors that identify the cumulative number of 18-kip ESALs (W_1) that will be applied to the pavement design lane during its life. Entries are made for the number of ESALs in all traffic lanes (W_o), for the percent of W_o in the design lane direction and for the percent of directional traffic that is in the design lane. W_1 , the product of these three values, is computed by the spreadsheet program and the result is displayed. The final traffic factor required is the annual rate of ESAL growth (r) that is used to project future traffic.

Category D contains environmental moisture and temperature factors that are inputs to certain of the COPES distress equations. Category E contains additional factors that are also required by one or another of these equations.

Finally, category F of table 29 contains all factors that are needed for spreadsheet calculation of annual costs and their present worth. The first entry is the annual interest rate (i). The second is for the unit cost of PCC construction, and is assumed to be the bid price (BP) in dollars per square yard.

The third cost factor required is the annual routine maintenance cost (\$/sy) when the serviceability level is $PSI = 2.5$. For the demonstration, the routine maintenance cost for year Y, RMC_y , is approximated based on its predicted PSI during year Y, PSI_y .

$$RMC_y (\$/sy) = m * (5 - PSI_y)^2 / 6.25 \quad (37)$$

The equation shows that RMC equals m, when $PSI = 2.5$.

It is assumed that vehicle operation costs for any particular year (Y) depend both on the pavement's serviceability level (P_y) and the number of ESAL (w_y) that it receives during the year. For the demonstration, it is

assumed that vehicle operating costs for year Y (VOC_y) are given by the equation:

$$VOC_y = q (0.00203 w_y) (1.397 - 0.088 P_y) \quad (38)$$

where q is the percentage of the total predicted VOC that is to be considered in determining the contractor's penalty or reward. This percentage was included to adjust the potentially large effect VOC can have on contractor payment. The VOC equation was derived from the results of a 1982 FHWA study of vehicle operating costs and required several simplifying assumptions as traffic distribution, vehicle loading, operating speed, etc.

There are obviously many alternatives for equations 37 and 38, but further research is needed to determine which alternatives are optimal for any particular PRS system.

TRAFFIC, SERVICEABILITY, DISTRESS, AND COST HISTORIES

After all inputs are entered in table 29, the spreadsheet program calculates and displays year-by-year histories for the design pavement traffic, serviceability, distress, and costs as shown in part A of table 30, then produces corresponding histories in part B for the as-constructed pavement. Although the example shows only the first 6 years, all histories cover a span of 30 years.

Two traffic histories are shown in category A, the annual design lane ESAL and the cumulative design lane ESAL. For any year, Y, the annual ESAL (w_y) is given by:

$$w_y \text{ (ESAL)} = W_1 (1 + r)^{Y-1} \quad (39)$$

where W_1 is the first year ESAL and r is the annual ESAL growth rate. Both W_1 and r are indicated in the traffic category of table 29.

The number (W_y) of cumulative ESAL through year Y is given by:

$$W_y \text{ (ESAL)} = \sum_{j=1}^Y w_j = W_1 [(1 + r)^Y - 1]/r \quad (40)$$

where the right side represents the sum of the geometric series that arises from substitution for w_y from equation 39.

The second set of histories in table 30 is for yearly levels of serviceability (P_y) that are given for PSI by the AASHTO design equation, and for PSR (present serviceability rating) levels that is given by the COPES equation (appendix B, relationship O).

After substitutions are made for $C_d = 1$, $J = 3.2$, and $k = 60 \text{ pci (1.66 kg/cm}^3\text{)}$ (see table 29), the AASHTO equation may be written in the form:

$$\log[(P_o - P_y)/3] = [f_1] [\log W_y - f_2 - (4.22 - 0.32 P_y)(f_3)] \quad (41)$$

Table 30. Traffic, serviceability, distress, and cost histories (part A).

A. DESIGN PAVEMENT (DES) YEARS,	0	1	2	3	4	5	6
Traffic Histories							
1. Design Lane ESAL, millions $w_y/1000$ from equation 39	--	0.225	0.236	0.248	0.260	0.273	0.287
2. Cumulative ESAL, millions $W_y/1000$ from equation 40	--	0.225	0.461	0.709	0.970	1.243	1.530
Serviceability Histories							
3. AASHTO PSI (P_o in table 29) (P_y from equation 41)	4.3	4.2	4.2	4.2	4.1	4.1	3.9
4. COPES PSR (4.5 initial) (PSR from relationship O, appendix B)	4.5	4.4	4.3	4.1	4.0	3.9	3.7
Distress Histories (from COPES Equations)							
5. Pumping (0 - 3) (relationship A, appendix B)	--	0	0	0	0	0	0
6. Faulting (in/fault) (relationship H, appendix B)	--	0.74	1.04	1.28	1.48	1.67	1.84
7. Joint Deterioration (jts/mi) (relationship J, appendix B)	--	0.05	0.27	0.69	1.34	2.24	3.41
8. Slab Cracking (ft/mile) (relationship B, appendix B)	--	4	8	13	18	24	30
Cost Histories							
9. PCC Construction (\$/sy) (Bid Price)	30.00	0.00	0.00	0.00	0.00	0.00	0.00
10. Routine Maintenance (\$/sy) (RMC_y from equation 37)	--	0.03	0.03	0.03	0.04	0.04	0.05
11. Vehicle Operating Cost (\$/sy) (VOC_y from equation 38)	-	47.03	49.42	51.94	55.05	57.85	61.31
12. Total Annual Cost (\$/sy) (TAC_y from equation 43)	30.00	47.06	49.45	51.97	55.09	57.89	61.36
13. Present Worth of TAC (\$/sy) (PWC_y from equation 44)	30.00	44.39	44.01	43.63	43.63	43.26	43.25
14. Cum. Present Worth (\$/sy) (CPW_y from equation 47)	30.00	74.39	118.4	162.0	205.7	248.9	292.2
15. CPW Factor (F_y from equation 52)	--	1.060	0.546	0.374	0.289	0.237	0.203
16. Equiv. Unif. Ann. Cost (\$/sy) ($EUAC_y = F_y * CPW_y$)	--	78.86	64.58	60.62	59.35	59.09	59.42
17. EUAC Min. Diff. Factor (G_y from equation 55)	--	0.943	1.832	2.674	3.460	4.219	4.926

Table 30. Traffic, serviceability, distress, and cost histories (part B).

B. CONSTRUCTED PVT. (CON) YEARS	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
Traffic Histories							
1. Design Lane ESAL, millions w _y /1000 from equation 39	--	0.225	0.236	0.248	0.260	0.273	0.287
2. Cumulative ESAL, millions W _y /1000 from equation 40	--	0.225	0.461	0.709	0.970	1.243	1.530
Serviceability Histories							
3. AASHTO PSI (P _o in Table 29) (P _y from equation 41)	4.0	3.9	3.8	3.7	3.6	3.5	3.3
4. COPEs PSR (4.5 initial) (PSR from relationship O, appendix B)	4.5	4.4	4.3	4.1	4.0	3.9	3.7
Distress Histories (from COPEs Equations)							
5. Pumping (0 - 3) (relationship A, appendix B)	--	0	0	0	0	0	0
6. Faulting (in/fault) (relationship H, appendix B)	--	0.74	1.04	1.28	1.48	1.67	1.84
7. Joint Deterioration (jts/mi) (relationship J, appendix B)	--	0.05	0.27	0.69	1.34	2.24	3.41
8. Slab Cracking (ft/mile) (relationship B, appendix B)	--	4	8	13	18	24	30
Cost Histories							
9. PCC Construction (\$/sy) (Bid Price)	30.00	0.00	0.00	0.00	0.00	0.00	0.00
10. Routine Maintenance (\$/sy) (RMC _y from equation 37)	--	0.06	0.07	0.08	0.09	0.11	0.12
11. Vehicle Operating Cost (\$/sy) (VOC _y from equation 38)	--	48.24	51.12	54.16	57.38	60.79	64.40
12. Total Annual Cost (\$/sy) (TAC _y from equation 43)	30.00	48.29	51.18	54.24	57.48	60.90	64.52
13. Present Worth of TAC (\$/sy) (PWC _y from equation 44)	30.00	45.56	45.55	45.54	45.53	45.51	45.49
14. Cum. Present Worth (\$/sy) (CPW _y from equation 47)	30.00	75.56	121.1	166.6	212.2	257.7	303.2
15. CPW Factor (F _y from equation 52)	--	1.060	0.546	0.374	0.289	0.237	0.203
16. Equiv. Unif. Ann. Cost (\$/sy) (EUAC _y = F _y * CPW _y)	--	80.09	66.06	62.35	61.23	61.17	61.65
17. EUAC Min. Diff. Factor (G _y from equation 55)	--	0.943	1.832	2.674	3.460	4.219	4.926

where:

$$f_1 = 1 + 16240000/(D_c + 1)^{8.46}$$

$$f_2 = 7.35 \log(D_c + 1) - 0.06$$

$$f_3 = \log [S'_c (D_c^{0.75} - 1.132)] / [690.02 (D_c^{0.75} - 51.266 E_c^{-0.25})]$$

To show the more specific nature of equation 41, the design pavement in part A of table 29 has $P_o = 4.3$, $D_c = 9.0$ in (228.6 mm), $S'_c = 614$ psi (43.2 kg/cm²), and $E_c = 4.17$ mpsi (293,000 kg/cm²). Substituting these values in equation 41 gives:

$$\log[(4.3 - P_y)/3] = 1.0563 [\log W_y - 7.29 - (4.22 - 0.32 P_y)(-0.0505)]$$

or,

$$\log(4.3 - P_y) + 0.0171 P_y = 1.0563 \log W_y - 6.9982 \quad (42)$$

Since equation 42 cannot be solved explicitly for P_y , the spreadsheet program includes a procedure for approximating P_y when W_y is given.

The PSI history produced by equation 42 with W_y inputs from equation 40 is tabulated in part A of table 30 and is shown graphically in figure 37. The figure also shows the PSI history for the as-constructed pavement, as tabulated in part B in table 30. The figure shows that the design pavement reached PSI = 2.5 after about 20 years and 8 million ESAL, whereas the as-constructed pavement life (at PSI = 2.5) was only about half that of the design pavement.

The spreadsheet program includes PSR values that are given by the COPES equation, mainly to show that rather large differences exist between the AASHTO and COPES equations. For example, table 30 shows that the COPES equation gives PSR = 3.7 at about 6 years and 1.5 million ESAL for both the design pavement and the as-constructed pavement, whereas the corresponding AASHTO values are 3.9 and 3.3, respectively. After 12 years, the difference would be even more significant. In terms of the NJDOT methodology, the AASHTO equation would give a load ratio of about 0.5 (4 million ESAL/8 million ESAL), whereas the COPES equation would give a load ratio of 1.0 for the example pavements. These observations imply that a PRS system may give different results not only for two different methodologies but also for different primary performance equations within a given methodology.

The next four histories in table 30 are for rigid pavement pumping, faulting, joint deterioration, and slab cracking as given by prediction equations that were developed in the COPES project.^(85.2) As was noted in box L of figure 33, annual maintenance costs might be based on levels that have been reached in a given year for one or more of these four distress types (option a), or they might simply be based on the serviceability level that has been reached (option b). Although option b has been selected for the demonstration specification, inclusion of distress histories makes it possible to incorporate option a as a future alternative. (Note: The COPES faulting equation obtained from reference 85.2 does have an apparent error that would need to be corrected before this option could be exercised.)

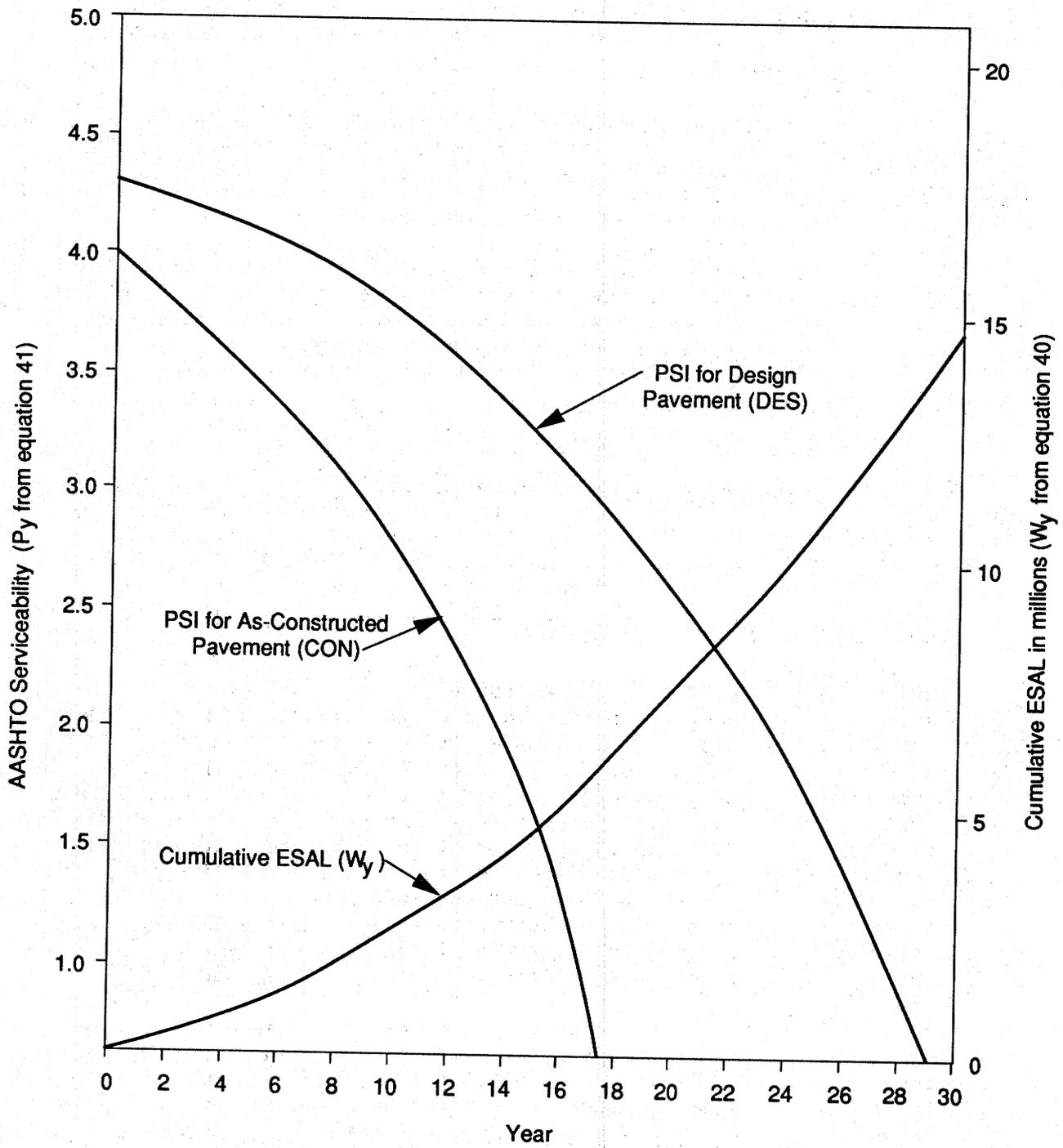


Figure 37. Serviceability and traffic histories.

Comparison of serviceability and distress histories for the design pavement (table 30, part A) and the as-constructed pavement (table 30, part B) shows that the COPES equations give identical annual serviceability and distress levels for both pavements. It follows that, unlike the AASHTO serviceability relationship, both pavements would have the same annual maintenance costs for any cost formula based on the COPES serviceability and/or distress formulas.

None of the COPES equations includes initial serviceability as a construction variable, but all include PCC thickness and flexural strength as determinants of serviceability and distress. Until further study is made, it is presumed that the AASHTO equation has greater sensitivity to changes in D_c and S'_c than do the COPES equations.

The final set of histories in table 30 contains year-by-year values for eight different cost factors. The first cost factor (history 9) is the unit cost of PCC construction that occurs only in year zero, i.e., before traffic has started. This cost is assumed to be the bid price for PCC pavement construction and has been entered as \$30.00/sy for both the design pavement and the as-constructed pavement.

Routine maintenance costs for each year are given in history 10 and are calculated from equation 37. History 11 is for annual vehicle operating costs as calculated from equation 38, and the total annual cost (history 12) is given by:

$$TAC_0 = \text{Bid Price for year zero}$$

$$TAC_y = RMC_y + VOC_y \text{ for } Y = 1, 2, \dots \quad (43)$$

Plots of RMC_y and VOC_y over time are shown in figures 38 and 39, respectively. Each figure contains the cost histories for both the design pavement and the as-constructed pavement, as given in table 30, part A and part B, respectively. For the demonstration PRS system, it can be seen that the relative cost differences between the two pavements are considerably larger for RMC than for VOC. Although not shown, a corresponding graph for total annual costs (TAC_y) would look much the same as figure 39, simply because the VOC in figure 39 are about two orders of magnitude greater than the RMC in figure 38.

As an adjunct to the sensitivity studies in this chapter, it is useful to display equation 43 in terms of all variables and input factors that determine TAC_y . Substitution from equations 37, 38, and 39 gives the following result:

$$TAC_y = 0.16 m Q_y^2 + 0.00203 q W_1 (1 + r)^{y-1} (0.957 + 0.088 Q_y) \quad (44)$$

where $Q_y = 5 - P_y$, $m = RMC_y$ when $P_y = 2.5$, $q =$ fraction of VOC_y used, $W_1 =$ initial year ESAL in the design lane, and $r =$ traffic growth rate.

For the input values given in table 29, equation 44 becomes:

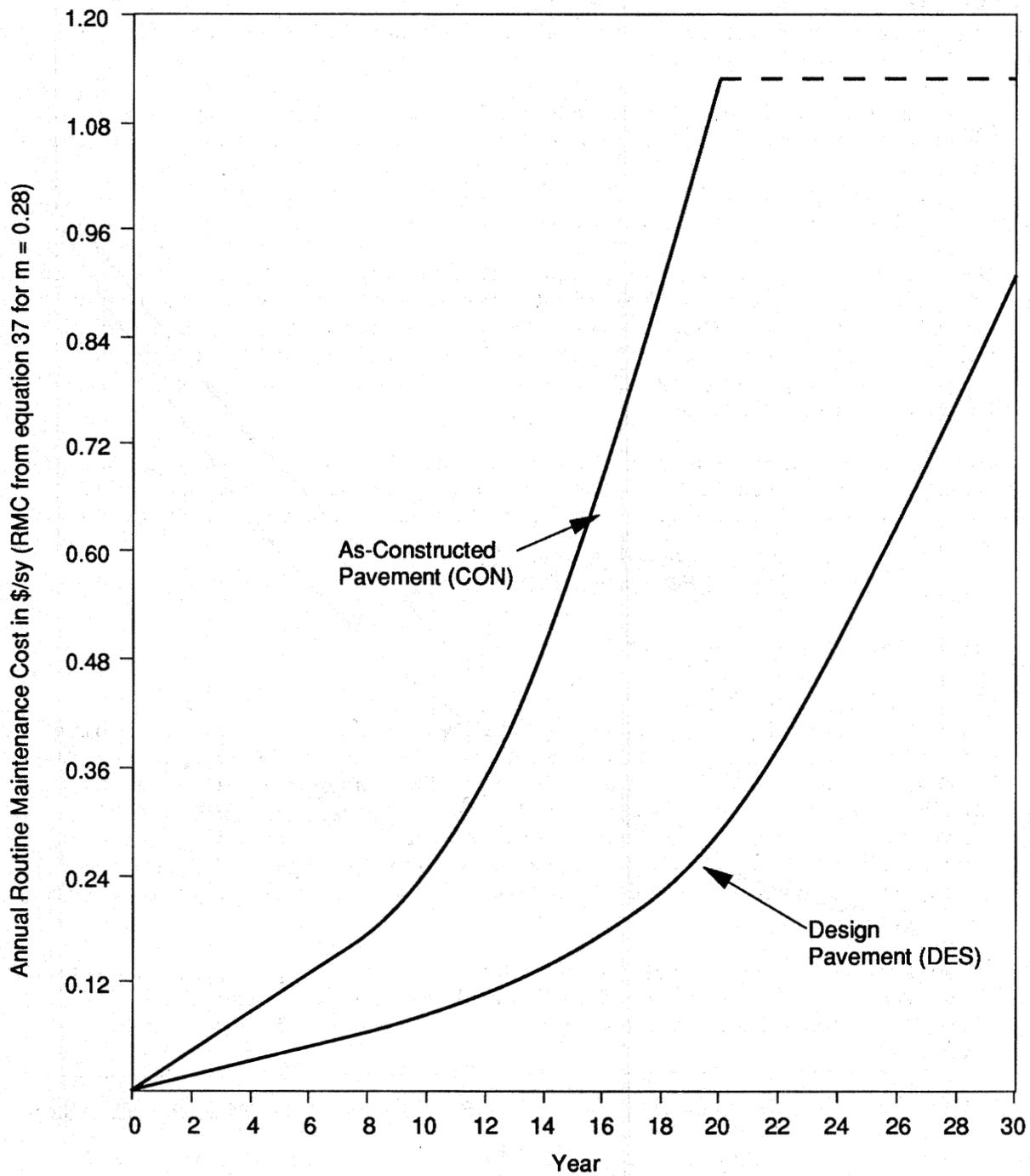


Figure 38. Routine maintenance cost histories.

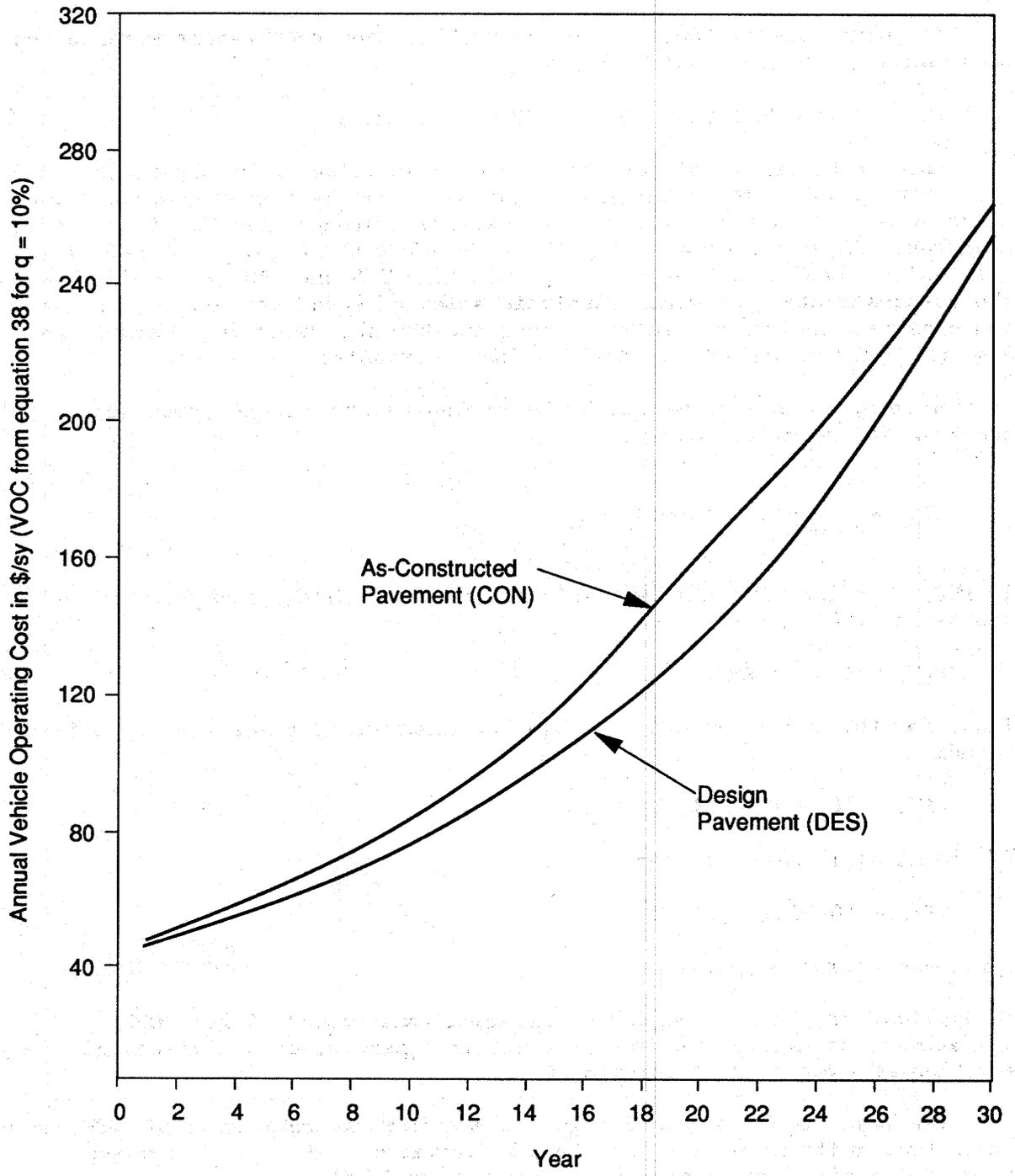


Figure 39. Vehicle operating cost histories.

$$TAC_y = 0.448 Q_y^2 + 45.675 (1.05)^{y-1} (0.957 + 0.088 Q_y) \quad (45)$$

and will produce the TAC_y cost histories shown in table 30.

The next line of table 30 (history 13) is for the present worth of each total annual cost and is defined by:

$$PWC_y (\$/sy) = TAC_y / (1 + i)^y \quad \text{for } Y = 0, 1, 2, \dots \quad (46)$$

For $Y = 0$, the total cost and its present value is simply the bid price (BP). For $Y > 0$, the total annual cost is given by either equation 43 or equation 44. If table 30 costs were depicted through year 30, it would be seen that PWC_y stays between approximately \$42 and \$44 (avg = \$42.85) for the design pavement and between approximately \$45 and \$50 (avg = \$47.29) for the as-constructed pavement. Analytic study of equations 41, 44, and 46, would be required to determine whether and how the relative constancy for CPW_y (or PWC_y) is dependent upon the input variables and levels.

History 14 is for the cumulative present worth at each year and is the summation of equation 46, i.e.,

$$CPW_y = \sum_{j=0}^Y PWC_j = BP + \sum_{k=1}^Y PWC_k \quad (47)$$

If PWC_k is relatively constant and has mean value MPWC, then equation 47 is approximated by:

$$CPW_y = BP + Y MPWC \quad (48)$$

Thus, for the two pavements in table 30, equation 48 gives the approximation formulas:

$$CPW_y \sim 30.00 + 42.85 Y \quad (49)$$

for the design pavement, and

$$CPW_y \sim 30.00 + 47.29 Y \quad (50)$$

for the constructed pavement.

In application, it will be found that equations 49 and 50 give CPW_y approximations that are essentially within 1 percent of the values given by equation 48 over years 1 through 30.

The last two histories in table 30 are for the conversion of CPW_y to an equivalent uniform annual cost ($EUAC_y$) for years 1, 2, ... The conversion formula was given in chapter 6, and is repeated below:

$$EUAC_y = F_y(i) * CPW_y \quad (51)$$

where

$$F_y(i) = [i(1 + i)^y] / [(1 + i)^y - 1] \quad (52)$$

The successive values of F_y are given in history 15 of table 30 and are common to both the design pavement and the as-constructed pavement. It can be shown that $F_y(i)$ is a hyperbola whose vertical asymptote is $Y = 0$ and whose horizontal asymptote is i (or 0.06 for the demonstration specification). Annual values for $EUAC_y$ are given in the final line (history 16) of table 30, and are plotted for both the design and constructed pavements in figure 40. Though not shown, very nearly the same graphs would be produced from the approximation equations 49 and 50.

At least for the demonstration specification, equation 48 gives a satisfactory approximation for CPW_y , as was shown above. If the approximation formula is used, then equation 51 becomes:

$$EUAC_y \sim F_y(i) * (BP + Y MPWC) \quad (53)$$

Analytical study of equations 52 and 53 shows that EUAC must first decrease, pass a minimum, then increase as years increase beyond $Y = 1$, provided that $MPWC > 0$, i.e., that not all TAC are zero.

ECONOMIC PERFORMANCE INDICATORS AND PAYMENT PLAN

This section illustrates the economic indicators of pavement performance and the associated payment plans that were addressed in chapter 6. Numerical results for the demonstration specifications are shown in table 31. Part A of the table gives results for the economic life methodology. For comparison purposes, part B of the table gives data that pertain to the load ratio methodology that is favored by the NJDOT.

Essential data for the economic life methodology are the coordinates (in figure 40) for the minimum point on the EUAC history of both the design pavement and the as-constructed pavement. The value of EUAC at the minimum point is denoted by MAC; the corresponding year is denoted by YMAC and is the pavement's economic life.

For both the design pavement and the as-constructed pavement, the spreadsheet program scans the EUAC histories to locate the minimum EUAC (MAC) and corresponding year (YMAC) for each pavement. More formally, YMAC is defined to be the smallest value of Y for which:

$$EUAC_y < EUAC_{(y+1)} \quad (54)$$

Table 31 shows that both DES YMAC and CON YMAC occur at $Y = 5$ years for the demonstration specification. Thus, both pavements have an economic life of 5 years. The corresponding minimum EUAC values are DES MAC = \$59.09/sy and CON MAC = \$61.17/sy.

As was discussed in chapter 6, the payment adjustment to the bid price for each construction lot is the present worth of the difference between the annual cost minimums (DES MAC - CON MAC), i.e., the present worth of -\$2.08/sy for the demonstration specification.

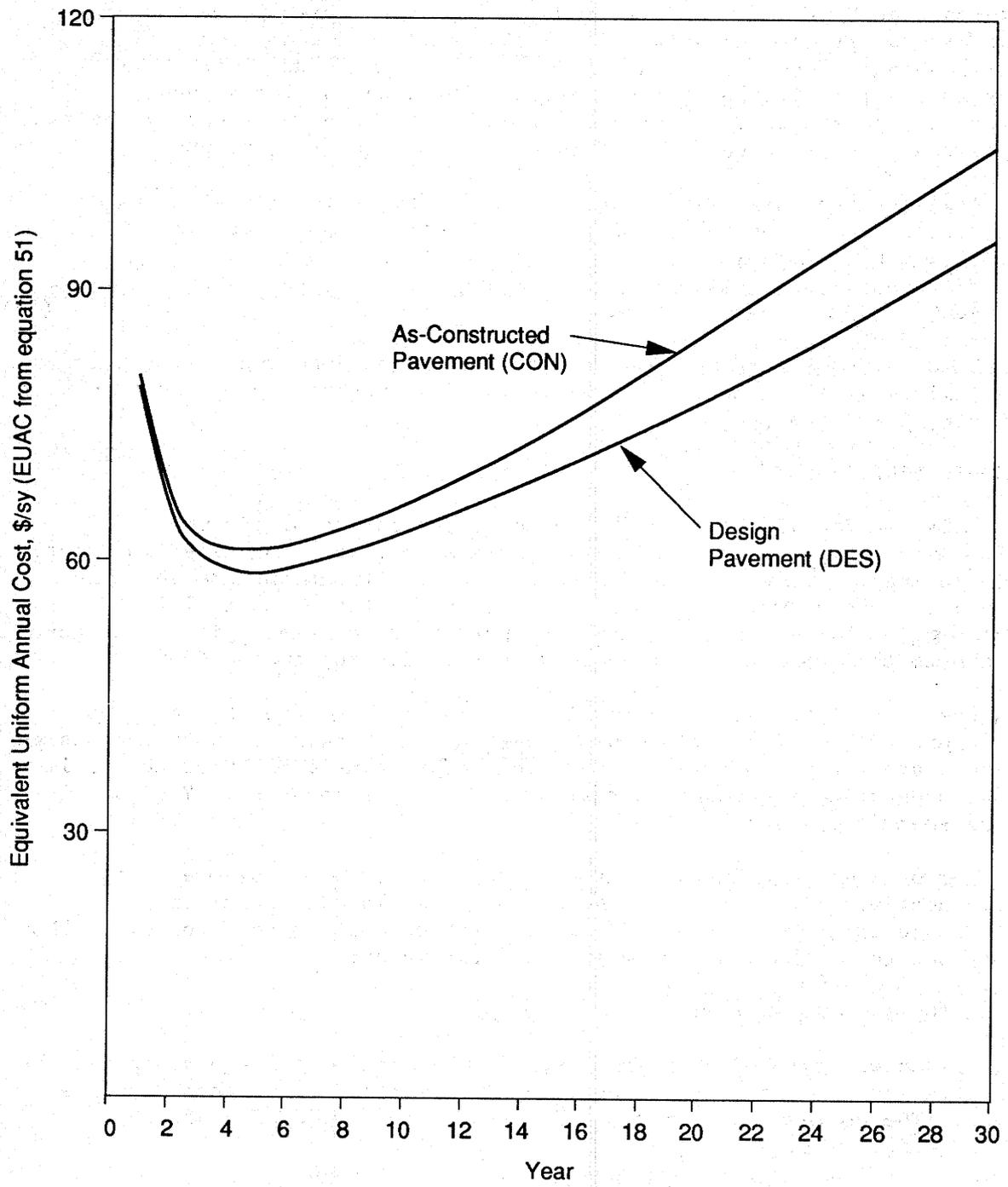


Figure 40. Histories for equivalent uniform annual costs (EUAC).

Table 31. Economic performance indicators and pay factors.

	<u>Design Pavement (DES)</u>	<u>Constructed Pavement (CON)</u>	<u>Difference (DES-CON)</u>
A. Economic Life Methodology			
1. Years at Min. EUAC (YMAC from equation 54)	5	5	0
2. Minimum EUAC (\$/sy) (MAC from equation 51)	\$59.09	\$61.17	-\$2.08
3. Pres. Worth Factor [G _y (YMAC) from equation 55]			4.219
4. Bid Price Adjustment [G _y (YMAC)*MAC Diff.]			-\$8.78
5. Bid Price (\$/sy)		\$30.00	
6. Payment (\$/sy) (BP + Adjustment)		\$21.22	
7. Pay Factor (Payment/Bid Price)		0.707	
	<u>DES</u>	<u>CON</u>	
B. Load Ratio Methodology			
1. Terminal PSI Level (Pt)	2.5	2.5	
2. Year for Pt (Yt)	21	11.5	
3. Cumulative ESAL (W _t) at Pt (W _t in millions of ESAL)	8.04	3.39	
4. Load Ratio (CON W _t / DES W _t)		0.422	
5. Bid Price (\$/sy)		\$30.00	
6. Payment (\$/sy, from equation 59)		\$26.34	
7. Pay Factor = Payment / BP		0.878	

The present worth factor for the MAC difference is the reciprocal of the present worth factor given by equation 52 for total annual costs, and is computed for the economic life of the as-constructed pavement. Thus, the present worth factor is given by:

$$G_y(\text{YMAC}) = 1/F_y(i) = [(1 + i)^{\text{CON YMAC}} - 1] / [i(1 + i)^{\text{CON YMAC}}] \quad (55)$$

and is shown in table 31 to be 4.219 for CON YMAC = 5 years. As was shown in chapter 6, the unit payment for each construction lot is given by:

$$\text{Payment} = \text{Bid Price} - G_y(\text{CON YMAC})(\text{CON MAC} - \text{DES MAC}) \quad (56)$$

where the second term is the payment adjustment to the bid price and can be either positive or negative. Table 31 shows that the demonstration adjustment is $-\$2.08 * 4.219$ or $-\$8.78/\text{sy}$ which is minus 29.2 percent of the bid price. Thus, the demonstration payment is $\$30.00 - \8.78 or $\$21.22/\text{sy}$. By definition, the specification pay factor is given by:

$$\text{Pay factor} = \text{Payment } (\$/\text{sy}) / \text{Bid Price } (\$/\text{sy}) \quad (57)$$

The pay factor for the demonstration is, therefore, $\$21.22 / \30.00 or 0.707.

If equation 53 is used to approximate EUAC, it can be shown that the minimum value for EUAC (MAC) occurs at the year YMAC that satisfies the equation:

$$[(j^{\text{YMAC}} - 1) / \ln j] - \text{YMAC} = \text{BP} / \text{MPWC} \quad (58)$$

where $j = 1 + i$ and $\ln j$ is the natural logarithm of j .

For the demonstration data, $\text{BP} = \$30.00/\text{sy}$ and MPWC from equations 49 and 50 is $\$42.85/\text{sy}$ for CON and $\$47.29$ for DES. Thus, the right side of equation 58 is 0.70 for DES and 0.63 for CON. It will be found that, to the nearest year, the corresponding YMAC is 5 years for both the DES and CON pavements as was shown in table 30. Equation 58 also shows that YMAC is about 10 years when MPWC is $\$8.00/\text{sy}$, and is about 20 years when MPWC is around $\$2.00/\text{sy}$. Thus, the YMAC year depends strongly upon the average present worth of annual costs.

If the YMAC relationship in equation 58 is substituted into equation 53, the minimum value of EUAC is given by:

$$\text{MAC} = (\text{MPWC}) * i * j^{\text{YMAC}} / \ln j \quad (59)$$

Equation 59 can then be written for both DES and MAC, and the results substituted into equation 56 to have the approximation formulas for the pay adjustment and pay factor. The new equations 58 and 59 not only provide good approximations for the specification pay factor, but also show that the major determinants of the pay factor are the mean present value of annual costs for DES and CON.

Part B of table 31 gives data that are required and results that might be obtained if the NJDOT load ratio methodology was used. As was discussed in chapter 6, these methods require the definition of a terminal

serviceability level for the pavement's first performance period. The selected value is shown to be PSI = 2.5 in table 31.

From the AASHTO PSI histories and the cumulative ESAL history, the number of ESAL at PSI = 2.5 (W_t) is determined for the design pavement (DES W_t) and the as-constructed pavement (CON W_t). For the demonstration, table 31 shows that DES $W_t = 8.04$ million ESAL at 21 years and (by interpolation) that CON $W_y = 3.39$ million ESAL at about 11.5 years. The load ratio is defined by:

$$\text{Load Ratio} = (\text{CON } W_t) / (\text{DES } W_t) \quad (60)$$

and is $3.39 / 8.04 = 0.422$ for the demonstration data. Equation 33 in chapter 6 provides the basis for calculating contractor payment. For the demonstration, it is assumed that $C_1 = \$8/\text{sy}$, $C_2 = \$7/\text{sy}$, and $\text{LOL} = 10$ years. Thus, equation 33 becomes:

$$\text{Payment} = \text{Bid Price} + \$16.77 (R^{\text{DES } Y_t} - R^{\text{CON } Y_t}) \quad (61)$$

where $R = (1 + \text{inflation rate}) / (1 + \text{interest rate})$, and DES Y_t and CON Y_t are the respective years at which the terminal PSI is reached by the two pavements. If R is assumed to be $1.00/1.06 = 0.943$, then for the demonstration example,

$$\begin{aligned} \text{Payment} &= \$30.00 + \$16.77 (0.292 - 0.509) \\ &= \$30.00 - 3.64 \\ &= \$26.36/\text{sy} \end{aligned}$$

and the specification pay factor is $\$26.36 / \$30.00 = 0.879$.

It is interesting to note that the load ratio pay factor would have been about 0.60, even if there was immediate failure (CON $Y_t = 0$) for the constructed pavement.

SENSITIVITY ANALYSES FOR THE DEMONSTRATION SPECIFICATION

This section describes sensitivity analyses that were run to illustrate how the demonstration specification pay factor changes when changes are made for selected input factors in table 29. Analyses were first made for pay factor sensitivity to only changes in the primary PCC specifications factors, and secondly for pay factor sensitivity to changes in economic factors in conjunction with PCC factor changes. Input data for the illustrative analyses are shown in table 32.

The first two columns of table 32 identify the design pavement (DES) and the as-constructed pavement (CON) for each of 47 runs of the spreadsheet demonstration PRS system. The next four columns give physical inputs for initial four-lane ESAL (W_o), initial serviceability (P_o), PCC thickness (D_o), and PCC compressive strength (F'_o). For all cases in the study (DES and CON), the initial design lane ESAL (W_1) is specified to be 45 percent of W_o .

The next four columns give economic inputs for discount or interest rate (i), bid price (BP in $\$/\text{sy}$), the maintenance cost parameter (m in $\$/\text{sy}$)

Table 32. Sensitivity analysis data for performance-related payment plan based on economic life.

CASE DESIGN PVT OR CONSTR. PVT	SEQ CODE	PHYSICAL INPUTS				ECONOMIC INPUTS				PERFORMANCE INDICATOR OUTPUTS				PAYMENT PLAN OUTPUTS			
		INITIAL		PCC		DISC. RATE (%)	BID PRICE (\$/yr)	ANN. COSTS @ PSI = 2.5 (\$/yr) MAINT.	% VOC** Considered	YEARS WHEN PSI = 2.5	MESAL WHEN PSI = 2.5	MIN EUAC (\$/yr)	YEARS AT MIN EUAC	BID PRICE ADJUSTMENTS		PAYMENT FOR LOT	
		ESAL (10 ³)	PVT. PSI	THK. (in)	fp _{c28} * (kpsi)									\$/yr	% B.P.	\$/yr	% B.P.
DES	1	500	4.3	9.0	4	6	30	0.28	10.00	21	8.04	59.09	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	-	-	-	>30	?	57.57	5	6.43	21.4	36.43	121.4
CON	B	-	-	9.5	4.0	-	-	-	-	28	13.1	57.65	5	6.06	20.2	36.06	120.2
CON	C	-	-	9.5	3.5	-	-	-	-	23	9.32	57.74	5	5.69	19.0	35.69	119.0
CON	D	-	-	9.0	4.5	-	-	-	-	27	12.3	57.65	5	6.06	20.2	36.06	120.2
CON	E	-	-	9.0	4.0	-	-	-	-	23	9.32	57.74	5	5.69	19.0	35.69	119.0
CON	F	-	-	9.0	3.5	-	-	-	-	19	6.60	57.92	5	4.94	16.5	34.94	116.5
CON	G	-	-	8.5	4.5	-	-	-	-	22	8.66	57.83	5	5.31	17.7	35.31	117.7
CON	H	-	-	8.5	4.0	-	-	-	-	18	6.33	58.01	5	4.56	15.2	34.56	115.2
CON	I	-	-	8.5	3.5	-	-	-	-	15	4.63	58.28	5	3.43	11.4	33.43	111.4
CON	J	-	4.3	9.5	4.5	-	-	-	-	30	14.95	58.92	5	0.75	2.5	30.75	102.5
CON	K	-	-	9.5	4.0	-	-	-	-	26	11.12	59.00	5	0.38	1.3	30.38	101.3
CON	L	-	-	9.5	3.5	-	-	-	-	21	8.04	59.09	5	0.00	0.0	30.00	100.0
CON	M	-	-	9.0	4.5	-	-	-	-	25	10.74	59.00	5	0.38	1.3	30.38	101.3
CON	N	-	-	9.0	3.5	-	-	-	-	17	5.57	59.36	5	-1.13	-3.8	28.87	96.2
CON	O	-	-	8.5	4.5	-	-	-	-	20	7.44	59.18	5	-0.38	-1.3	29.62	98.7
CON	P	-	-	8.5	4.0	-	-	-	-	15	5.32	59.36	5	-1.13	-3.8	28.87	96.2
CON	Q	-	-	8.5	3.5	-	-	-	-	13	3.99	59.81	5	-3.02	-10.1	26.98	89.9
CON	R	-	4.0	9.5	4.5	-	-	-	-	27	12.3	60.27	5	-4.87	-16.6	25.03	83.4
CON	S	-	-	9.5	4.0	-	-	-	-	23	9.32	60.36	5	-5.34	-17.8	24.66	82.2
CON	T	-	-	9.5	3.5	-	-	-	-	19	6.60	60.45	5	-5.72	-19.1	24.28	80.9
CON	U	-	-	9.0	4.5	-	-	-	-	23	9.32	60.36	5	-5.34	-17.8	24.66	82.2
CON	V	-	-	9.0	4.0	-	-	-	-	18	6.33	60.54	5	-6.10	-20.3	23.90	79.7
CON	W	-	-	9.0	3.5	-	-	-	-	15	4.66	60.72	5	-6.86	-22.9	23.14	77.1
CON	X	-	-	8.5	4.5	-	-	-	-	18	6.33	60.54	5	-6.10	-20.3	23.90	79.7
CON	Y	-	-	8.5	4.0	-	-	-	-	15	4.66	60.72	5	-6.86	-22.9	23.14	77.1
CON	Z	-	-	8.5	3.5	-	-	-	-	12	3.39	61.17	5	-8.76	-29.2	21.24	70.8
A ESAL																	
DES	2.1	400	4.3	9.0	4.0	6	30	0.28	10.00	24	8.01	48.70	6	0	0	30	100
CON	A	400	4.6	9.5	4.5	-	-	-	-	>30	?	47.41	6	6.35	21.2	36.35	121.2
CON	Z	400	4.0	8.5	3.5	-	-	-	-	14	3.36	50.16	5	-6.15	-20.5	23.85	79.5
A DISC. RATE																	
DES	2.2	600	4.3	9.0	4.0	6	30	0.28	10.00	19	7.92	69.62	4	0	0	30	100
CON	A	600	4.6	9.5	4.5	-	-	-	-	30	17.9	67.76	5	7.83	26.1	37.83	126.1
CON	Z	600	4.0	8.5	3.5	-	-	-	-	10	3.40	71.73	4	-7.33	-24.4	22.67	75.6
A BID PRICE																	
DES	3.1	500	4.3	9.0	4.0	4	30	0.28	10.00	21	8.04	58.81	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	4	-	-	-	>30	?	57.28	5	6.83	22.8	36.83	122.8
CON	Z	-	4.0	8.5	3.5	4	-	-	-	12	3.39	60.91	5	-9.33	-31.1	20.67	68.9
A MAINT. COSTS																	
DES	3.2	500	4.3	9.0	4.0	8	30	0.28	10.00	21	8.04	61.53	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	8	-	-	-	>30	?	57.87	5	6.07	20.2	36.07	120.2
CON	Z	-	4.0	8.5	3.5	8	-	-	-	12	3.39	61.45	5	-8.24	-27.5	21.76	72.5
A % VEH. OPN. COST																	
DES	4.1	500	4.3	9.0	4.0	6	35	0.28	10.00	21	8.04	60.28	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	35	-	-	>30	?	58.75	5	6.43	18.4	41.43	118.4
CON	Z	-	4.0	8.5	3.5	-	35	-	-	12	3.39	62.36	5	-8.76	-25.0	26.24	75.0
A % VEH. OPN. COST																	
DES	4.2	500	4.3	9.0	4.0	6	25	0.28	10.00	21	8.04	57.91	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	25	-	-	>30	?	56.38	5	6.43	25.7	31.43	125.7
CON	Z	-	4.0	8.5	3.5	-	25	-	-	12	3.39	59.79	4	-6.53	-26.1	18.47	73.9
A MAINT. COSTS																	
DES	5.1	500	4.3	9.0	4.0	6	30	0.22	10.00	21	8.04	59.09	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	-	0.22	-	>30	?	57.56	5	6.41	21.4	36.41	121.4
CON	Z	-	4.0	8.5	3.5	-	-	0.22	-	12	3.39	61.16	5	-8.72	-29.1	21.28	70.9
A MAINT. COSTS																	
DES	5.2	500	4.3	9.0	4.0	6	30	0.34	10.00	21	8.04	59.10	5	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	-	0.34	-	>30	?	57.57	5	6.45	21.5	36.45	121.5
CON	Z	-	4.0	8.5	3.5	-	-	0.34	-	12	3.39	61.19	5	-8.80	-29.3	21.20	70.7
A % VEH. OPN. COST																	
DES	6.1	500	4.3	9.0	4.0	6	30	0.28	5.00	21	8.04	32.72	7	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	-	-	5.00	>30	?	31.87	7	4.75	15.8	34.75	115.8
CON	Z	-	4.0	8.5	3.5	-	-	-	5.00	12	3.39	33.92	6	-5.62	-19.7	24.08	80.3
A % VEH. OPN. COST																	
DES	6.2	500	4.3	9.0	4.0	6	30	0.28	15.00	21	8.04	84.69	4	0	0	30	100
CON	A	-	4.6	9.5	4.5	-	-	-	15.00	>30	?	82.55	4	7.38	24.6	37.38	124.6
CON	Z	-	4.0	8.5	3.5	-	-	-	15.00	12	3.39	87.48	4	-9.70	-32.3	20.30	67.7

* Converted to f₂₈ = 1.22 f₁₇ = 1.22[131.2 + 0.093 f_{c28}]

** Vehicle Oper. Costs (\$/mile) = 14.32 x (Tot ESALs) x (1.40 - 0.088 x PSI)

in equation 37, and the vehicle operating cost parameter (q in percent) in equation 38.

In the right half of table 32, four performance indicator outputs and four payment plan outputs are shown. These correspond to the demonstration outputs that were listed in table 31. The four performance indicators are the year at which $PSI = 2.5$, the corresponding cumulative ESAL, the year (YMAC) at which EUAC is minimum, and the corresponding minimum value (MAC) of EUAC.

Payment plan outputs in the last four columns of table 32 are the bid price adjustments, both in $\$/sy$ and as a percent of bid price, and the resultant payment, both in $\$/sy$ and as a percent of bid price. The last column contains 100 times the pay factor for the as-constructed pavement lot. Values in the last column, after division by 100, have been used as the dependent variable in the illustrative sensitivity analyses. More extensive analyses would involve one or more of the remaining output variables, and would perhaps include load ratios that are determined by the ESAL for which $PSI = 2.5$.

The physical input columns show that all eleven designs have the same PCC specifications, namely, $P_o = 4.3$, $D_o = 9.0$ in (22.9 cm), and $F'_c = 4000$ psi (281 kg/cm²). Moreover, nine of the designs have $W_o = 500,000$ ESAL ($W_1 = 225,000$ ESAL), whereas DES 2.1 and DES 2.2 have $W_o = 400,000$ ESAL and $W_o = 600,000$ ESAL, respectively. The remaining pairs of designs differ from DES 1 only with respect to either the interest rate (DES 3.1 and DES 3.2), the bid price (DES 4.1 and DES 4.2), the maintenance cost parameter (DES 5.1 and DES 5.2), or the vehicle operating cost factor (DES 6.1 and DES 6.2).

For DES 1, the associated as-constructed pavements (CON A through CON Z) differ from DES 1 only with respect to the three primary specification factors (P_o , D_o , and F'_c). Taken together, DES 1 and CON A through CON Z represent a 3 by 3 by 3 factorial study of P_o values at 4.6, 4.3, and 4.0, D_o values at 9.5 in (24.1 cm), 9.0 in (22.9 cm), and 8.5 in (21.6 cm), and F'_c values at 4,500 psi (316 kg/cm²), 4,000 psi (281 kg/cm²), and 3,500 psi (246 kg/cm²). It can be seen that DES 1 is at the middle level for all three factors, that CON A is at the high levels, and that CON Z is at the low levels. Levels and increments of the three PCC factors have been selected so that DES 1 represents an average AASHO Road Test rigid pavement section. The CON A pavement is assumed to be as much superior to DES 1 as might be expected in construction practice. The CON Z pavement is assumed to be the minimum level of construction that would be tolerated by the specification acceptance plans. The final column of table 32 shows that, relative to DES 1, pay factors calculated by the spreadsheet program range from 1.214 for CON A to 0.708 for CON Z.

For each remaining design in table 32, CON A and CON Z have the same PCC factor levels as for DES 1, and the same level for non-PCC factors as the design with which they are compared. For example, DES 6.1 and DES 6.2 and the corresponding CON A and CON Z differ from DES 1 only with respect to the vehicle operating cost parameter ($q = 5$ percent for DES 6.1 and $q = 15$ percent for DES 6.2).

In summary, DES 1 with CON A through CON Z provide sensitivity data for a complete factorial of PCC factors. The remaining five pairs of design pavements, together with CON A and CON Z, provide sensitivity data for five different non-PCC factors when all remaining non-PCC factors are at the DES 1 level.

Analysis of variance (ANOVA) and regression analysis were performed for the 26 pay factors in the top part of table 32 for CON A through CON Z relative to DES 1. Results of the analyses are given in table 33. The dependent variable for the analyses is the decimal value of the pay factor and not the percentage values that are given in table 32. The three independent variables are P_o , D_c , and F'_c , where each is expressed as a deviation from its central (DES 1) value divided by its half-range. Thus, for example, P_o becomes $P_{od} = (P_o - 4.3)/0.3$. The deviation variables thus have values of -1, 0, and +1 for their linear form. Corresponding quadratic forms are, for example $(3P_{od}^2 - 2)$, and have values of 1, -2, and 1 at the three values of P_o .

As shown in the ANOVA portion of table 33, the 27 observations give two degrees of freedom for each main effect, one for the linear term and one for the quadratic term. Each two-factor interaction has 4 degrees of freedom and can be separated into linear x linear, linear x quadratic, quadratic x linear, and quadratic x quadratic. Finally, the three-factor interactions provide 8 degrees of freedom and are considered to represent unexplained variation (experimental error).

The middle ANOVA column shows that 95.5 percent of the total variation is explained by the linear component of P_o , and that 2.32 percent and 1.62 percent, respectively, of the total variation is explained by the linear components of D_c and F'_c . Thus, the three linear components account for 99.44 percent of the total variation among the 27 pay factors. The ANOVA shows that the quadratic effect of D_c , the quadratic effect of F'_c , and two interaction effects are also highly significant at less than the 1 percent level, i.e., $SL = (00)$. The regression coefficients for the seven significant terms are shown in the last column of table 33, including the constant term for the equation which is the mean value of the pay factor (0.9851). The R-square for the regression analysis is 0.999, and the root-mean-square residual is $RMS = 0.0053$. Thus, the pay factor regression equation gives an exceptionally close fit to the pay factor data, and predicts all observed values to within 0.01.

If the significant quadratic and interaction terms are ignored, the resulting approximation equation in terms of the original factors is:

$$\text{Pay Factor (PF)} = -2.5254 + 0.6435 P_o + 0.0602 D_c + 0.0503 F'_c \text{ (in ksi)} \quad (62)$$

whose R-square is 0.994 and whose RMS residual is 0.013. Thus, the approximation equation predicts all observed pay factors to within about 0.02.

Table 33. Variance and regression analysis for pay factor dependence on primary PCC specifications.

PCC SPECIFICATIONS AND REGRESSION VARIABLES	df	ANOVA Percent of Total SS	Signif. Level	REGRESSION EQUATION FOR PAY FACTOR
Initial PSI = P_o	2	95.50%	(00)	---
Lin: $P_{od} = (P_o - 4.3)/0.3$	1	95.50%	(00)	0.1931
Quad: $(3 P_{od}^2 - 2)$	1	0.00%	(NS)	---
PCC Thick. (in) = D_c	2	2.40%	(00)	---
Lin: $D_d = (D_c - 9.0)/0.5$	1	2.32%	(00)	0.0301
Quad: $(3 D_d^2 - 2)$	1	0.07%	(00)	-0.0031
PCC Compr. Str. (ksi) = F'_c	2	1.67%	(00)	---
Lin: $F_d = (F'_c - 4.0)/0.5$	1	1.62%	(00)	0.0252
Quad: $(3 F_d^2 - 2)$	1	0.04%	(00)	-0.0024
$P_o * D_c$ Interactions	4	0.01%	(NS)	---
$P_o * F'_c$ Interactions	4	0.03%	(NS)	---
$D_c * F'_c$ Interactions	4	0.36%	(00)	---
$D_d * F_d$	1	0.33%	(00)	-0.0138
$D_d * (3 F_d^2 - 2)$	1	0.03%	(05)	0.0024
Other	2	0.00%	(NS)	---
$P_o * D_c * F'_c$ Interactions (Error Term)	8	0.02%		
			MS = 0.218	

Mean Pay Factor = 0.9851

R-square = 0.999

RMS residual = 0.0053

Approximation Regression Equation Without Quadratic and Interaction Terms:

$$\text{Pay Factor (PF)} = - 2.5254 + 0.6435 P_o + 0.0602 D_c + 0.0503 F'_c$$

R-Square = 0.994 RMS Residual = 0.013

The practical importance of the foregoing regression analysis is that it provides a way to derive pay factor equations for the economic life methodology. Although the pay factor is given in principle by equations 55, 56, and 57, these equations involve CON YMAC, the year at which EUAC is a minimum for the as-constructed pavements. Although CON YMAC is indeed a function of P_o , D_c and F'_c , it is virtually impossible to derive an explicit relationship among CON YMAC and its determinants.

On the other hand, it would be quite feasible to extend the PRS spreadsheet program to include the derivation of a pay factor regression equation that covered specified ranges for P_o , D_c , and F'_c . If desired, the procedures could be further extended to include other construction variables.

It is noted that the NJDOT load ratio methodology involves a pay factor equation (equation 34) whose determinants are the years to terminal serviceability for the design pavement and the as-constructed pavement. Thus, in conjunction with the AASHTO equation 41, the pay factor equation 62 is "almost explicit" with respect to P_o , D_c , and F'_c .

Sensitivities of the specification pay factor to selected changes in PCC and non-PCC factors are shown in table 34. In part A, PF sensitivities to P_o , D_c , and F'_c are calculated from the approximation pay factor equation 62. For this linear equation with no-cross products, the PF change per unit of any determinant is simply the determinant's coefficient. For example, the change in PF per inch of PCC thickness is the coefficient for D_c . Thus, the equation predicts that PF will change by about 0.06 per inch (0.0236 per cm) of thickness change.

The first three lines of table 34 give base levels for each PCC factor, 10 percent changes in the base levels, and PF changes that result from the factor changes. It can be seen that a 10 percent change in P_o produces almost five times as much PF change as does a 10 percent change in D_c , and over ten times the change produced by a 10 percent change in F'_c .

The PF change for 10 percent change in D_c is over twice as much as for a 10 percent change in F'_c . Thus, it appears that the relative impact of P_o , D_c , and F'_c changes on PF are in the approximate ratio of 10-2-1. These relative sensitivities are of course related to the relative effects of the three factors on the PSI histories (table 30 and figure 37), and the subsequent effects on maintenance and vehicle operating costs.

The last two lines in table 34 show how much each factor must be changed to produce a 0.01 change in the pay factor. The required percent changes from base levels are 0.4 percent for P_o , 1.9 percent for D_c , and 5 percent for F'_c . Pay factor sensitivity to changes in non-PCC factors is also illustrated in table 34. The results shown are derived from the data given in the bottom part of table 32.

The five non-PCC factors are initial 4-lane ESAL rate (W_o), interest rate (i), bid price (BP), annual maintenance cost (m) at $PSI = 2.5$, and the percent (q) of vehicle operating costs to be considered. For each factor, base levels are those for DES 1 in table 32, as shown in the top line of table 34. The next line shows a 10 percent increase for each base level,

Table 34. Sensitivity of specification pay factor to selected changes in PCC and non-PCC factors.

A. Illustrative Pay Factor Sensitivities to Changes in Primary PCC Factors

PRIMARY PCC SPECIFICATIONS FACTORS			
	Initial PSI (P _o)	PCC Thickness (D _o)	PCC Compressive Strength (F' _c)
Base Level for PCC Factor	4.3	9.0 in.	4.0 ksi
10% Change in Base Level	0.43	0.9 in.	400 psi
Corresponding Pay Factor Change	0.276	0.054	0.020
Factor Change Needed to Produce 0.01 PF Change	0.02	0.17 in	200 psi
Corresponding Percent of Base Level	0.4% of 4.3	1.9% of 9.0 in	5% of 4 ksi

B. Selected Pay Factor Sensitivities to Changes in Non-PCC Factors

SELECTED NON-PCC FACTORS					
	Initial ESAL (W _o)	Interest Rate (i)	Bid Price	Maint. Cost (m)	% (q) for VOC
Base Level for Non-PCC Factor	500,000 ESAL	6%	\$30.00/sy	\$0.28/sy	10%
10% Increase in Base Level	50,000 ESAL	0.6%	\$3.00/sy	\$0.028/sy	1%
Corresponding Change in Pay Factor for CON A	+0.012	-0.004	+0.022	+0.0002	+0.009
Corresponding Change in Pay Factor for CON Z	-0.010	+0.005	-0.003	-0.0005	-0.013
Estimated Factor Change to Produce +0.01 PF Change	For CON A +40,800 ESAL	-1.5%	+1.37/sy	+\$1.20/sy	+1.1%
	For CON Z -51,300 ESAL	+1.1%	- \$9.09/sy	-\$0.60/sy	-0.8%

and the following two lines show corresponding PF changes for the CON A and CON Z as-constructed pavements. In table 32, it was shown that the CON A pavement has $P_o = 4.6$, $D_c = 9.5$ in (24.1 cm), $F'_o = 4500$ psi (316 kg/cm²), and that the CON Z pavement has $P_o = 4.0$, $D_c = 8.5$ in (21.6 cm), and $F'_o = 3500$ psi (246 kg/cm²). Thus, table 34 shows PF sensitivities to non-PCC factor changes for the two extremes of change for CON Z. For the interest rate (i), the CON A change is negative, but is positive for the remaining factors. In magnitude, the only pay factor change that equals the corresponding changes for PCC factors is the bid price effect (+0.02) on as-constructed pavements.

For the 10 percent change in base levels of the non-PCC factors, the third and fourth lines of table 34 show corresponding pay factor changes that range in magnitude from 0.00 to 0.02, approximately. For all five factors, the pay factor change for CON A is opposite in sign to the PF for CON A. Next lower in magnitude are the effects of changes in initial ESAL (W_o) and the percent (q) of VOC used. These changes produce PF changes of around 0.01 in magnitude. A 10 percent interest rate (i) change produces about 0.005 change in the pay factor, and virtually no PF change is induced by 10 percent change in the annual maintenance cost parameter (m). Since initial ESAL and bid price cannot be controlled in practical applications, it appears that only the VOC factor needs further attention in further development of PRS systems.

COMMENTS ON THE DEMONSTRATION SPECIFICATION ALGORITHMS AND RESULTS

The following comments relate to the procedures and results that were presented in the first four sections of this chapter. For the most part the comments are extensions of statements that were made in the course of the chapter presentation, and are generally listed in the order of their occurrence within the chapter.

1. It is quite clear that further development is needed for the algorithms that have been used for routine maintenance costs (equation 37) and for vehicle operating costs (equation 38). The present formulations appear to give imbalance between the two types of costs, and may give unduly low weight to RMC and unduly high weight to VOC. A better rationale needs to be determined for annual costs.

2. The annual cost rationale might be related to estimated costs for various distress levels and types (option a in figure 33). However, the only available distress prediction equations appear to be those produced in the COPES project.^(85.2) It has not been fully determined to what extent the predicted distress levels are sensitive to the PCC factor changes. Thus, costs based on the COPES equations might produce the same economic lives for both the design and constructed pavement, and therefore pay factors of 1.00 for all types of construction.

3. If serviceability levels are used to produce cost histories, economic life and load ratios will depend strongly upon the serviceability prediction equation that is used. It was demonstrated, for example, that pay factors based on COPES serviceability predictions would give the same pay factors for all constructed pavements in table 32.

4. The demonstration specification gave what appears to be an unusually small number of years (YMAC) at which minimum EUAC was reached (see table 32). For most cases the minimum occurs when serviceability is quite high after only 4 to 6 years of performance. It is thus questionable as to whether YMAC years are indeed turning points in rigid pavement life.

5. Since the AASHTO PSI equation is much more sensitive to initial profile in terms of P_0 than to any other M&C factor, it may be advisable to derive one payment plan for P_0 and another for the remaining PCC specification factors. Perhaps the overall pay factor could be the product of the individual pay factors.

6. In the economic life methodology, the difference between minimum EUAC for the CON and DES pavements is multiplied by the G_y factor (equation 55) whose variables are the interest rate and the CON economic life. Strong consideration should be given to the use of DES economic life in the G_y factor. Thus, G_y would be a constant multiplier for all EUAC differences throughout the construction project. Otherwise, it appears that G_y is relatively low for poorly constructed lots and relatively high for superior lots.

7. More study needs to be made of the conditions for which the cumulative present worth of total annual costs (equation 47) is relatively constant over all years, as was the case for the demonstration specification. If this constancy prevails for all practical cases, then a much simpler algorithm (equation 53) can be used to calculate EUAC. Such simplification would greatly enhance further study of the mathematical properties (e.g., VOC derivatives with respect to years) of the EUAC function.

8. No effort was made in the present study to determine practical limits for the pay factor (equation 57). Further study should produce a rationale for such limits, as has been done for the NJDOT load ratio methodology.

9. Further attention should be paid to the question of incorporating inflation rate into the discount factor, as has been done in the NJDOT methodology (see equation 30).

10. It would be quite useful to extend the demonstration specification algorithms to include (1) factorial pay factor data as in the top portion of table 32, and (2) regression analysis as shown in table 33. It seems likely that the resulting regression equation would be quite adequate for all applications of the pay factor function, including the development of its associated operating characteristic. Although these procedures are unnecessary to the load ratio methodology, they represent the only practical way to derive a pay factor function for the economic life methodology.

11. The importance of sensitivity analyses for the specification payment plan cannot be overestimated. Only when these analyses have been fully developed for a given plan can both the contractor and SHA be well informed on how the pay factor changes with changes in any input variable.

12. Sensitivity analyses have not been fully developed for the demonstration PRS system. For example, no effort was made to determine pay

factor sensitivity to changes in the subgrade modulus (k) or to simultaneous changes in PCC and non-PCC factors. Further development should not only extend to additional variables but should produce factorial pay factor data for all factors within the same matrix. It will thus be possible to learn of curvilinear and interacting effects of the input factors.

13. Although a formidable task, differential calculus should be applied to the whole set of equations that are used to determine the specification pay factor. If this task can be accomplished, sensitivities can be calculated from pay factor derivatives with respect to all input factors. The results can be used to make improvements in the specification rationale.

14. At present, there are three competing methodologies for rigid pavement PRS development: (1) economic life methods as used in NCHRP Project 10-26A, (2) pavement life or load ratio as used in the NJDOT approach, and (3) life-cycle cost methods as conceptualized in this study. Further research is needed to determine major pros and cons for the three methods. Many of the needed comparisons can be done through sensitivity analyses.

15. The validity of any PRS methodology depends greatly upon the validity of the primary performance equations that are used to estimate pavement condition and life. Validation of existing equations and derivation of new equations must perhaps await the results of SHRP and other field studies of pavement performance.

CHAPTER 8
SUMMARY AND RECOMMENDATIONS FOR FURTHER RESEARCH

SUMMARY

Much emphasis has been placed recently on the development of performance-related specifications (PRS) for both rigid and flexible pavements; so much so that the Federal Highway Administration has made it one of its High-Priority National Program Areas. This emphasis has come about as a result of a recognized need to better control the materials and construction (M&C) factors that have the most effect on performance while simultaneously relaxing (or de-emphasizing) the controls over those M&C factors that have little effect on performance. In other words, procedures are needed to design and construct better and more cost-effective pavements by inducing highway engineers and contractors to focus more attention on such factors as slab thickness, PCC strength and initial smoothness and less attention (perhaps) to such factors as subbase thickness and strength.

These PRS procedures (or systems) have been and are being programmed to reward contractors for constructing better pavements than specified and penalize them for building poorer pavements. The basis for the rewards and penalties are selected pavement performance prediction models that consider measured values for various M&C factors (as provided by the contractor) in estimating future performance. Thus, if the contractor builds a pavement that does not provide the expected (predicted design) performance because of failure to meet one or more M&C specifications, a penalty would be assessed by receiving only a fraction of the bid amount. Conversely, if a pavement is built that exceeds the expected performance, the contractor would receive an amount higher than the original bid. The amount of the penalty (disincentive) or reward (bonus/incentive) depends upon how far the contractor was below or above the various M&C specifications. In M&C specifications that are highly performance-related, a contractor would be penalized heavily for falling a half-inch (1.27 cm) below the specification for a significant factor like slab thickness but only minimally for falling an inch (2.54 cm) or so below the specification for a less significant factor like subbase thickness. Obviously, the ideal PRS system would account for the levels of all the M&C factors delivered by the contractor (on a day-to-day or lot-to-lot basis) in assessing the penalty or reward.

In keeping with the objectives of the contract, this study has furthered the development of a PRS system for PCC pavements (such as that described above) in several key areas:

Framework for Development of Performance-Related Specifications

Under NCHRP Project 10-26, "Data Bases for Performance-Related Specifications," completed in 1985, it was basically concluded that existing data bases were not adequate for direct derivation of the necessary relationships for PRS.^(85.4) As a result, a key member of the NCHRP Panel, derived an approach that overcame the biggest problem with using existing data bases and performance prediction relationships for PRS development.^(89.4) His approach became the basis for PRS development under NCHRP Project 10-26A for asphalt concrete pavements as well as this study for PCC

pavements. (87.4, 88.1) Chapter 2 of this report provides a refinement and summary of that framework as it applies to PCC pavements.

Identification of Relationships Available to Establish the Connection Between M&C Factors and Various Measures of Pavement Performance

Since there are no existing pavement performance prediction relationships that expressly consider the multitude of M&C factors, a two-stage mechanism was established which allowed the existing performance prediction relationships to account for the effects of more M&C factors. The first stage consisted of the actual pavement performance prediction relationships (referred to as primary prediction relationships or PPRs) which did include some M&C factors such as slab thickness, concrete strength and initial serviceability (riding quality). To allow other M&C factors to have an influence on these PPRs, a second stage of relationships (referred to as secondary prediction relationships or SPRs) was identified. Using these SPRs, the effect of additional M&C variables could be considered by predicting what are now independent variables in the PPRs. Although these SPRs could be used to estimate the value of more than one independent variable in the PPRs, the focus in this study was on those that could be used to predict various properties of the hardened concrete, particularly, strength and elastic modulus.

Chapters 3 and 4 document the work that was accomplished in this study by identifying the available primary and secondary prediction relationships and in evaluating existing data bases that could be used to develop these relationships. Appendixes A through E are also part of this documentation.

Development and Conduct of Laboratory/Field Test Program(s) to Quantify New Relationships Needed for PRS Development

Because of its extensive use as the measure of concrete strength in most of the available rigid pavement performance prediction relationships, PCC flexural strength was selected in this study as the one key determinant of rigid pavement distress/performance to use in demonstrating the two-stage approach to PRS development. Although flexural strength is considered an M&C variable, it (unlike slab thickness) is not directly controllable. Since it is dependent on so many other M&C factors (i.e., water content in mix, cement content, aggregate type, air content, etc.), it was the ideal choice for the demonstration PRS system.

Based upon a thorough review of existing secondary prediction relationships, it was determined that there were none available which had more than two M&C factors as independent variables. This included those relationships for PCC flexural strength. In addition, none of the relationships provided any of the basic measures of statistical accuracy (i.e., coefficient of determination, standard error of estimate, etc.) needed to treat their variability or lack-of-fit. Consequently, a laboratory study was planned and executed in order to provide a basis for developing new prediction relationships. A fractional factorial of seven factors (each at two levels) was used in the experiment design. The seven factors included: (1) coarse aggregate type, (2) coarse aggregate maximum size, (3) fine aggregate modulus, (4) air entraining agent quantity, (5) coarse aggregate quantity, (6) cement quantity, and (7) water quantity.

Tests on the plastic concrete (just after mixing) included slump and air content. Tests on the hardened PCC specimens consisted of compressive strength (7- and 28-day), flexural strength (7-day), elastic modulus (28-day) and splitting (indirect) tensile strength. In addition, measurements were made to determine the average unit weight and yield of each batch. Using standard analysis of variance and statistical regression procedures, a series of relationships was derived, most of which were directly applicable to the new demonstration PRS system.

The results of the laboratory studies and experiments are presented in chapter 5 of this report. Because of the comprehensive nature of the lab study, many of the relationships derived have wider application (for engineering purposes) than for PRS development.

With respect to field studies, none were conducted. It was initially envisioned that some field measurements and performance data could be collected to either verify or calibrate existing PPRs. As the study progressed, however, it was decided that the data necessary to improve any existing PPRs would be expensive to obtain and not very cost-effective. Thus, the primary efforts were directed towards the laboratory experiment while field studies were deferred to future research.

Demonstration Performance-Related Specification System

The last major objective of the study was the development of a demonstration PRS for PCC pavements that was to be as parallel as possible to the PRS for asphalt concrete pavements developed under NCHRP Project 10-26A.^(88.1) The specific procedures and algorithms needed for PRS development were covered in chapter 6 of this report while the development, application, and sensitivity of a computerized demonstration PRS for PCC pavements was presented in chapter 7. It is important to note that although the demonstration PRS is very comparable to the NCHRP 10-26A system, it did have some shortcomings (from both a conceptual and application standpoint) that led to the identification of two other approaches for assessing contractor penalties and rewards. One of these was the method developed at New Jersey DOT; the other is a method conceptualized based on the life-cycle cost analysis model presented in the 1986 AASHTO Guide.^(84.6,86.3)

The demonstration PRS for PCC pavements was developed in the form of a Lotus 1-2-3 spreadsheet. It allows users to determine the fraction of a contractor's bid price that should be received for the pavement that is actually built (on a lot-by-lot basis). If the contractor builds a pavement that is projected to perform better than the design (specified) pavement, the fraction will be greater than 1 and a bonus will be received. If the contractor builds a pavement that will perform worse than the design pavement, the fraction will be less than 1 and a penalty will be assessed. The demonstration PRS considers the three primary M&C factors in the AASHTO rigid pavement performance prediction equation; slab thickness, initial serviceability and, of course, PCC flexural strength. Thus, it is possible for the contractor to not meet the specification for one factor (while exceeding the other two) and still be rewarded.

The rest of this chapter is devoted to providing recommendations for further research in the development and enhancement of performance-related

specifications. All of these recommendations apply to PRS systems for PCC pavements, however, some also apply to asphalt concrete pavements.

FURTHER LABORATORY STUDIES FOR SECONDARY PREDICTION RELATIONSHIPS

This study identified initial pavement profile (serviceability), slab thickness and PCC flexural strength as the principal determinants of rigid pavement distress and performance. This assessment was based on an examination of the existing primary prediction relationships (PPRs), particularly the AASHTO Road Test rigid pavement performance equation which has serviceability as its performance criteria. To permit consideration of the effects of other M&C variables on rigid pavement distress and performance, several new secondary prediction relationships (SPRs) were derived to relate other PCC mix factors to flexural strength. The derivation of the new SPRs was based on a small but statistically sophisticated laboratory study. The results of the laboratory experiments and statistical analyses turned out to be quite good, however, they by no means constitute a comprehensive study. Rigid pavements can exhibit excessive distress and poor performance as a result of (1) low freeze-thaw durability, (2) high concrete shrinkage, (3) high concrete thermal coefficient, (4) high permeability, and (5) rapid loss of skid resistance. There are laboratory tests for each of these and some of them are even found in some of the available primary prediction relationships, both empirical and mechanistic. Thus, it is recommended that effort in a future laboratory study be directed towards measuring these concrete properties (as a function of the various M&C factors related to the PCC mix) as well as all the tests that were conducted in this study (both on the plastic and hardened mix).

Besides the limitation on the amount of PCC testing carried out under this project, only two levels for each of the seven experimental variables was considered in the factorial design. Other factors such as cement type, fine aggregate type and air-entraining agent type were held at only one level. Also, to better treat curvilinearity that may exist between the dependent variable and some of the independent variables, some of the experimental factors ought to be run at three levels in a future lab study. Candidates for three level variables to consider this potential nonlinearity include water content, cement content and air-entraining agent content. Coarse aggregate type, fine aggregate type and cement type are examples of factors that could be run at three or more levels because of the number of different types of each there are. Following are recommendations on the extent of a future laboratory test program:

- Coarse aggregate type (CAT) - Five levels: Choose the most commonly used types from a list that includes limestone, sandstone, quartzite, granite, syenite, and dolomite. Only a siliceous river gravel and a crushed limestone were considered in this study.
- Coarse aggregate maximum size (CAM) - Two levels: Experimental results indicated that this was significant in some cases, but not enough to justify more than two levels.
- Fine aggregate modulus (FAM) - Two levels: The grain size of the fine aggregate appeared to be more significant in some cases than the

maximum size of the coarse aggregate (CAM), but again not enough to warrant more than two levels.

- Air-entraining agent quantity (AEQ) - Three levels: Of the two levels this was run at in this study, one was zero (none). It is a significant factor and it is not likely that factors such as air content and concrete strength are linearly proportional to the line that connects the "none" endpoint to the "some" endpoint. Thus, a midlevel between the two would be very useful.
- Coarse aggregate quantity (CAQ) - Two levels: Although this was a significant factor in the analysis, from an engineering standpoint it is not worth examining at more than two levels unless the funds are available.
- Cement quantity (CEQ) - Three levels: This has a large effect on PCC mix properties (particularly strength) and is definitely worth studying at more than two levels.
- Water quantity (WAQ) - Three levels: Like cement quantity (CEQ), this had a large effect on the various PCC mix properties and should be studied at more than two levels.
- Fine aggregate type (FAT) - Two levels: A commonly used quartzitic sand was used for this experiment. Since its size did have some effect on various properties of the mix, it may be worth studying the effects of one other commonly used fine aggregate (perhaps a manufactured sand).
- Consolidation (CSL) - Two levels: All samples tested for this study were prepared according to ASTM specifications. Since proper consolidation is not always achieved in the field, it may be desirable to examine the effects of poor consolidation in a future laboratory study. Two levels of rodding should be sufficient to provide a high and a low entrapped air content.
- Mineral Admixture (Fly Ash) - Three levels: One beneficial admixture not considered in this laboratory study is fly ash. In addition to its cost effectiveness, pozzolans such as fly ash can improve concrete physical characteristics such as workability, strength, and durability. Consequently, fly ash admixtures are being used more and more frequently by SHAs. The suggested levels of fly ash are zero (none), and an optimum substitution of cement with either an ASTM Type F or Type C fly ash. This is probably not a high-priority factor compared to those identified above.
- Cement type (CET) - Two levels: The most commonly used cement (Type I) was used to prepare the PCC test specimens for this experiment. This probably would be sufficient for use in a performance-related specification system, however the common use of Type II and Type III cements with varying chemical and physical properties may make it worthwhile to consider two types of cement. This is probably a low-priority factor.

- High-range water reducer (HRWR) - Three levels: Although potentially significant, this study did not examine the effects of high-range water reducers (also known as superplasticizers) on the various PCC mix properties. Because of its significance, it may be desirable to examine the effects of an ASTM C494 Type G high-range water reducer in a future laboratory study. The three levels indicated would include zero (none), and two dosage levels (concentrations) as recommended by the manufacturer of the selected high-range water reducer. This is probably a low-priority factor.

A full factorial of all these combinations would produce an experiment of $(5 \times 2^6 \times 3^5)$ or 77,760 cells. Eliminating the last three factors, taking advantage of fractional factorial experiment design techniques, and the fact that some tests would not need to be run in all the cells would probably still produce an experiment that would be too costly to conduct. Thus, it would likely be necessary to do some further prioritizing and "cutting back" of factors and/or levels to achieve the most cost-effective results in the recommended future laboratory study.

FIELD STUDIES RELATED TO PERFORMANCE-RELATED SPECIFICATIONS

The principal objective of any future field studies would be to verify existing relationships and/or derive new relationships for relating the principal determinants of rigid pavement distress and performance to rigid pavement response under a single load and to rigid pavement distress after repeated (known) loadings. These relationships would further the development of PRS since they would likely provide a better means of treating various M&C factors in the analysis process.

The experiment design for such a field study would use the secondary prediction relationships to identify the most appropriate M&C factors and levels that determine PCC strength, durability, thickness and initial pavement profile (serviceability) of the experimental pavement test sections. For example, 32 test sections could be constructed as a one-half replicate of a 2^6 experiment design. The six experimental factors, each at two levels would be as follows:

- Initial pavement profile.
- Slab thickness.
- Coarse aggregate type.
- Cement content.
- Water content.
- Air content.

The last four of these factors all have a measurable effect on the flexural strength of the concrete.

From a construction standpoint, all other pertinent factors for the 32 experimental sections (i.e., soil support, subbase type, thickness and

strength, reinforcement, joint spacing, load transfer, etc.) would be held constant. The test sections would be evaluated for response (i.e., deflection and strain) under a load of varying magnitude and then for fatigue cracking, faulting, serviceability (roughness) and other types of distress after repeated uniform loadings. The results would demonstrate the role and relative importance of each experimental factor in the prediction of pavement response and performance. They would thus provide improved primary prediction relationships for the development of better PRS systems.

It is important to note that several of these factors will be implemented in SHRP SPS-2, but not to the detail that is recommended above for the pavement field studies.

FURTHER DEVELOPMENT OF PERFORMANCE-RELATED SPECIFICATIONS

One of the major accomplishments of this study was the development of a demonstration PRS system for PCC pavements. It was based in large part on the conceptual system outlined in NCHRP Project 10-26A for asphalt concrete pavements.^(88.1) Where the NCHRP 10-26A work fell short in some of the key areas related to PRS development for rigid pavements, methods and techniques were adopted from PRS developed by NJDOT.^(84.6) The new PRS does produce results, but as observed in chapter 7, there are some problems that need to be overcome before it can be reasonably applied for rigid pavements. Below is a list of the key areas where further work is needed to overcome these problems. They are not in order of importance, but in the order they are addressed in chapter 6.

1. Development of specific acceptance plans. As indicated in chapter 6, this refers to the set of rules and definitions that govern acceptance and rejection of the contractor's work for a given lot. The acceptance developed by NJDOT are quite good and make an excellent starting point in developing plans for wider application.

2. Treatment of material and construction variability. One factor that was not treated in the demonstration PRS system was the effect of variability in the material properties and construction characteristics delivered by the contractor. At present, if two contractors deliver pavements that have the same predicted (design) performance, but vastly different variabilities in M&C factors, both would receive the same penalty (or reward). Obviously, the one with the least variability ought to receive better consideration in terms of payment. One approach is in the development of specific acceptance/rejection plans as described above. However, an alternate or additional approach would be to consider the effects of measured variability on reliability, i.e., the probable distribution of performance. Using this latter approach, a contractor who exercises good quality control (and achieves low variability) would receive more favorable consideration.

3. Development of optimum M&C variables to be evaluated during construction. Three M&C variables are used in the demonstration PRS system to evaluate construction: initial profile, slab thickness, and 28-day compressive strength. The 28-day compressive strength is used to estimate the 28-day flexural strength and elastic modulus used as primary determinants of distress, but it would be better to use 7-day compressive

strength, since this is a more common measurement. Unfortunately, only the 7-day flexural strength was measured in the lab study.

4. Selection of optimum distress variables. The current demonstration PRS system only considers serviceability history in the analysis process, however, it does calculate and display pavement distress values using the COPES equations. At some point, it may be better to consider more types of distress than just serviceability. Since the COPES equations are based on the analysis of an observational data base, they should be studied more carefully before incorporating them into the system.

5. More rational selection of cost evaluation procedures. Chapter 6 provides a description of the conceptual cost evaluation procedure recommended under NCHRP Project 10-26A for PRS development. As observed, however, this is one key area where, in application, the NCHRP 10-26A methodology develops problems. In order to identify the economic life using "real" numbers for both the design and constructed pavement, it was necessary to consider vehicle operating costs. (Future maintenance costs were not enough to produce the "upturn" in the EUAC curve). Once vehicle operating costs were introduced, however, they so overwhelmed the initial construction cost that the associated economic life (at minimum EUAC) was only one year. Thus, in order to produce reasonable results, only a fraction of the vehicle operating cost could be considered. Conceptually, it also did not seem correct to penalize (or reward) the contractor based upon the difference in costs incurred over economic life of the as-constructed pavement. (It ought to be the economic life of the design pavement). These kinds of problems need to be thoroughly examined and corrected to produce more reasonable and defensible results in future PRS systems.

6. Critical comparison of methodologies. In addition to the review of the NCHRP 10-26A methodology, chapter 6 also identifies and describes certain features of two alternative approaches to developing a PRS system for PCC pavements. One was the fully operational method developed and currently used by NJDOT. Like the demonstration PRS presented in chapter 7, it is based on the AASHTO Road Test rigid pavement performance equation and considers initial serviceability (profile), slab thickness, and PCC strength as its primary distress determinants. Future costs are calculated based on a fixed (10-year cyclic) overlay policy which begins once the initial pavement (design and as-constructed) is projected to reach terminal serviceability. Although they could easily be adapted, the NJDOT PRS does not currently consider future costs associated with maintenance or user operation in assessing contractor penalty/reward. The method has been thoroughly tested, however, and is complete in the sense that it has its own set of acceptance and payment plans.

The other alternative method for PRS system development is one that has been conceptualized as part of this study and which is based on the rigid pavement design and life-cycle cost analysis procedure presented in the latest AASHTO Guide for Design of Pavement Structures.^(86.3) It would also consider initial serviceability, slab thickness and PCC strength as its primary distress determinants. The life-cycle (future) costs that would be considered in assessing contractor penalty/reward currently include maintenance and rehabilitation costs incurred over a specified analysis

period. Rehabilitation costs would be calculated based upon the projected needs of both the design or as-constructed pavement to last the analysis period. (Thus, if the contractor builds a pavement that does provide the life associated with the design pavement, part of his penalty would be determined based upon the earlier timing and need for a thicker overlay). Like the NJDOT PRS, this one could be easily adapted to consider the difference in user costs associated with the difference between the design and as-constructed pavement.

Based upon the practical difficulties associated with the current NCHRP 10-26A approach (at least for rigid pavements), it is strongly recommended that additional effort be directed in the future towards examining these three PRS methods (as a minimum) to derive the one that is best-suited for concrete pavements. An essential tool for the comparisons is extensive sensitivity analyses for the dependence of pay factors on all pay factor determinants.

7. Development of operating characteristic curves for payment plans that consider prediction equation uncertainties. Chapter 6 describes how the operating characteristic curves were developed for use in the NJDOT PRS. Although, this represents the state of the art as far as its application to pavements, it lacks the consideration of the effects of prediction equation uncertainties (as well as other uncertainties) in assessing contractor penalty/reward. Consideration of uncertainty (reliability/risk) would ultimately make both the client and the contractor feel better about the output of the PRS, therefore, it should not be overlooked in future research efforts.

APPENDIX A SELECTED STRESS PREDICTION RELATIONSHIPS

This appendix provides some additional select relationships and models that may be used to predict the response (i.e., stress, strain, and deformation) of PCC pavements subject to various kinds of loading. It is subdivided according to two of the four major types of PCC pavement response prediction models identified in chapter 3:

- Multilayered Elastic Solid.
- Elastic Plate on Dense Liquid Subgrade.

Selected examples of the other two types of prediction models are not provided here for two reasons: (1) additional empirical models were overlooked because of their limited application in a PRS system and (2) finite element models are well covered in chapter 3.

MULTILAYERED ELASTIC SOLID

Chapter 3 addresses many of the computer programs available for the prediction of pavement response using elastic layer theory concepts. In addition to these programs, there are two other methods for estimating response, graphical solution techniques and approximation functions.

Graphical Solution Technique

Several researchers have been involved with the evolution of graphical techniques to solve for elastic layer responses. These methods are applicable to one-, two- and three-layer pavements and increase in complexity with the inclusion of each additional layer. Yoder and Witczak provide an excellent summary of the available graphical methods which identifies many of the primary problems associated with them^(75.1):

- Superposition is required to treat the effects of more than one load.
- There is error associated with reading coefficients from graphs.
- There is error associated with linear interpolation for radii or depth factors not included in tables and graphs.
- Time and labor requirements are excessive.

These problems make the use of graphical techniques impractical; however, they do provide an indication of the complexity of the elastic layer solution process.

Approximation Functions

Computer programs are by far the best means for estimating pavement response due to load. However, there are some specific areas where even more rapid solutions may be required. In the specific cases where some of the independent variables can be fixed (such as type of response, number of layers, Poisson's ratios, and range of layer elastic moduli), statistically derived approximation functions may be developed to replace the standard

computer programs. Examples of these specific cases include nondestructive testing based back-calculation techniques, systems-oriented pavement structural design programs and even performance-related specification systems.

The following are regression equations developed as part of a recent study for the Trucking Research Institute.^(89.1) These equations are for a three-layer rigid pavement structure in which the load configuration is a simulated dual-tired single axle. All are based on interior loading conditions, i.e., away from the slab edge or corner.

Maximum Surface Deflection:

$$\begin{aligned} \text{LDEFL} = & - 1.870 + 0.0114 \cdot \text{LD1} \cdot \text{LE1} \cdot (\text{LE3})^2 + 0.998 \cdot \text{LLOAD} \\ & - 0.117 \cdot \text{LE1} \cdot \text{LE3} - 0.513 \cdot \text{LD1} \cdot \text{LE3} \end{aligned}$$

$$R^2 = 0.994, \quad \text{SEE} = 0.0298, \quad n = 3^6 = 729$$

Maximum Principal Slab Stress:

$$\begin{aligned} \text{LSTRS} = & - 2.200 + 0.00476 \cdot (\text{LD1})^2 \cdot \text{LE3} + 0.914 \cdot \text{LLOAD} \\ & + 0.427 \cdot \text{LE1} - 0.0593 \cdot \text{LD2} \cdot \text{LRAT} \\ & - 0.231 \cdot \text{LD1} \cdot \text{LE1} - 0.0270 \cdot (\text{LE3})^2 \end{aligned}$$

$$R^2 = 0.998, \quad \text{SEE} = 0.0129, \quad n = 3^6 = 729$$

Maximum Roadbed Soil Vertical Strain:

$$\begin{aligned} \text{LSTRN} = & - 0.510 + 0.00598 \cdot (\text{LD1})^2 \cdot \text{LE1} \cdot (\text{LE3})^2 \\ & + 0.00251 \cdot \text{LLOAD} \cdot (\text{LD1})^2 \cdot (\text{LE3})^2 - 0.787 \cdot \text{LD1} \cdot \text{LE3} \\ & - 0.699 \cdot \text{LE1} + 0.902 \cdot \text{LLOAD} \end{aligned}$$

$$R^2 = 0.994, \quad \text{SEE} = 0.282, \quad n = 3^6 = 793$$

Definitions of the variables used in these equations are as follows:

LDEFL = log of maximum surface deflection (inches).
 LSTRS = log of maximum principal slab stress (psi).
 LSTRN = log of maximum roadbed soil vertical strain.
 LD1 = log of slab thickness (inches).
 LD2 = log of base/subbase thickness (inches).
 LE1 = log of slab elastic modulus (psi).
 LE3 = log of roadbed soil elastic modulus (psi).
 LRAT = log of the ratio between the subbase elastic modulus to that of the roadbed soil.
 LLOAD = log of wheel load magnitude (lb).

Note: All logs are in base 10.

ELASTIC PLATE ON DENSE LIQUID SUBGRADE

The following are equations developed by Ioannides for the prediction of slab stress and surface deflection for interior, edge and corner loading conditions.^(84.9, 85.6) These equations were derived based on an in-depth

study of the equations developed by other researchers and an analysis of the ILLI-SLAB finite element program.^(78.3)

Maximum Deflection, Interior (Circular) Load:

$$\text{DEFIC} = [P/(8*k*\ell^2)] * \\ (1 + (0.5*\pi)*[\ln(0.5*a/\ell) + \text{EUL} - 5/4]*(a/\ell)^2)$$

Maximum Bending Stress, Ordinary Theory, Interior (Circular) Load:

$$\text{BSIOT} = \{ [3*P*(1+\nu)] / (2*\pi*h^2) \} * \\ [\ln(2*\ell/a) + 0.5 - \text{EUL}] + \text{BSI2OT}$$

$$\text{BSI2OT} = \{ [3*P*(1+\nu)] / (64*h^2) \} * (a/\ell)^2$$

Maximum Bending Stress, Special Theory, Interior (Circular) Load:

$$\text{BSIST} = \{ [3*P*(1+\nu)] / (2*\pi*h^2) \} * \\ [\ln(2*\ell/b) + 0.5 - \text{EUL}] + \text{BSI2ST}$$

$$\text{BSI2ST} = \{ [3*P*(1+\nu)] / (64*h^2) \} * (b/\ell)^2$$

Maximum Bending Stress, Interior (Square) Load:

$$\text{BSISQ} = \{ [3*P*(1+\nu)] / (2*\pi*h^2) \} * \\ [\ln(2*\ell/c') + 0.5 - \text{EUL}] + \text{BSI2SQ}$$

$$\text{BSI2SQ} = \{ [3*P*(1+\nu)] / (64*h^2) \} * (c'/\ell)^2$$

Maximum Deflection, Edge (Circular) Load:

$$\text{DEFEIC} = P * [(2 + 1.2*\nu)^{0.5}] * [1 - (0.76 + 0.4*\nu)*(a/\ell)] / (E*h^3*k)^{0.5}$$

Maximum Deflection, Edge (Semicircular) Load:

$$\text{DEFEIS} = P * [(2 + 1.2*\nu)^{0.5}] * [1 - (0.323 + 0.17*\nu)*(a_2/\ell)] / (E*h^3*k)^{0.5}$$

Maximum Bending Stress, Edge (Circular) Load:

$$\text{BSEIC} = \{ 3*(1+\nu)*P / [\pi*(3+\nu)*h^2] \} * \\ \{ \ln[E*h^3/(100*k*a^4)] + 1.84 - 4*\nu/3 + [(1-\nu)/2] + \\ 1.18*(1 + 2*\nu)*(a/\ell) \}$$

Maximum Bending Stress, Edge (Semicircular) Load:

$$\text{BSEIS} = \{ 3*(1+\nu)*P / [\pi*(3+\nu)*h^2] \} * \\ \{ \ln[E*h^3/(100*k*a_2^4)] + 3.84 - 4*\nu/3 + \\ 0.5*(1 + 2*\nu)*(a_2/\ell) \}$$

Maximum Deflection, Corner (Square) Load:

$$\text{DEFCS} = (P/k*\ell^2)*(1.205 - 0.69*c/\ell)$$

Maximum Bending Stress, Corner (Square) Load:

$$BSCSQ = (3*P/h^2)*[1.0 - (c/\ell)^{0.72}]$$

The variables used in these equations are as follows:

- P = Total applied load (lb).
- E = Slab elastic modulus (psi).
- ν = Slab Poisson's ratio.
- h = Slab thickness (in).
- k = Modulus of subgrade reaction (pci).
- a = Radius of circular load (in).
- b = a, if $a \leq 1.724*h$,
= $(1.6*a^2 + h^2)^{0.5} - 0.675*h$, if $a > 1.724*h$.
- a_2 = Radius of semicircular load (in).
- b_2 = a_2 , if $a_2 \leq 1.724*h$,
= $(1.6*a_2^2 + h^2)^{0.5} - 0.675*h$, if $a_2 > 1.724*h$.
- c = Side length of square load (in).
- c' = $\{e^{[(\pi/4)-1]}/2^{0.5}\}*c$.
- EUL = Euler's constant (0.57721566490).
- π = 3.141592654.
- ℓ = Radius of relative stiffness (in).
= $\{E*h^3/[12*(1-\nu^2)*k]\}^{0.25}$.

APPENDIX B
SELECTED DISTRESS/PERFORMANCE PREDICTION RELATIONSHIPS
FOR RIGID PAVEMENTS

This appendix gives details for distress/performance prediction relationships that were discussed in chapter 3. The 18 entries, coded A through R, represent selections from the research literature and are intended to give representative coverage of all reported relationships for the various types of rigid pavement distress.

A. Prediction of Pumping. [From COPES Report]^(85.2)

For JPCP:

$$\text{PUMP} = (\text{ESAL}^{0.443}) [-1.224 + 0.0605 \text{SUMPREC}^{0.50} \\ + 0.00027 \text{FI}^{1.205} + 54.65/\text{THICK}^{1.747} \\ - 0.255 \text{SOILCRS}]$$

$$R^2 = 0.68, \quad \text{SEE} = 0.42, \quad n = 289 \text{ pavement sections}$$

For JRCP:

$$\text{PUMP} = (\text{ESAL}^{0.670}) [-22.82 + 13.224 \text{SUMPREC}^{0.0395} \\ + 6.834 (\text{FI} + 1)^{0.00805} + 26102/\text{THICK}^{5.0} \\ - 0.129 \text{DRAIN} - 0.118 \text{SOILCRS}]$$

$$R^2 = 0.57, \quad \text{SEE} = 0.52, \quad n = 481 \text{ pavement sections}$$

PUMP = 0 (none), 1 (low), 2 (medium), 3 (high severity).
ESAL = Accumulated 18-kip equivalent single axle loads.
SUMPREC = Average annual precipitation (cm).
FI = Freezing index.
THICK = Slab thickness (in).
DRAIN = 0 (no subdrain), 1 (subdrain pipes).
SOILCRS = 0 (fine grained soil), 1 (coarse roadbed soil).

B. Prediction of Cracking [From COPES Report]^(85.2)

For JPCP:

$$\begin{aligned} \text{CRACKS} = & (\text{ESAL}^{2.76}) [3092 (1 - \text{SOILCRS}) \text{RATIO}^{10.0}] \\ & + (\text{ESAL}^{0.50}) [1.233 (\text{TRANGE}) \text{RATIO}^{2.87}] \\ & + (\text{ESAL}^{2.42}) [0.23 (\text{FI}^{1.53}) \text{RATIO}^{7.31}] \end{aligned}$$

$$R^2 = 0.69, \quad \text{SEE} = 176 \text{ feet per mile}, \quad n = 303 \text{ pavement sections}$$

For JRCP:

$$\begin{aligned} \text{CRACKS} = & (\text{ESAL}^{0.897}) [7130 \text{JTSP}/(\text{ASTEEL} * \text{THICK})^{5.0}] \\ & + (\text{ESAL}^{0.10}) [2.281 \text{PUMP}^{5.0}] \\ & + (\text{ESAL}^{2.16}) [1.81/(\text{BASETYP} + 1)] \\ & + (\text{AGE}^{1.3}) [0.0036 (\text{FI} + 1)^{0.36}] \end{aligned}$$

$$R^2 = 0.41, \quad \text{SEE} = 280 \text{ ft per mi}, \quad n = 314 \text{ pavement sections}$$

CRACKS = Total crack length (ft per mi):
for JPCP - all cracks;
for JRCP - medium and high-severity temperature and shrinkage cracks.

ESAL = Accumulated 18-kip equivalent single axle loads.

FI = Freezing index.

THICK = Slab thickness (in).

PUMP = 0 (none), 1 (low), 2 (medium), 3 (high severity).

SOILCRS = 0 (fine grained soil), 1 (coarse roadbed soil).

AGE = Years since construction.

TRANGE = (Maximum July temperature) - (minimum January temperature).

RATIO = (Westergaard 9-kip wheel load edge stress)/(PCC modulus of rupture).

JTSP = Transverse joint spacing (ft).

ASTEEL = Area of reinforcing steel (inches per foot of width).

BASETYP = 0 (granular base), 1 (stabilized base).

C. Prediction of Load Cycles to Concrete Fatigue Failure (flexural beam break). [From FHWA Zero-Maintenance Report]^(77.3)

$$\log N = 16.61 - 17.61 * (\sigma/f)$$

$$R^2 = 0.50 \text{ (est)}, \quad \text{SEE (log)} = 0.40 \text{ (est)}, \quad n = 140 \text{ plain PCC beams}$$

N = Number of stress applications to beam failure.

σ = Repeated flexural stress (psi).

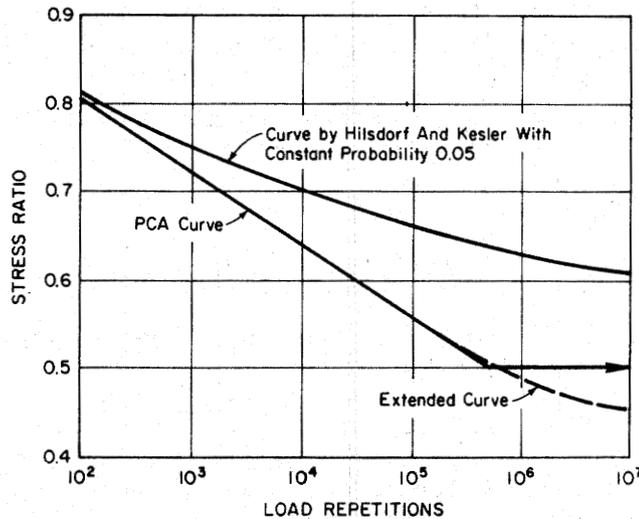
f = Concrete modulus of rupture (psi).

D. Prediction of Load Cycles to Fatigue Failure (exact failure criteria - unknown) [From PCA thickness design manual]^(84.4)

$R^2 = ??$

SEE = ??

$n = ??$



STRESS RATIO = Wheel load flexural stress divided by 28-day modulus of rupture.

LOAD REPETITIONS = Allowable number of load repetitions corresponding to stress level.

E. Prediction of Load Cycles to Pavement Fatigue Failure (Class 3&4 Cracking). [From initial FHWA rehabilitation design study]^(77.4)

$\log N = 4.37 + 3.21 \cdot \log(f/\sigma)$

$R^2 = 0.83$, SEE (log) = ??, $n = ??$ AASHO Road Test Sections

N = Number of stress applications to onset of class 3&4 cracking.

f = Concrete modulus of rupture (psi).

σ = Maximum wheel load slab stress (psi).

F. Prediction of Load Cycles to Pavement Fatigue Failure (cracking index of 50). [From Center for Transportation Research Report]^(81.2)

$$\log N = 4.66 + 3.00 \cdot \log(f/\sigma)$$

$$R^2 = ??, \text{ SEE (log)} = ??, \text{ n} = (\text{all}) \text{ AASHO Road Test Sections}$$

N = Number of stress applications to AASHO cracking index of 50 feet per 1000 square feet.

f = Modulus of rupture (psi).

σ = Maximum slab stress (psi).

G. Prediction of CRCP Shrinkage Cracking. [From AASHTO Guide]^(86.4)

To predict subsequent spacing between cracks:

$$\begin{aligned} \text{CRACKSPACE} = & 1.32 [(1 + \text{TSTRG}/1000)^{6.70}] * [(1 - \text{THERMRAT}/2)^{1.15}] \\ & * [(1 + \text{BDIAM})^{2.19}] * [(1 + \text{TSTRS}/1000)^{-5.20}] \\ & * [(1 + \text{PCTST})^{-4.60}] * [(1 + 1000 \text{SHRN})^{-1.79}] \end{aligned}$$

$$R^2 = ??, \text{ SEE} = ??, \text{ n} = ??$$

To predict subsequent crack width:

$$\begin{aligned} \text{CRACKWIDTH} = & 0.00932 [(1 + \text{TSTRG}/1000)^{6.53}] * [(1 + \text{BDIAM})^{2.20}] \\ & * [(1 + \text{TSTRS}/1000)^{-4.91}] * [(1 + \text{PCTST})^{-4.55}] \end{aligned}$$

$$R^2 = ??, \text{ SEE} = ??, \text{ n} = ??$$

CRACKSPACE = Mean distance (ft) between cracks.

CRACKWIDTH = Mean crack width (in).

TSTRG = Concrete indirect tensile strength (psi).

THERMRAT = Ratio of steel thermal coefficient to concrete thermal coefficient.

TSTRS = Tensile stress (psi) due to wheel load.

PCTST = Percent steel.

SHRN = Concrete shrinkage at 28 days.

H. Prediction of Faulting. [From COPES Report]^(85.2)

For JPCP:

$$\text{FAULT} = (\text{ESAL}^{0.144}) [-0.298 + 0.2671/(\text{THICK}^{0.318}) \\ - 0.0285 \text{ BASETYP} + 0.00406 (\text{FI} + 1)^{0.3598} - 0.0462 \text{ EDGSUP} \\ + 0.2384 (\text{PUMP} + 1)^{0.0109} - 0.0340 (\text{DOW})^{2.0587}]$$

$$R^2 = 0.79, \quad \text{SEE} = 0.02 \text{ inches}, \quad n = 259 \text{ pavement sections}$$

For JRCP:

$$\text{FAULT} = (\text{ESAL}^{0.4731}) [-3.8536 - 1.536 \text{ SOILCRS} \\ + 197.1 (\text{THICK} * \text{DOW}^{2.0})^{-1.784} + 0.00024 \text{ FI} \\ + 0.0986 \text{ JTSP} + 0.2412 \text{ PUMP}^{2.0}]$$

$$R^2 = 0.69, \quad \text{SEE} = 0.06 \text{ inches}, \quad n = 384 \text{ pavement sections}$$

FAULT = Mean transverse joint faulting (in).
PUMP = 0 (none), 1 (low), 2 (medium), 3 (high severity).
ESAL = Accumulated 18-kip equivalent single axle loads.
FI = Freezing index.
THICK = Slab thickness (in).
SOILCRS = 0 (fine grained soil), 1 (coarse roadbed soil).
AGE = Years since construction.
JTSP = Transverse joint spacing (ft).
BASETYP = 0 (granular base), 1 (stabilized base).
DOW = Dowel bar diameter (in).
EDGSUP = 0 (for AC shoulder), 1 (for tied PCC shoulder).

I. Prediction of Faulting. [From AASHTO Guide, Volume 3]^(86.4)

$$\ln (\text{FAULT} + 1) = [\ln (\text{ESAL} + 1)] [-0.09013 + 0.00014 \text{ BSTRESS}]$$

$$R^2 = ??, \quad \text{SEE} = ??, \quad n = ??$$

FAULT = Mean transverse joint faulting (in).
ESAL = Accumulated 18-kip equivalent single axle loads.
BSTRESS = Maximum bearing stress of dowel bar (depends on PCC thickness and modulus, dowel diameter and modulus, and roadbed modulus).

J. Prediction of Joint Deterioration. [From COPES Report]^(85.2)

For JPCP:

$$\text{DETJT} = (\text{AGE}^{1.695}) (0.9754 \text{ DCRACK}) + (\text{AGE}^{2.841}) (0.01247 \text{ UNITUBE}) \\ + (\text{AGE}^{3.038}) (0.00135 \text{ INCOMP})$$

$$R^2 = 0.59, \quad \text{SSE} = 16 \text{ joints per mile}, \quad n = 252 \text{ pavement sections}$$

For JRCP:

$$\text{DETJT} = (\text{AGE}^{0.756}) (2.437 \text{ DCRACK} + 2.744 \text{ REACTAG}) \\ + (\text{AGE}^{2.152}) (0.052 + 0.0000254 \text{ FI} + 0.0111 \text{ TJSD} \\ - 0.00338 \text{ KI*JTSP} - 0.000645 \text{ K2*JTSP})$$

$$R^2 = 0.61, \quad \text{SEE} = 15 \text{ joints per mile}, \quad n = 319 \text{ pavement sections}$$

DETJT = Number of medium and high-severity deteriorated joints/mile.
ESAL = Accumulated 18-kip equivalent single axle loads.
PUMP = 0 (none), 1 (low), 2 (medium), 3 (high severity).
AGE = Years since construction.
JTSP = Transverse joint spacing (ft).
DCRACK = 1 (D cracks), 0 (none).
UNITUBE = 1 (Unitube joints), 0 (none).
INCOMP = 1 (visible incompressibles in joint), 0 (none).
REACTAG = 1 (reactive aggregates), 0 (none).
TJSD = 1 (medium or high-severity joint real damage), 0 (none or low).
K1 = 1 (if JTSP at least 27 ft), 0 (if not).
K2 = 1 (if JTSP at least 39 ft), 0 (if not).

K. Prediction of CRCP Slab Distress. [From study for Texas SDHPT]^(81.6)

$$N = -0.381 - 0.0356 X1 + 0.000131 X2^2 + 0.0461 X3 (X2-X1) \\ + 0.0000494 X2 X4 + X5$$

$$R^2 = 0.67, \quad \text{SEE} = 2.44, \quad n = 147$$

N = Number of defects (punchouts + patches) per mile at future time chosen for prediction.
X1 = Pavement age at time of condition survey (months).
X2 = Pavement age at future time for distress prediction (months).
X3 = Number of defects at time of condition survey.
X4 = Texas SDHPT District Temperature Constant.
X5 = -5.840 + 0.0988 X2 (for pit run gravel subbase aggregate),
0 (for other subbase aggregates).

L. Prediction of Swells and Depressions. [From FHWA Cost-Allocation Report]^(84.3)

For JPCP:

$$(D + S)/10 = AGE [0.0016 IMOIST - 0.00045 CBR - 0.0155 BASETYP + 0.00706 F2 + 0.00171 F3 + 0.02375]$$

$$R^2 = 0.78, \quad SEE = 0.56, \quad n = 65 \text{ pavement sections}$$

For JRCP:

$$(D + S)/10 = AGE [0.00035 SUMPREC - 0.0074 BASETYP - 0.01785]$$

$$R^2 = 0.68, \quad SEE = 0.78, \quad n = 50 \text{ pavement sections}$$

D + S = Number of depressions and swells per mile (medium and high-severity).

AGE = Years since construction.

BASETYP = 0 (granular base), 1 (stabilized base).

IMOIST = Thornthwaite Moisture Index.

F2 = 1 (pavement in cut), 0 (not in cut).

F3 = 1 (pavement in fill), 0 (not in fill).

SUMPREC = Average annual precipitation (cm).

M. Prediction of Skid Number Loss. [From FHWA Cost-Allocation Report]^(84.3)

$$\ln (70 - SN) = - 2.372 + 0.258 \ln TRUCKS + 0.137 \ln ESAL - 0.033 \ln AXLES$$

$$R^2 = 0.70, \quad SEE = ??, \quad n = 33 \text{ pavement sections}$$

SN = Skid number at 40 mi/h (skid trailer).

TRUCKS = Total number of truck passes (excepting pickups and panels).

AXLES = Total number of axle passes in traffic lane.

N. Prediction of Serviceability Loss. [From AASHTO Guide]^(86.3)

$$PSIL = PO - PW = 3 * [W / (RHO * ADJ)]^{BETA}$$

$$BETA = 1 + 0.0563 / [(THICK + 1)^{8.46}]$$

$$RHO = [(THICK + 1)^{7.35}] - 0.06$$

$$ADJ = (NUM / DEN)^{(4.22 - 0.32 PW)}$$

$$NUM = MODRUP * DRACO * [(THICK^{0.75}) - 1.132]$$

$$DEN = 215.63 * JFACT * [(THICK^{0.75}) - 18.42 (RAT^{0.25})]$$

$$RAT = (KVAL / EMOD)$$

- PSIL = Serviceability loss from PSI = PO to PSI = PW.
PO = As-constructed present serviceability index (PSI).
PW = PSI when accumulated 18-kip single axle load applications (ESAL) equals W.
THICK = PCC slab thickness (in).
MODRUP = PCC modulus of rupture (psi).
DRACO = Drainage coefficient (ranges from 0.70 = poor to 1.25 = excellent).
JFACT = Load transfer coefficient (ranges from 2.3 = excellent to 4.4 = poor).
EMOD = PCC modulus of elasticity (psi).
KVAL = Roadbed modulus of subgrade reaction (pci).

Note: The standard error of estimate (SEE) of this relationship has not been evaluated from an observational data base but is likely to be at least 0.34 PSI units.

O. Prediction of Serviceability Loss. [From COPES Report]^(85.2)

For JPCP:

$$\begin{aligned} \text{PSRL} &= (4.5 - \text{PSRE}) \\ &= 1.486 (\text{ESAL}^{0.1467}) - 0.4963 [(\text{ESAL}^{0.265})/(\text{RATIO}^{0.5})] \\ &\quad + 0.01082 (\text{ESAL}^{0.644}) [(\text{SUMPREC}^{0.91})/(\text{AVGMT}^{1.07})] \\ &\quad * (\text{AGE}^{0.525}) \end{aligned}$$

$$R^2 = 0.69, \quad \text{SEE} = 0.25, \quad n = 316 \text{ pavement sections}$$

For JRCP:

$$\begin{aligned} \text{PSRL} &= (4.5 - \text{PSRE}) \\ &= (\text{ESAL}^{0.424}) [-0.00188 + 14.417 (\text{RATIO}^{3.58}) + 0.0399 \text{PUMP} \\ &\quad + 0.002153 \text{JTSP} + 0.1146 \text{DCRACK} + 0.05903 \text{REACTAG} \\ &\quad + 0.00004156 \text{FI} + 0.00163 \text{SUMPREC} - 0.070535 \text{BASETYP}] \end{aligned}$$

$$R^2 = 0.78, \quad \text{SEE} = 0.30, \quad n = 377 \text{ pavement sections}$$

- PSRE = Panel present serviceability rating (PSR) when pavement has received ESAL equivalent standard load applications.
PSRL = Initial serviceability (assumed to be 4.5) minus PSRE.
ESAL = Accumulated 18-kip equivalent single axle loads.
FI = Freezing index.
AVGMT = Average monthly temperature (°F).
PUMP = 0 (none), 1 (low), 2 (medium), 3 (high severity).
SUMPREC = Average annual precipitation (cm).
AGE = Years since construction.
RATIO = (Westergaard 9-kip wheel load edge stress)/(PCC modulus of rupture).
JTSP = Transverse joint spacing (ft).
BASETYP = 0 (granular base), 1 (stabilized base).
DCRACK = 1 (D cracks), 0 (none).
REACTAG = 1 (reactive aggregates), 0 (none).

P. Prediction of Load Applications to Pavement Failure (terminal serviceability of 2.5). [From NCHRP Report 97]^(70.1)

$$\log N = 5.352 - 4 (\sigma/f)$$

$$R^2 = ??, \quad \text{SEE} (\log) = ??, \quad n = 29 (?) \text{ AASHO Road Test Sections}$$

- N = number of stress applications to terminal serviceability of 2.5.
f = concrete modulus of rupture (psi).
 σ = maximum wheel load slab stress (psi).
-

Q. Prediction of Load Cycles to Pavement Failure (AASHO Serviceability of 2.0). [From follow-up FHWA rehabilitation design study]^(83.7)

$$\log N = 4.35 + 4.29 \cdot \log(f/\sigma)$$

$$R^2 = 0.92, \text{ SEE (log)} = 0.23, \text{ n} = 99 \text{ AASHO Road Test Sections}$$

N = number of wheel load stress applications to terminal serviceability of 2.5.

f = concrete modulus of rupture (psi).

σ = maximum wheel load slab stress (psi).

R. Prediction of Pavement Condition Rating. [From Resource International Inc Study for FHWA]^(84.2)

	Non PCC Terms	PCC Terms
PCR = -96.7	$+ 96.7/\log(C+3)$ $+ 0.0474 (C+3)/\log(C+3)$ $+ 0.560 D * \log(C+3)$ $+ 2.27 [\log(B+3)]/[\log(C+3)]$ $+ 43.9 F/D$ $- 18.2 F / \text{root } C$ $- 0.0174 C * G$ $+ 0.479/G * G$ $- 6.65/\log F$ $- 6.70 \log F$ $+ 0.0497 A * G$ $- 0.220 D * G$ $+ 1.90 [\log(C+3)] / \log F$	$+ 0.634 H$ $+ 0.637 D * I$ $- 0.0786 I * I / \log F$ $+ 0.000446 L * I$ $- 0.507 A * L * I$ $+ 36.3 G / I$ $+ 0.803 / [F * (I^{0.75})]$ $- 0.000026 E * L$ $- 0.972 K$ $- 0.000251 D * L$ $- 0.155 J$

$$R^2 = 0.634, \text{ SSE} = ??, \text{ n} = 734 \text{ pavement sections (max)}$$

Variables	Mean Value	Std. Dev.
PCR = Pavement Condition Rating (100 max.)	72 (est.)	4 (est.)
A = Annual rainfall (in)	45.3	9.8
B = Frost penetration (in)	12.0	9.6
C = Freeze Index	148	178
D = Subgrade CBR (percent)	8.1	1.8
E = Joint spacing (ft)	44.0	22.1
F = Cumulative ESAL (millions)	4.4	3.6
G = Age (years)	11.3	3.8
H = Design thickness (in)	9.1	0.5
I = Actual thickness (in)	9.3	0.7
J = Min PCC pour temperature (deg F)	58.6	10.3
K = Air entrained (percent)	5.6	1.0
L = Core compressive strength (psi)	5450	1076

APPENDIX C
SUMMARY OF SECONDARY PREDICTION RELATIONSHIPS

The following secondary relationships (coded A through U) among M&C variables have been selected from the research and engineering literature. Some relationships are expressed in algebraic form, others are presented in graphical or tabular form.

A. Modulus of Rupture. [ACI 318-83]^(84.8)

$$SF = 7.5*(SC)^{0.5}$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

SF = PCC modulus of rupture (psi).
SC = PCC compressive strength (psi).

B. PCC Flexural Strength. [Jones and Kaplan via Neville]^(73.2)

See figure 41 - Relation between flexural and compressive strengths for concretes made with different aggregates.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

C. PCC Flexural Strength. [Walker and Bloem via Mindess]^(81.4)

See figure 42 - Relation between flexural strength and compressive strength for concretes made with different maximum size coarse aggregate.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

D. PCC Modulus of Rupture. [Wright via Neville]^(73.2)

See figure 43 - Relation between modulus of rupture at 28 days and water/cement ratio.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

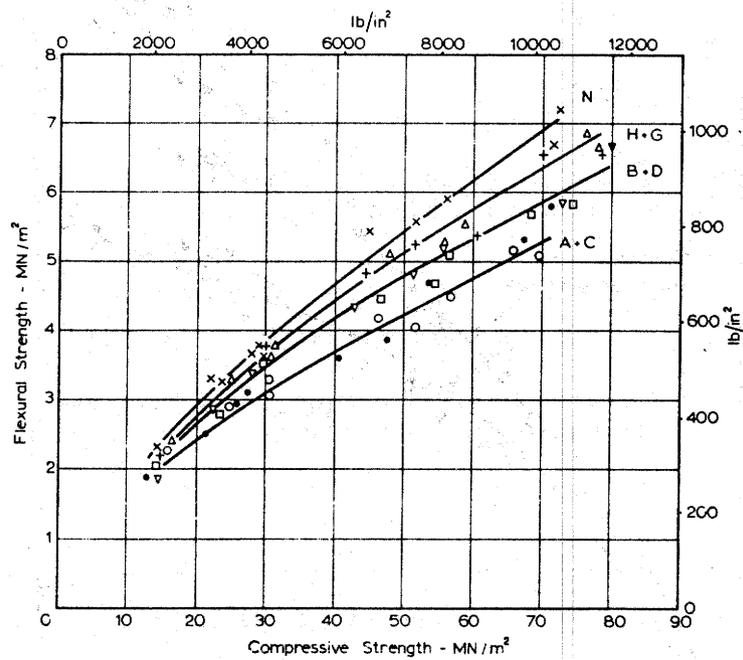


Figure 41. Relationship B: Relation between flexural and compressive strengths for concretes made with different aggregates. (73.2)

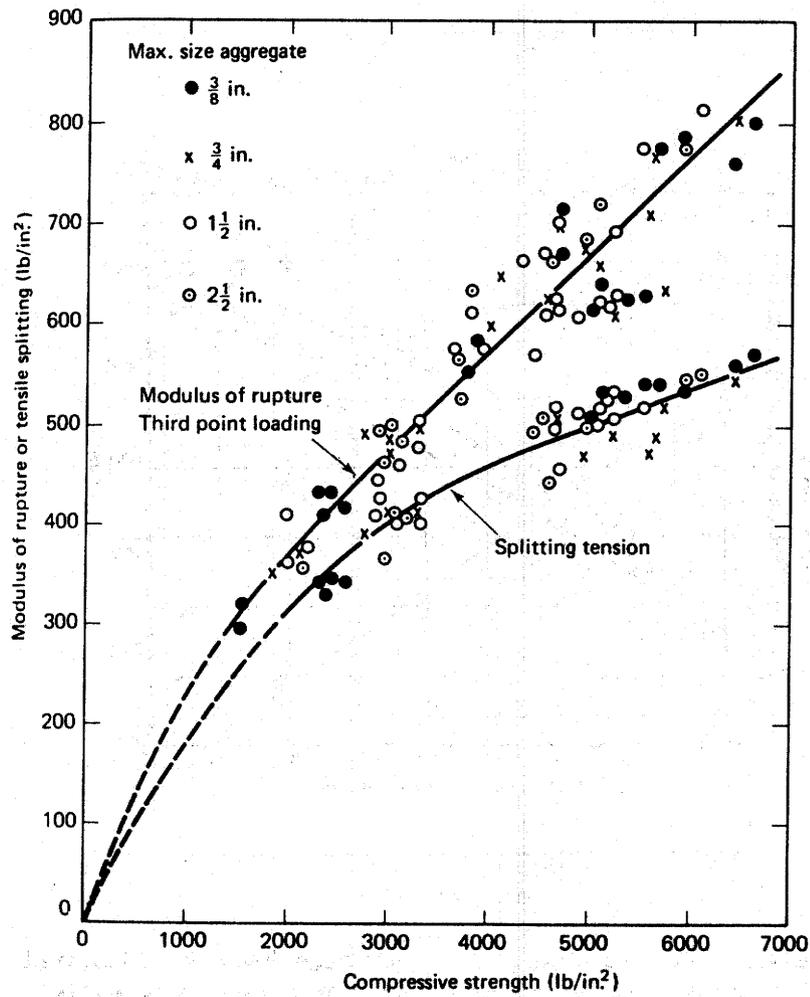


Figure 42. Relationships C and G: Relationships of compressive to flexural and tensile strengths of concrete. ^(81.4)

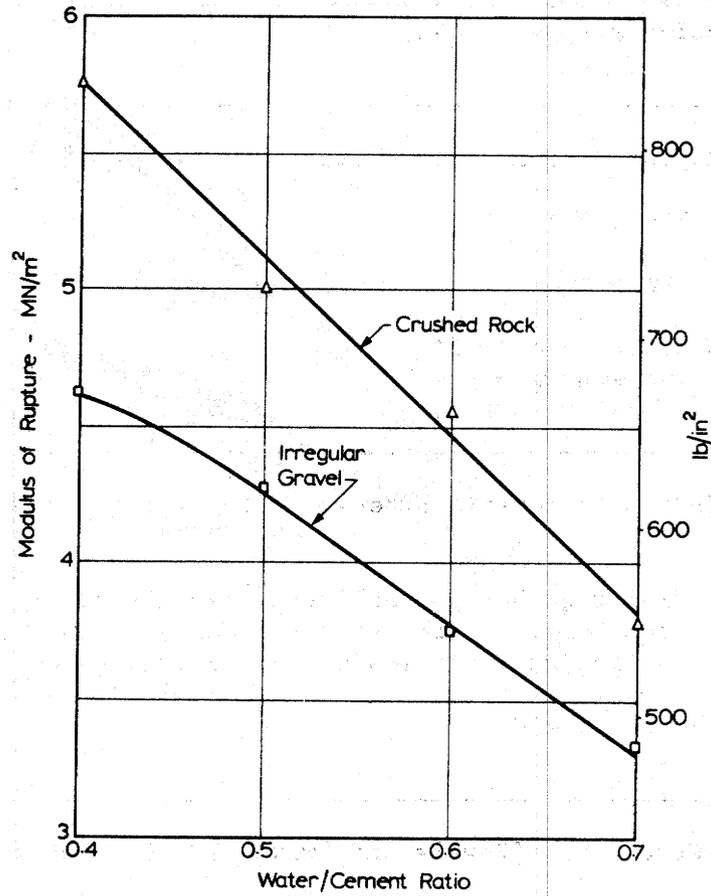


Figure 43. Relationship D: Relation between modulus of rupture at 28 days and water/cement ratio. (73.2)

E. PCC Modulus of Rupture. [European Concrete Committee via Neville]^(73.2)

$$SF = 9.5*(SC)^{0.5}$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

SF = PCC modulus of rupture (psi).
SC = PCC compressive strength (psi).

F. PCC Modulus of Rupture. [Univ. of Illinois via Neville]^(73.2)

$$SF = 3000/(4 + 12000/FC)$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

SF = PCC modulus of rupture (psi).
SC = PCC compressive strength (psi).

G. PCC Splitting (Indirect) Tensile Strength. [Walker and Bloem via Mindess]^(81.4)

See figure 42 - Relation between splitting tensile strength and compressive strength for concretes made with different maximum size coarse aggregate.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

H. PCC Compressive Strength. [Powers and Brownyard via Mindess]^(81.4)

$$SC = 34000*(GR)^3$$

$$GR = (0.68*A)/(0.32*A + WC)$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

SC = PCC compressive strength (psi).
GR = Gel/space ratio.
A = Degree of cement hydration, i.e., fraction of cement that is hydrated.
WC = Water/cement ratio.

I. PCC Compressive Strength. [Singh via Neville]^(73.2)

See figure 44 - Influence of water/cement ratio and aggregate/cement ratio on 7-day compressive strength.

$R^2 = ??$, $SEE = ??$, $n = ??$

J. PCC Compressive Strength. [Portland Cement Association via Flinn and Trojan]^(81.5)

See figure 45 - Compressive strengths for air-entrained and non-air-entrained concretes as related to curing time.

$R^2 = ??$, $SEE = ??$, $n = ??$

K. PCC Compressive Strength. [Bureau of Reclamation via Mindess]^(81.4)

See figure 46 - 28-day compressive strength in relation to cement content for air-entrained and non-air-entrained concrete of constant slump.

$R^2 = ??$, $SEE = ??$, $n = ??$

L. PCC Compressive Strength. [American Concrete Institute via Mindess]^(81.4)

See figure 47 - Influence of aggregate size on 28-day compressive strength of concretes with different cement contents.

$R^2 = ??$, $SEE = ??$, $n = ??$

M. PCC Compressive Strength. [Bureau of Reclamation via Mindess]^(81.4)

See figure 48 - Effect of entrained air content on compressive strength.

$R^2 = ??$, $SEE = ??$, $n = ??$

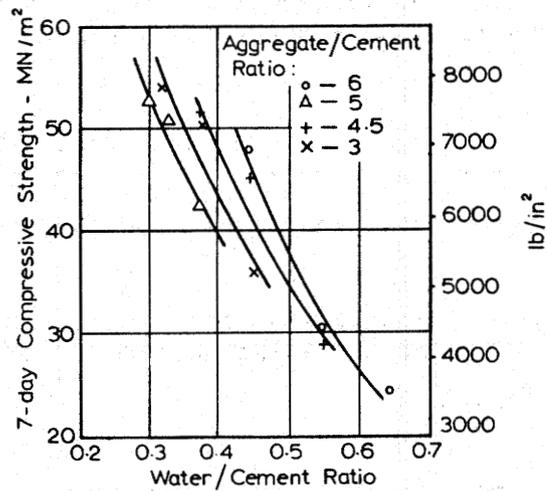


Figure 44. Relationship I: The influence of the aggregate/cement ratio on strength of concrete. (73.2)

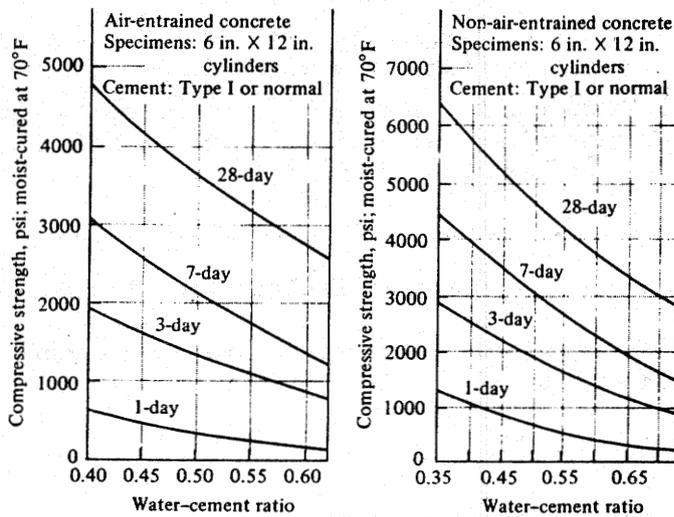


Figure 45. Relationship J: Typical compressive strengths for air-entrained and non-air-entrained concretes as related to curing time. (81.5)

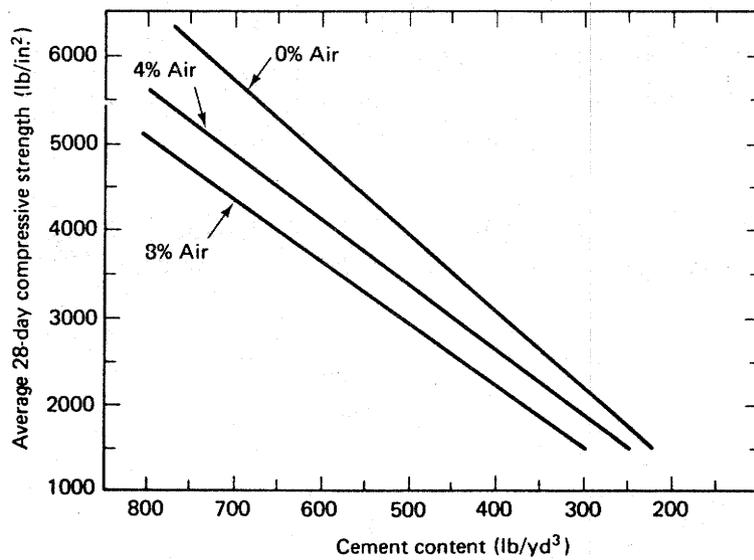


Figure 46. Relationship K: Strength in relation to cement content for air-entrained and non-air-entrained concrete of constant slump. ^(81.4)

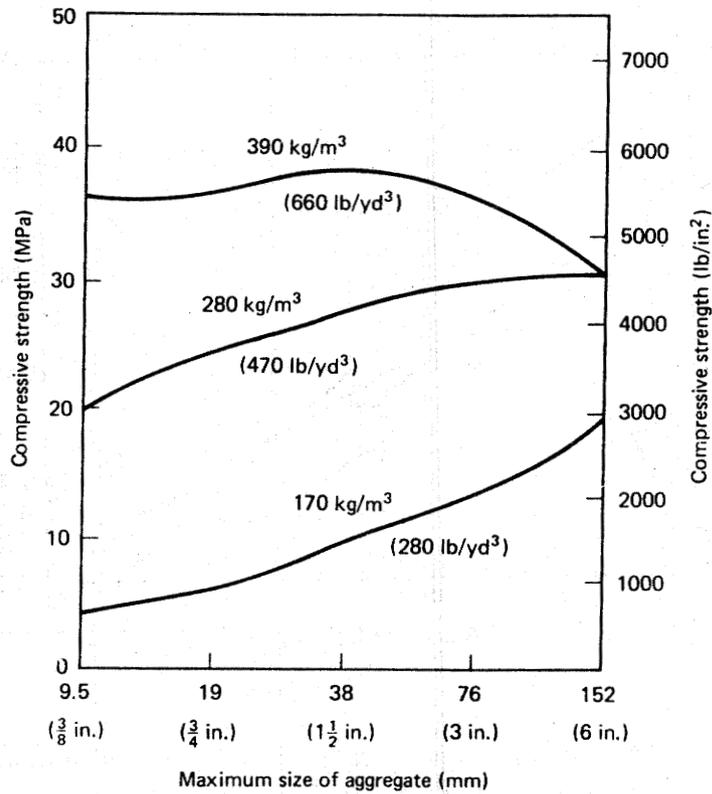


Figure 47. Relationship L: Influence of aggregate size on 28-day compressive strength of concretes with different cement contents. (81.4)

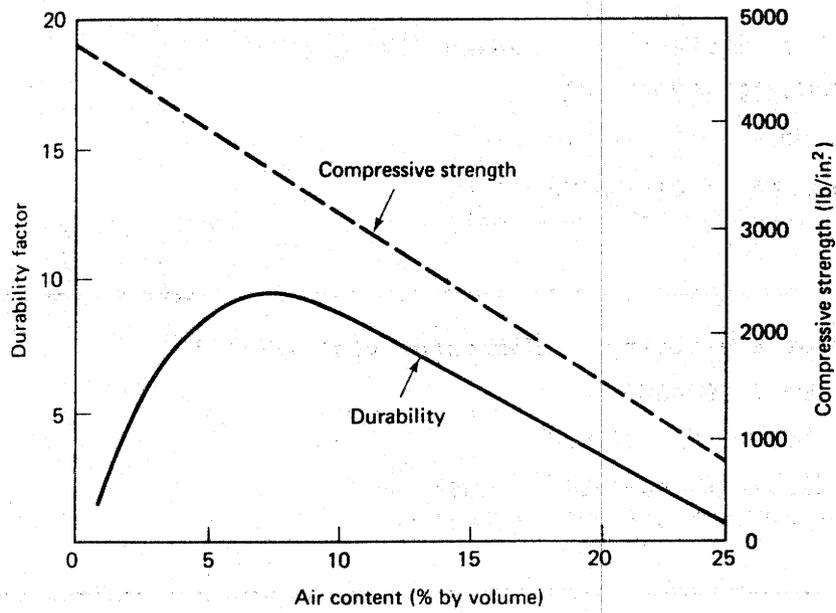


Figure 48. Relationship M: Effect of entrained air on durability. ^(81.4)

N. PCC Modulus of Elasticity. [ACI Code 318-83]^(84.8)

$$EC = UW^{1.5} * 33 * (SC)^{0.5}$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

EC = PCC modulus of elasticity (psi).

UW = PCC unit weight (pcf).

SC = PCC compressive strength (psi).

O. PCC Modulus of Elasticity. [Jensen via Lin]^(63.1)

$$EC = 6000000 / (1 + 2000 / SC)$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

EC = PCC modulus of elasticity (psi).

SC = PCC compressive strength (psi).

P. PCC Modulus of Elasticity. [Hognestad via Lin]^(63.1)

$$EC = 1800000 + 460 * SC$$

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

EC = PCC modulus of elasticity (psi).

SC = PCC compressive strength (psi).

Q. PCC Modulus of Elasticity. [Ishai via Mindess]^(81.4)

See figure 49 - Effect of aggregates on the modulus of elasticity of concrete.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

R. PCC Shrinkage. [Nawy]^(85.5)

See figure 50 - Water/cement ratio and aggregate content effect on concrete drying shrinkage.

$$R^2 = ??, \quad SEE = ??, \quad n = ??$$

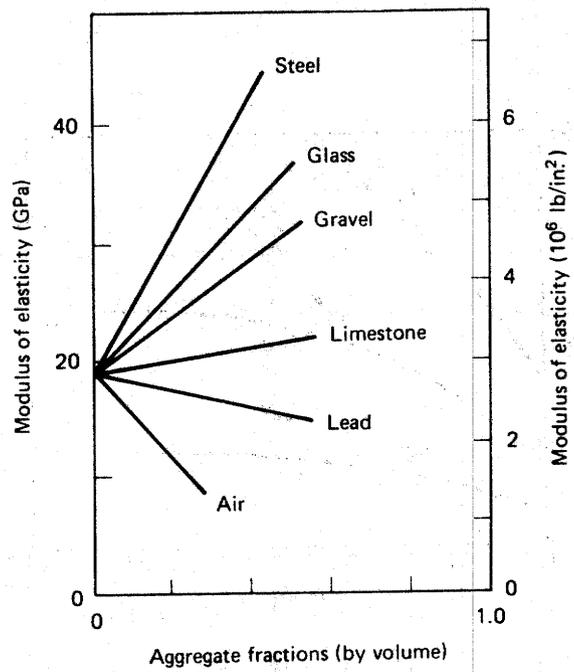


Figure 49. Relationship Q: Effect of aggregates on the modulus of elasticity of concrete. ^(81.4)

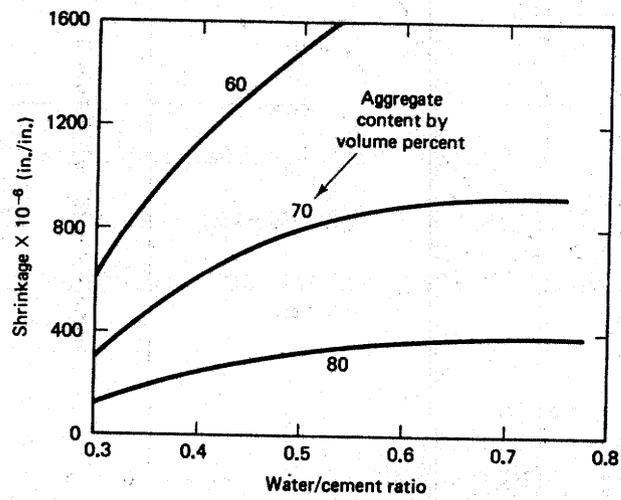


Figure 50. Relationship R: . Water/cement ratio and aggregate content effect on drying shrinkage. ^(85.5)

S. PCC Thermal Coefficient. [Seeds, McCullough and Carmichael]^(82.7)

See table 35 - Thermal coefficients for concretes made with different types of coarse aggregate.

$$R^2 = ??, \text{ SEE} = ??, \text{ n} = 14$$

T. PCC Durability. [Bureau of Reclamation via Mindess]^(81.4)

See figure 48 - Effect of entrained air on concrete freeze-thaw durability.

$$R^2 = ??, \text{ SEE} = ??, \text{ n} = ??$$

U. Water/Cement Ratio. [Federal Highway Administration]^(84.2)

$$\begin{aligned} \text{WC} = & 1.24 + 0.0223*(\text{SL})^2 + 0.00069*[(\text{SL})^2*(\text{AE})^2] - 2174*(\text{SL}/\text{SC}) \\ & + 19279/(\text{AE}*\text{SC}) - 13637/(\text{SL}*\text{AE}*\text{SC}) - 0.000115*(\text{SC}) \end{aligned}$$

$$R^2 = 0.73, \text{ SEE} = 0.06, \text{ n} = 558$$

WC = Water/cement ratio.
SL = Concrete slump (inches).
AE = Entrained air content (percent).
SC = PCC compressive strength (PSI).

Table 35. Relationship S: Thermal coefficients for concretes made with different types of coarse aggregate. ^(82.7)

Coarse Aggregate Type	Mean Value of PCC Thermal Coefficient (10 ⁻⁶ in/in/°F)	Number of Specimens
Syenite	4.7	2
Dolomite	4.0	2
Limestone	4.4	2
Sandstone	5.7	4
Gravel - A	5.8	2
Gravel - B	7.2	2

1 in = 25.4 mm
 °C = 5(°F-32)/9

APPENDIX D RII DATA BASE ANALYSIS

The Resource International Inc (RII) data base was generated as part of a previous study for the Federal Highway Administration.^(84.2) The purpose of this study was to examine the interrelationships between quality indicators and the performance of portland cement concrete (PCC) pavements. Historical, construction and condition data relative to selected quality variables were collected for 104 concrete pavement projects in five different States (Florida, Louisiana, Maryland, New York, and Ohio). A number of statistical analyses were then performed to establish prediction relationships between PCC pavement performance and the available quality indicator data.

Since the focus of the study was on development of primary prediction relationships, the thrust of it was to evaluate the potential for using the RII data base in developing secondary prediction relationships. Basically, it was believed that all potential avenues of primary prediction relationship development had been explored by the original researchers.

The data that was used in the analysis was extracted from the RII SECTION file. The SECTION file contains data for 733 sections within the 104 projects. Data for up to 66 variables is provided for each section. The names and descriptions of the 66 variables are presented in table 36.

With the primary objective of analyzing the data to develop secondary prediction relationships, a statistical approach consisting of four basic steps was derived. These four steps are:

- Correlation matrix.
- Frequency histograms.
- Scatter plots.
- Multiple regression analyses.

Each of these steps is discussed below.

CORRELATION MATRIX

After examining the pertinent factors and the extent to which they exist to the data base, only a small subset were chosen for inclusion in what was termed a "working data base". Table 37 identifies these pertinent factors as well as their corresponding summary statistics. Observations worth noting in this table include:

- The distress-performance and traffic/time factors were included more as a means for testing the reasonableness of the strength and M&C factor data than for developing primary prediction relationships.
- As observed in the original RII SECTION data base, flexural and compressive strengths each have four data "slots" allocated. These slots were included in order to account for the age at which the

Table 36. Description of variables in RII SECTION data file.

No.	Variable	Description
1.	STATE	State Code
2.	CO	County Code
3.	RTYP	Route Type
4.	ROUTE	Route No.
5.	PROJ	Project No.
6.	BEGST	Begin Station
7.	ENDST	End Station
8.	PRTYP	Project Type (1=CRC, 2=JCP)
9.	NSPP	No. of sections/project
10.	PCRCL	PCR Class (ignore)
11.	CLIM	Climate (W=wet, WF=wet freeze)
12.	BSTYP	Base Type (GRANULAR or STABILIZED)
13.	SGTYP	Subgrade Type (G=good, F=fair, P=poor)
14.	JTYP	Joint Type (DOW=dowel bars, PLAI=plain)
15.	JTSP	Joint Spacing (applicable to JCP only)
16.	TRAF	Cumulative (millions of 18-kip ESALs)
17.	AGE	Age of pavement, years
18.	PDT	Pavement design thickness, inches
19.	PCR	Pavement condition rating
20.	RCI	Ride comfort index
21.	STR	Structural deduct value
22.	SURF	Surface deduct value
23.	JOIN	Joint deduct value
24.	SUPP	Support deduct value
25.	CRAK	Cracking deduct value
26.	LFT	Lineal ft of transverse cracking/200 ft.
27.	SQP	Square yards of patching/200 ft.
28.	NP	Number of punchouts/200 ft.
29.	MINT	Minimum pour temperature, °F
30.	MAXT	Maximum pour temperature, °F
31.	SLUMP	Concrete slump, inches
32.	PAIR	Percent entrained air
33.	AIR	Air content, percent
34.	WT	Unit weight, lb/ft ³
35.	WC	Water-cement ratio
36.	YIELD	Cubic feet of concrete per sack of cement
37.	FS3	Flexural strength, psi (0-3 days)
38.	FS7	Flexural strength, psi (4-7 days)
39.	FS15	Flexural strength, psi (8-15 days)
40.	FS	Flexural strength, psi (> 15 days)
41.	CS3	Compressive strength, psi (0-3 days)
42.	CS7	Compressive strength, psi (4-7 days)
43.	CS15	Compressive strength, psi (8-15 days)
44.	CS	Compressive strength, psi (> 15 days)
45.	COREST	Core compressive strength, psi
46.	CAGE	Core age at testing, months
47.	DTHICK	Design thick-core thick, inches
48.	SMINT	Std. Deviation of MINT (No. 29)

Table 36. Description of variables in RII SECTION data file (continued).

No.	Variable	Description
49.	SMAXT	Std. Deviation of MAXT (No. 30)
50.	SSLUMP	Std. Deviation of SLUMP (No. 31)
51.	SPAIR	Std. Deviation of PAIR (No. 32)
52.	SAIR	Std. Deviation of AIR (No. 33)
53.	SWT	Std. Deviation of WT (No. 34)
54.	SWC	Std. Deviation of WC (No. 35)
55.	SYIELD	Std. Deviation of YIELD (No. 36)
56.	SFS3	Std. Deviation of FS3 (No. 37)
57.	SFS7	Std. Deviation of FS7 (No. 38)
58.	SFS15	Std. Deviation of FS15 (No. 39)
59.	SFS	Std. Deviation of FS (No. 40)
60.	SCS3	Std. Deviation of CS3 (No. 41)
61.	SCS7	Std. Deviation of CS7 (No. 42)
62.	SCS15	Std. Deviation of CS15 (No. 43)
63.	SCS	Std. Deviation of CS (No. 44)
64.	SCORES	Std. Deviation of COREST (No. 45)
65.	SCAGE	Std. Deviation of CAGE (No. 46)
66.	SDTHIK	Std. Deviation of DTHICK (No. 47)

Table 37. Summary statistics for key factors contained in "working" RII data base.

SELECTED VARIABLES	RII CODE	NO. OF CASES	MEAN VALUE	STD. DEV.	5th %tile	95th %tile
Distress-Performance Factors						
PCR (Pavement Condition Rating)		734	74.24	7.55	62	87
RCI (Riding Comfort Index)		727	7.33	0.80	6.1	8.0
Traffic/Time Factors						
TRAF (Cumulative ESAL)	N16	733	4.40	3.60	1.0	12.0
AGE (Years)	N17	733	11.25	3.85	5.1	17.1
Surfacing Factors						
PDT (Pavement Design Thickness)		733	9.13	0.52	8.0	10.0
FS7 (7-Day Flexural Strength)		179	627	123	422	812
CS7 (7-Day Compressive Strength)		336	3369	620	2400	4400
COREST (Core Strength)	N46	458	5455	1142	3462	7065
PCC M&C Factors						
SLUMP	N32	531	2.05	0.64	1.0	3.0
PAIR (% Air Entrained)	N33	495	5.45	0.98	4.0	7.0
WC (Water Content Ratio)	N36	262	0.43	0.17	0.0	1.0

specimens were tested. Only the 7-day strengths were considered in this study, since in both cases, they were the ones that contained the largest numbers of cases.

- Like the distress-performance factors, pavement design thickness was included more as a means of testing the data than for developing primary or secondary prediction relationships.

[Note: The microcomputer version of the Statistical Package for the Social Sciences program, SPSS/PC+ (V2.0), was used to conduct all statistical analyses of the working data base.]

Using the SPSS package, a correlation matrix was generated to examine the magnitude of the correlations between the various factors. This matrix is shown in table 38. The top number in each cell of the matrix represents the actual Pearson correlation coefficient while the bottom number (shown in parentheses) represents the number of pairwise cases that went into the determination of the corresponding correlation coefficient.

In examining the magnitudes of the correlation coefficients, it is helpful to recognize that an absolute value of greater than 0.7 generally indicates a strong degree of linear correlation. Furthermore, absolute values in the range of 0.4 to 0.7 are considered to be reasonably well correlated such that a meaningful relationship could be derived. Correlation coefficients that approach 0 indicate no linear correlation between the two variables. With this as a background, one can begin to understand the strong degree of correlation between the 7-day flexural strength (FS7) and the long-term concrete core strength (COREST) and also the strong negative correlation between water/cement ratio (WC) and concrete core strength (COREST). On the other hand, there should be some reason for concern because of the strong negative correlation between water/cement ratio (WC) and concrete slump and also the strong positive relationship between concrete slump and pavement age. Both of these correlations between the variables are examined in further detail in the following sections.

FREQUENCY HISTOGRAMS

Frequency histograms provide information about the distribution of values for each variable within their observed range. Figures 51 through 61 provide frequency histograms for each of the selected variables contained in the working data base. Note that several of the histograms (i.e., traff, age, WC and PDT), are far from being normally distributed. This should cast some doubt on the interpretation of the values of standard deviation provided. Furthermore, it should also be recognized when conducting further statistical analyses that assume normally distributed variables.

SCATTER PLOTS

Several "scatter" plots were prepared to visually examine the relationships between many of the key variables. Figures 62 through 65

Table 38. Correlation matrix of factors in "working" RII data base.

SEL- ECTED VAR- IABLE	DISTRESS PERFORMANCE		TRAFFIC/TIME		SURFACING FACTORS				PCC M&C FACTORS		
	PCR	RCI	TRAF	AGE	PDT	FS7	CS7	COREST	SLUMP	PAIR	WC
PCR		.378 (727)	.0433 (733)	-.131 (733)	-.0586 (733)	.0667 (179)	.0041 (336)	-.335 (458)	-.0764 (531)	-.0458 (495)	.167 (262)
RCI			.119 (726)	.221 (726)	.176 (726)	.363 (179)	.0584 (336)	.0950 (455)	.0859 (525)	.156 (489)	-.177 (262)
TRAF				.445 (733)	.399 (733)	.0822 (179)	.282 (336)	.0020 (458)	.426 (531)	.366 (495)	-.491 (262)
AGE					.490 (733)	.668 (179)	.521 (336)	.533 (450)	.735 (531)	.540 (495)	-.661 (262)
PDT						.207 (179)	.0782 (336)	.308 (458)	.332 (531)	.243 (495)	-.180 (262)
FS7							.479 (103)	.767 (125)	.496 (175)	.630 (151)	.0194 (54)
CS7								.544 (233)	.340 (335)	.299 (318)	-.286 (220)
COREST									.430 (352)	.275 (345)	-.762 (203)
SLUMP										.539 (495)	-.693 (259)
PAIR											-.660 (259)
WC											

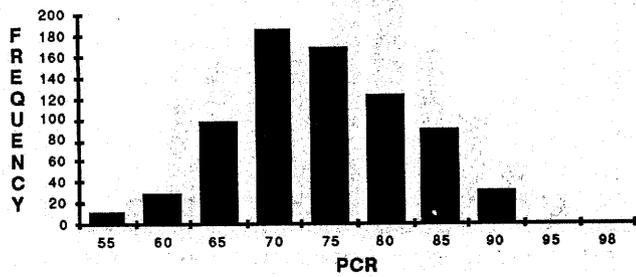


Figure 51. Pavement condition rating.

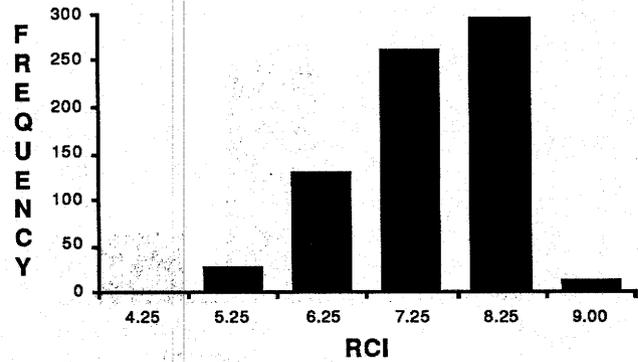


Figure 52. Ride condition index.

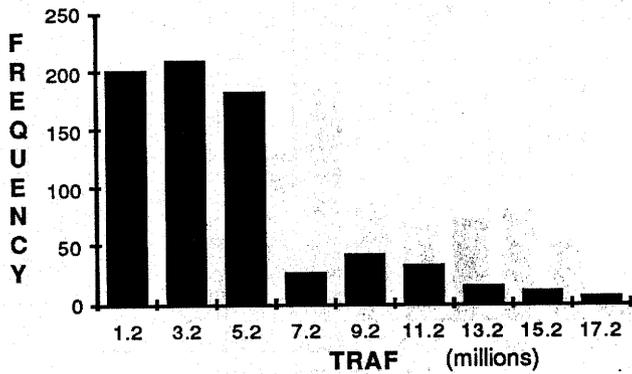


Figure 53. Cumulative 18-kip ESAL traffic.

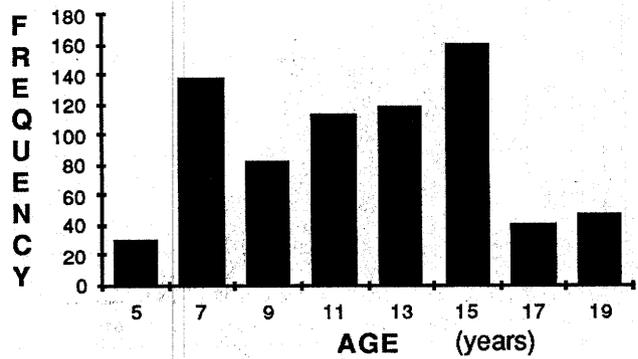


Figure 54. Pavement age.

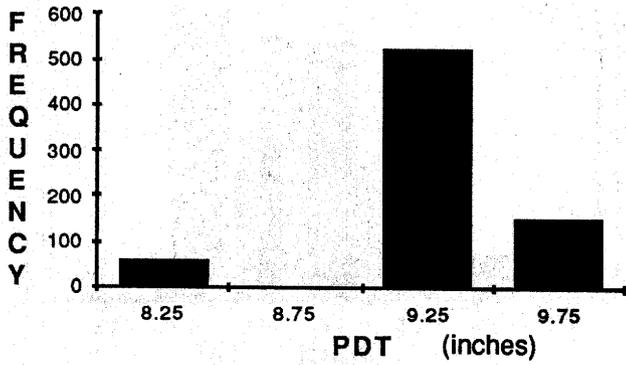


Figure 55. Pavement design thickness.

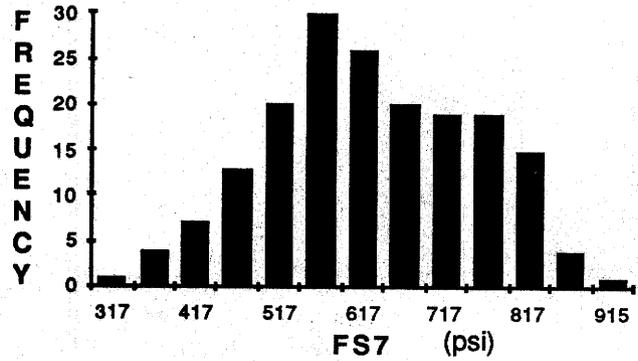


Figure 56. Concrete flexural strength at 7 days.

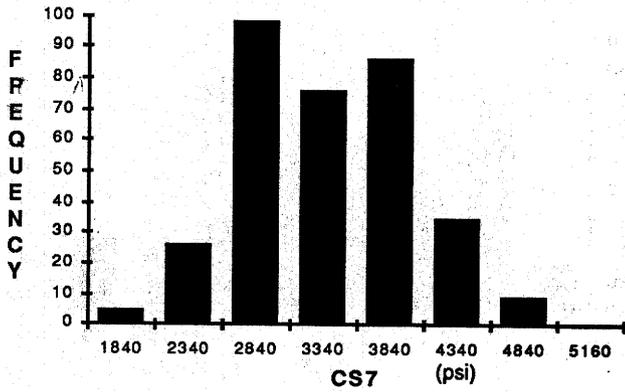


Figure 57. Concrete compressive strength at 7 days.

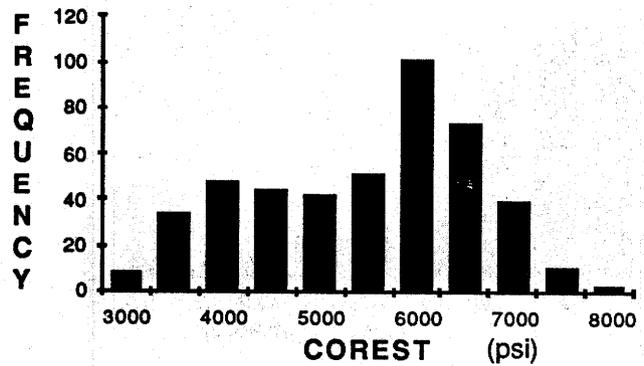


Figure 58. Concrete core strength.

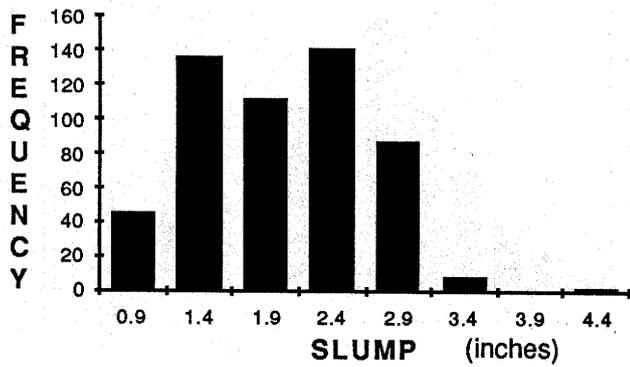


Figure 59. Concrete slump.

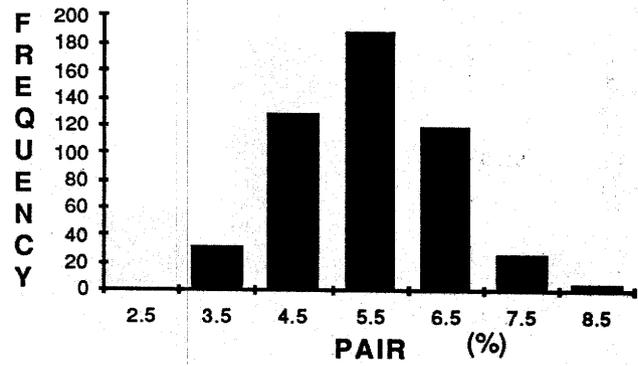


Figure 60. Percent air in mix.

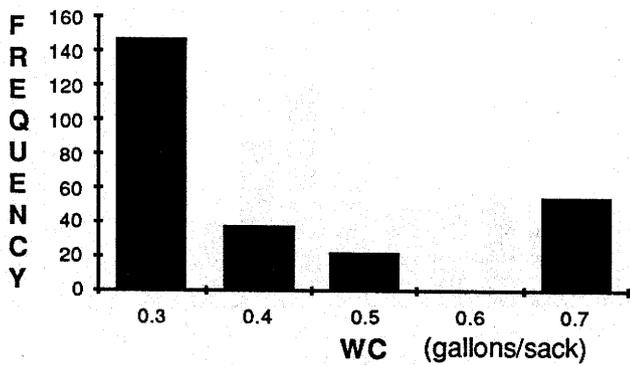
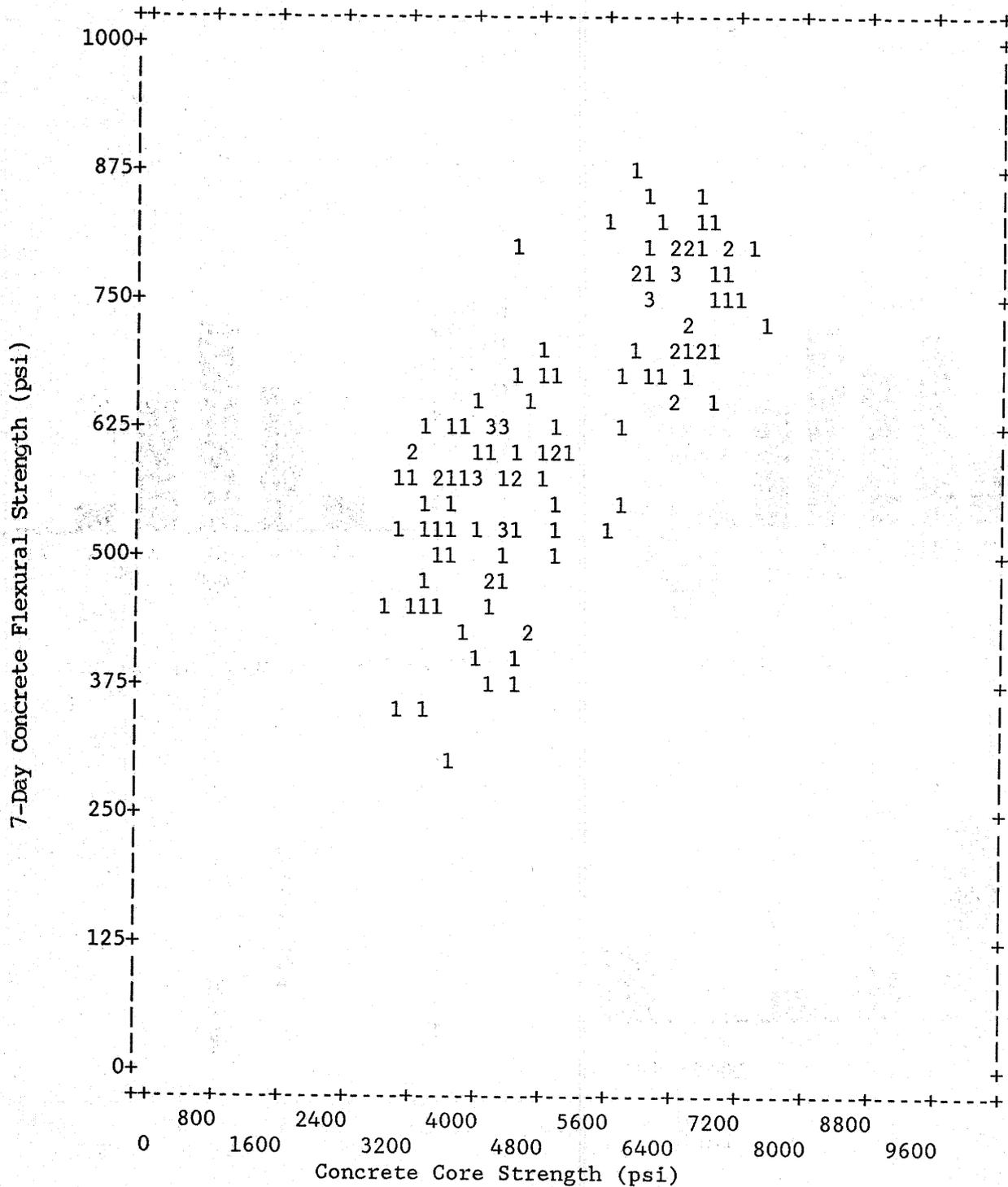
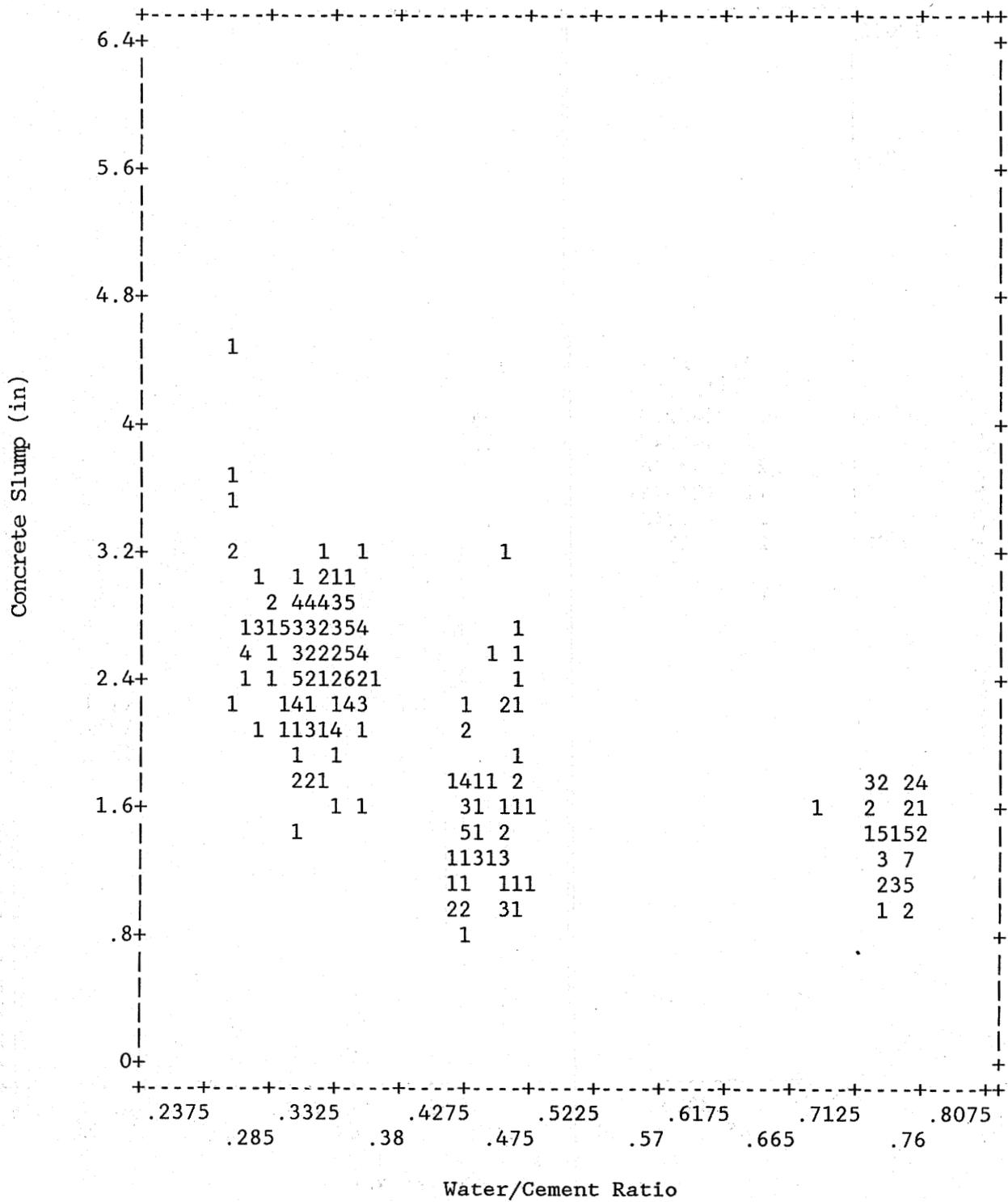


Figure 61. Water/cement ratio.



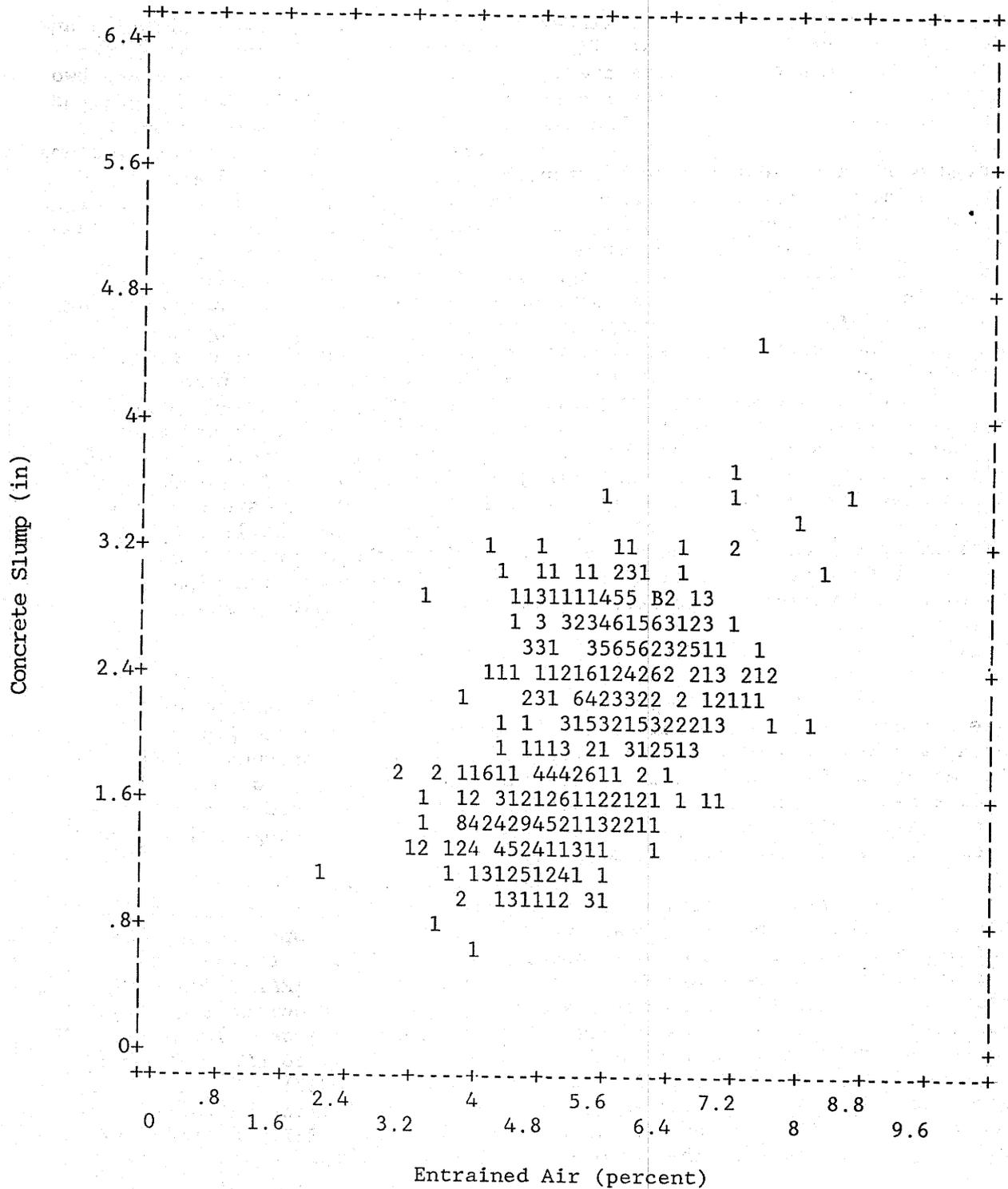
125 cases plotted.

Figure 62. Concrete flexural strength versus concrete core compressive strength.



259 cases plotted.

Figure 64. Concrete slump versus water/cement ratio.



495 cases plotted.

Figure 65. Concrete slump versus entrained air percentage.

illustrate some of the better linear correlations between the surfacing and PCC M&C factors. In contrast, figures 66 and 67 provide examples of the factors that are poorly correlated. Figure 68 provides a plot between two factors that have a poor linear correlation but that may be better related in some non-linear fashion. The next two plots depict rather strong but unsettling relationships between 7-day flexural strength and concrete slump (figure 69) and 7-day flexural strength and percent entrained air (figure 70). From an engineering standpoint, it is illogical to believe that high-slump concrete produces high strength. Similarly, it is also not reasonable to believe that high air entrainment results in high strength (although it is generally acknowledged that high air entrainment does produce more workable concrete that is also more durable from a freeze-thaw standpoint). The reasons for this are perhaps explained by the plots in figures 71, 72 and 73. Each plot indicates that the characteristics of the concrete are highly dependent on the year the pavement was constructed. Thus, the relationships here may be more reflective of changes in design/construction practice rather than the true relationships between strength and mix factors. For example, over the years more and more contractors have gone to the use of high-production slip-form pavers that require low-slump concrete. It is also likely that better quality control procedures have permitted contractors to lower their target concrete strength and still satisfy minimum specifications. Unfortunately, these kinds of problems make it difficult to develop practical secondary prediction relationships that could be used in developing a performance-related specification system.

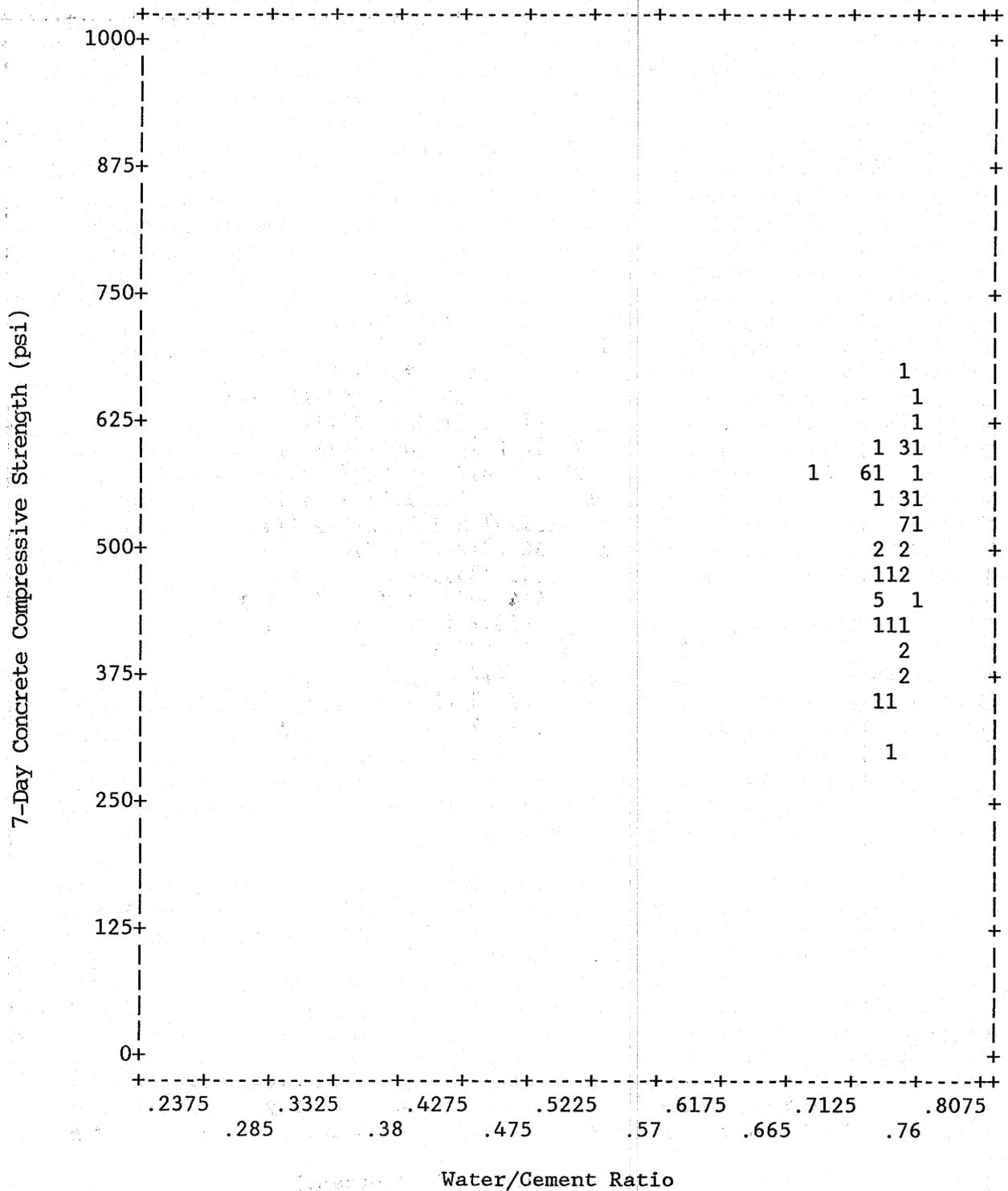
MULTIPLE REGRESSION ANALYSES

The final step in the process of analyzing the RII SECTION data base was the development of prediction relationships between the key variables using standard stepwise linear regression analysis techniques. Initially, it was hoped that the resulting relationships could be used in the final specifications system. However, because of the confounded nature of the data, the results now show the danger associated with using observational data bases to develop prediction equations.

Five separate prediction relationships were derived: two primary and three secondary. The two primary relationships had (respectively) pavement condition rating (PCR) and ride condition index (RCI) as the dependent variables with pavement age (AGE), cumulative traffic (TRAF), 7-day concrete flexural strength (FS7), concrete slump (SLUMP), water/cement ratio (WC), and entrained air percentage (PAIR) as potential independent variables. The three secondary relationships had (respectively) 7-day concrete flexural strength (FS7), 7-day concrete compressive strength (CS7) and concrete core strength (COREST) as dependent variables with water/cement ratio (WC), concrete slump (SLUMP), and entrained air percentage (PAIR) as potential independent variables. The results are presented below.

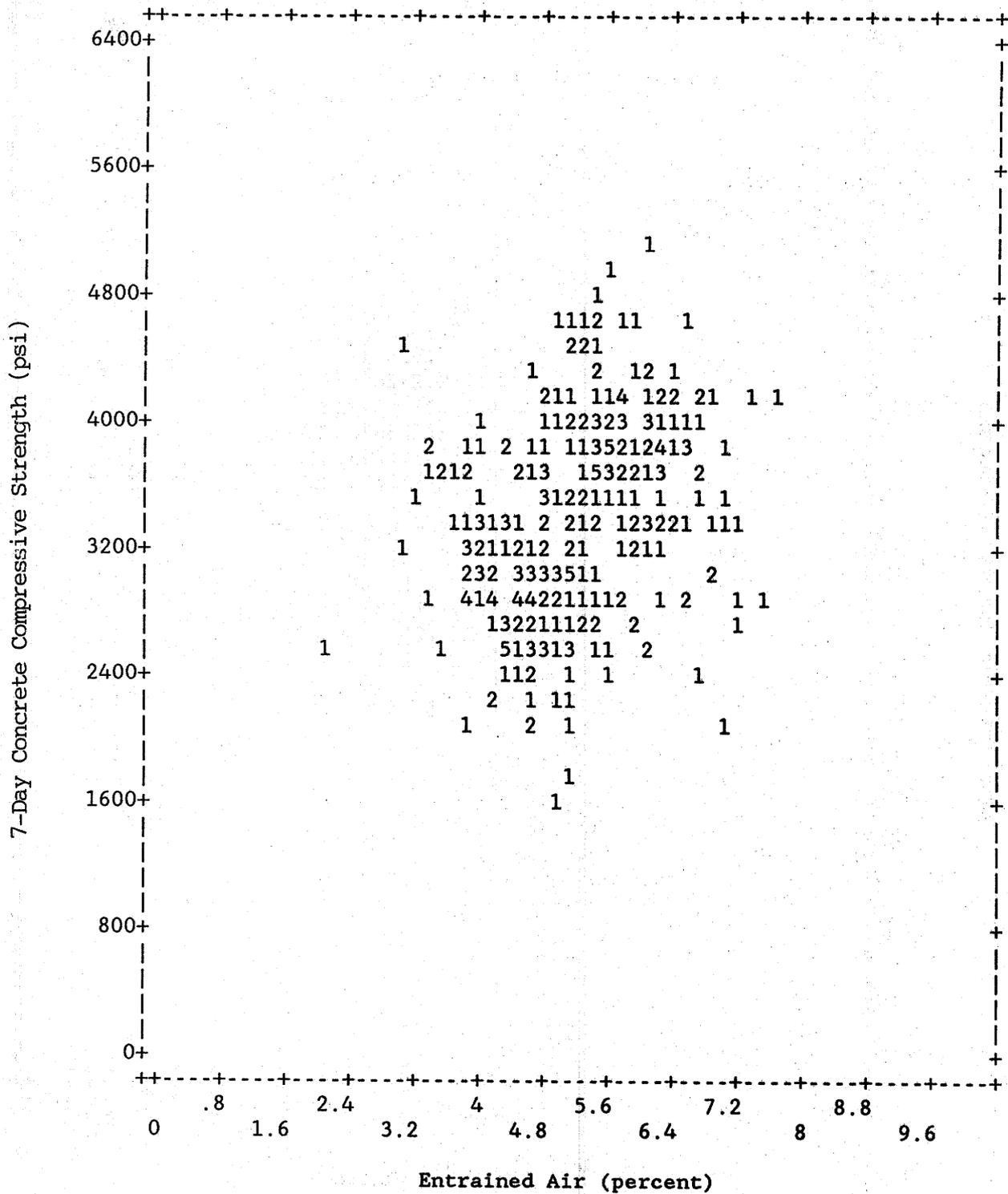
Primary Relationships:

1. $PCR = 137.0 + 10.2(AGE) - 15.2(TRAF) - 146.7(WC)$
 $R^2 = 0.82, n = 53, S.E.E. = 3.93$



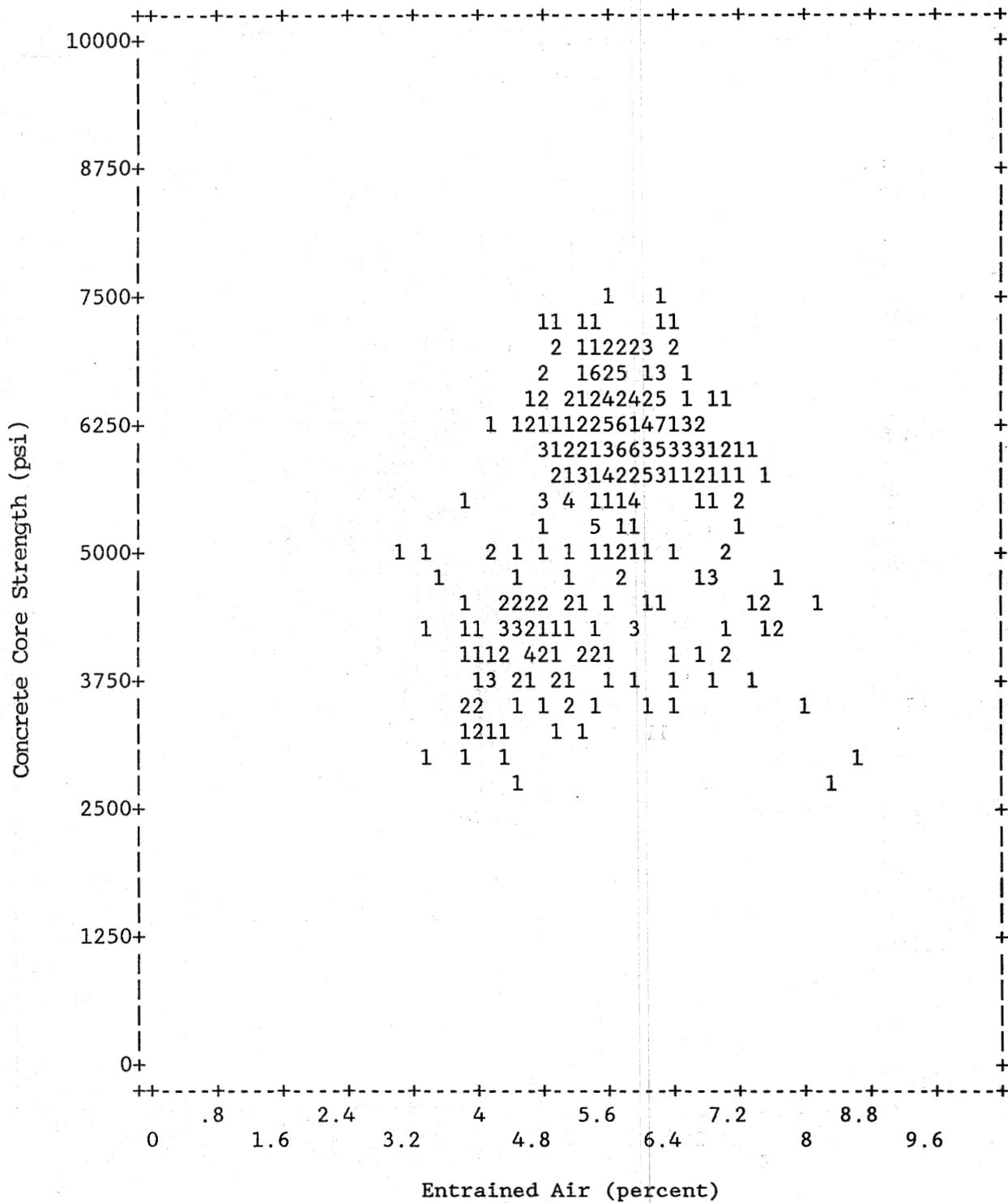
54 cases plotted.

Figure 66. Concrete flexural strength versus water/cement ratio.



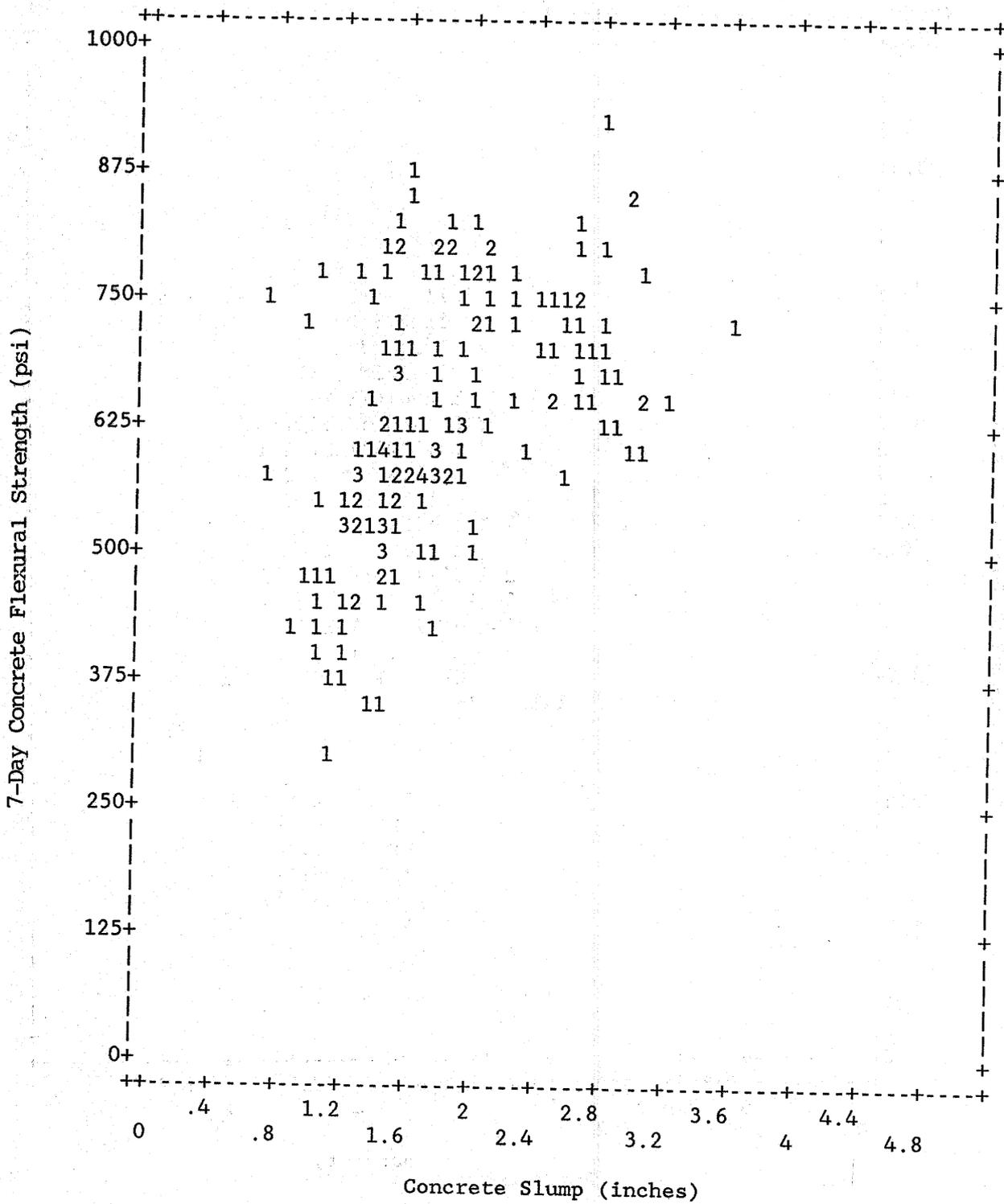
318 cases plotted.

Figure 67. Concrete compressive strength versus entrained air percentage.



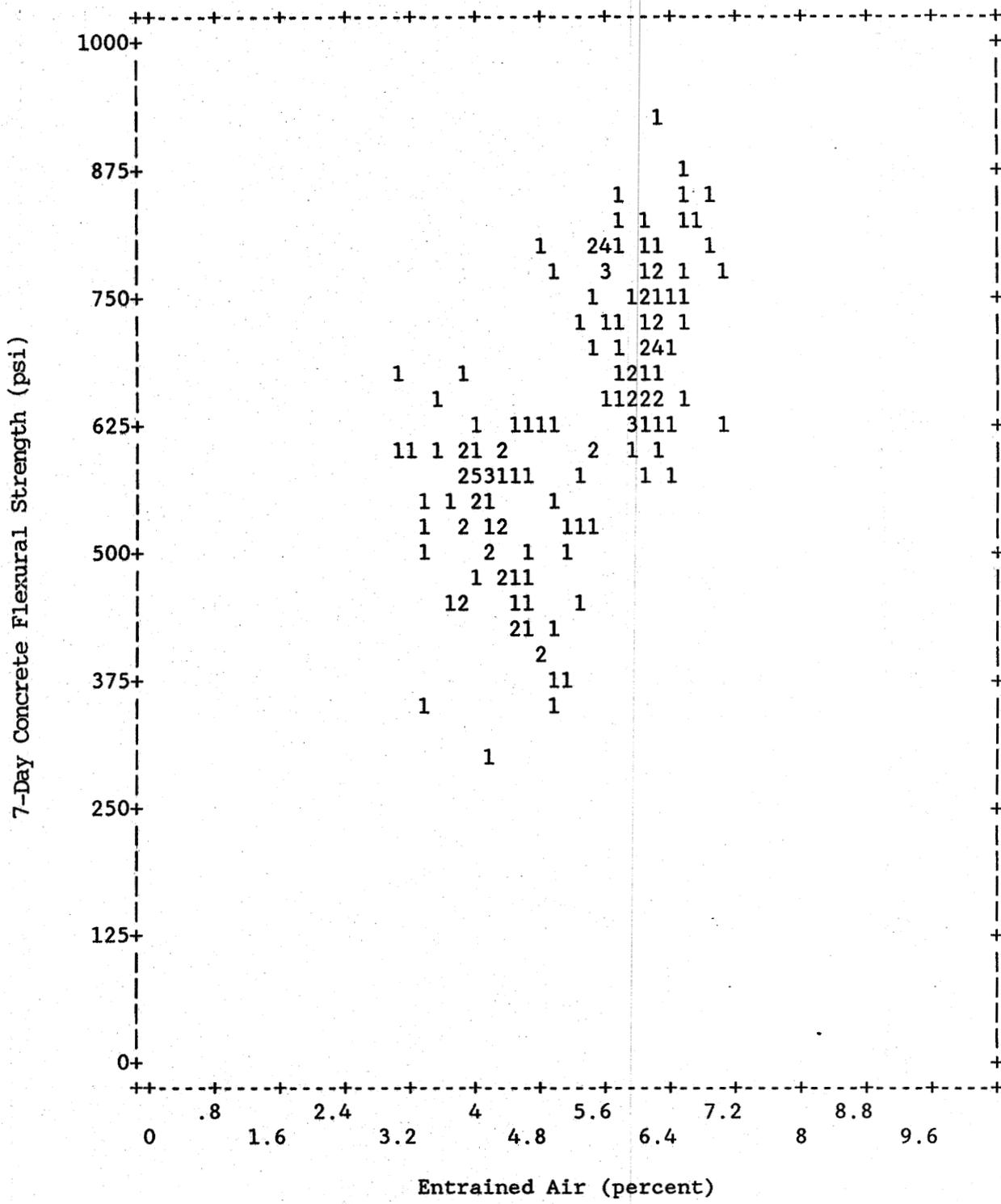
345 cases plotted.

Figure 68. Concrete core strength versus entrained air percentage.



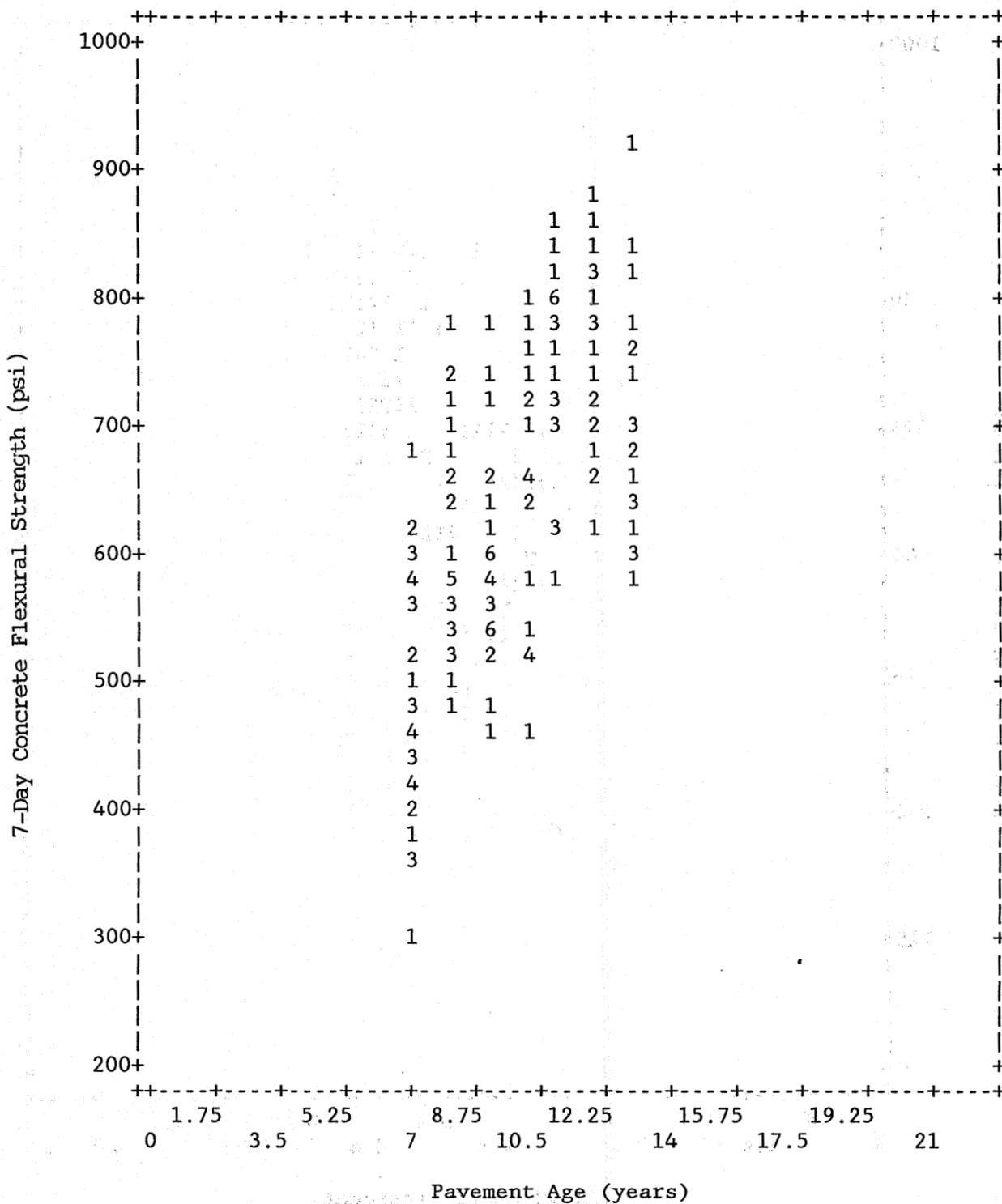
175 cases plotted.

Figure 69. Concrete flexural strength versus slump.



151 cases plotted.

Figure 70. Concrete flexural strength versus entrained air percentage.



179 cases plotted.

Figure 71. Concrete flexural strength versus pavement age.

$$2. \text{ RCI} = 11.0 - 1.46(\text{TRAF}) - 0.509(\text{PAIR}) + 0.734(\text{SLUMP})$$

$$R^2 = 0.71, n = 53, \text{ S.E.E.} = 0.40$$

Secondary Relationships:

$$3. \text{ FS7} = 437 + 167(\text{SLUMP}) - 38.7(\text{PAIR})$$

$$R^2 = 0.42, n = 53, \text{ S.E.E.} = 64.0$$

$$4. \text{ CS7} = 3890 - 965(\text{WC})$$

$$R^2 = 0.08, n = 219, \text{ S.E.E.} = 552$$

$$5. \text{ COREST} = 7370 - 4448(\text{WC})$$

$$R^2 = 0.59, n = 199, \text{ S.E.E.} = 657$$

Initial examination of the relationships shows that although the available independent variables did have a reasonable to good correlation with the dependent variable, not all entered the equation. The reason for this is that much of the variability observed in the dependent variables is explained by almost any one of the independent variables. This phenomenon is detectable by stepwise regression analysis methods so that redundant terms can be ignored.

Following are some other observations and comments that can be made about the relationships. Some of these (indicated below) are the logical consequences of some of the data inconsistencies discussed previously:

1. For the PCR equation, the limits of PCR are from 0 (very bad) to 100 (very good):
 - a. 82 percent of the variability observed in PCR was predictable using three independent variables AGE, TRAF, and WC. However, of the 733 sections, there were only 53 in which all 4 factors were available to derive the relationship.
 - b. The coefficient on the AGE term is a +10. This means that for every year the pavement ages, its predicted PCR increases by 10. Obviously, this is offset by the traffic term; however, for low levels of traffic, the equation would predict that the pavement gets better with time.
 - c. The range of water/cement ratio (WC) observed in the data was from 0.3 to 0.7. With this kind of range, the effect of the WC term is relatively large but not unreasonable.
2. In the RCI equation, the range of RCI is from 0 (poor) to 10 (good):
 - a. 71 percent of the observed variation in RCI is explained by three independent variables; in this case TRAF, PAIR, and SLUMP. But again, only 53 of the available 733 sections could be used to derive the relationship.
 - b. The coefficient in the PAIR term is very significant, but opposite to practical expectations. Increasing the entrained air

in concrete should improve its durability and, therefore, its performance.

- c. The coefficient on the SLUMP term is also very significant and contrary to practical expectations. To improve projected future ride condition, one would not recommend high slump concrete.
3. 7-day flexural strengths (FS7) in the SECTION data base are on the order of 400 to 900 psi (28 to 63 kg/cm²):
 - a. 42 percent of the observed variation in FS7 is explained by SLUMP and PAIR. Again, the equation is based on only 53 sections.
 - b. The equation is not practical, however, since the positive coefficient on SLUMP indicates that increasing concrete slump increases its strength.
 4. The observed range of 7-day compressive strength (CS7) is between about 2,000 and 5,000 psi (140 and 350 kg/cm²):
 - a. Apparently none of the available independent variables could predict the variation observed in CS7. Using 219 of the available 773 sections, WC explained only 8 percent.
 - b. Although the equation does not explain very much of the variation in the data, it does, interestingly, seem to produce reasonable results.
 5. The observed range of COREST, the long-term compressive strength of the concrete (as determined from pavement cores), is between 3,000 and 8,000 psi (210 and 560 kg/cm²):
 - a. 59 percent of the variation of COREST could be explained by just one variable, WC. Furthermore, this equation is based on a more reasonable number (percentage-wise) of the total number of sections.
 - b. The equation does seem to provide reasonable results; unfortunately, core strength has not typically been used as factor for predicting pavement performance.

APPENDIX E COPES DATA BASE ANALYSES

The Concrete Pavement Evaluation System (COPES) was developed under NCHRP Project 1-19.^(85.2) COPES provides a framework and procedure for collecting field data on the characteristics and performance of inservice portland cement concrete (PCC) pavements. As part of the research study, data were collected in six States using standard COPES data collection procedures. These data were subsequently analyzed by the researchers to develop PCC pavement distress prediction relationships and also to evaluate various design, construction, maintenance and rehabilitation aspects of PCC pavements. Because of the extent of the data available in COPES, it seemed a likely candidate for the development of additional relationships (primarily secondary) that might be useful in this project for the development of a performance-related specifications system for PCC pavements.

The COPES data available for this research effort represented only five of the original six States. Included were California, Georgia, Louisiana, Minnesota, and Utah. Illinois data could not be considered, since it had not been reconfigured in time for these analyses. There are 1182 observations (pavement sections) in the five-State data base that was analyzed. Of these, 994 observations make up Minnesota's recently updated portion.

Upon extracting the desired information from the COPES data base, the data was placed in ASCII format for use with SPSS/PC+ (V2.0), a microcomputer-based statistical analysis program. A list of the key variables is presented in table 39. The data base analyses consisted of four basic steps:

- Correlation matrix.
- Frequency histograms.
- Scatter plots.
- Multiple regression analyses.

Each of these is discussed below.

CORRELATION MATRIX

Pearson correlation coefficients were generated for all pairwise combinations of the variables in the COPES database. A partial correlation matrix is presented in table 39. The top number in each cell is the actual Pearson correlation coefficient while the lower number (shown in parentheses) represents the number of pairwise observations included in the determination of the coefficients. Several noteworthy observations can be made based upon inspection of the table.

Three M&C variables are well correlated with 28-day modulus of rupture, an explicit performance predictor. These factors and their corresponding

Table 39. Correlations for COPES data base.

VARIABLES	MEAN ST DEV	CASES	D105	D101A	WTOC	D101B	D101C	D107A	D101D	D102A	RIR	AGE	TCUMR	D104A	D41	S37RH
Type of cement used (i.e. 1 or 2) D105	1.04 0.33	1059	×	-0.067 (1002)	0.038 (731)	0.0870 (1001)	0.001 (998)	-0.154 (1035)	-0.006 (732)	-0.723 (163)	-	-0.044 (1059)	0.241 (152)	0.418 (1035)	0.011 (1059)	-0.007 (1058)
Amount of coarse aggregate (lb/cy) D101A	2156 231	1078	×	-0.464 (772)	-0.051 (1077)	0.168 (772)	0.346 (1074)	0.066 (1068)	-0.249 (772)	0.256 (139)	-	0.341 (1028)	-0.030 (128)	0.091 (1063)	-0.136 (1028)	0.010 (1077)
Water to cement ratio WTOC	0.46 0.04	772		×	0.168 (772)	×	-0.582 (772)	-0.060 (764)	0.579 (772)	-0.499 (127)	-	-0.379 (745)	0.190 (127)	-0.186 (763)	-0.065 (745)	-0.029 (771)
Amount of fine aggregate (lb/cy) D101B	1020 119	1077			×	×	0.146 (1074)	-0.385 (1068)	-0.025 (772)	-0.246 (138)	-	0.052 (1027)	0.236 (127)	0.241 (1063)	-0.055 (1027)	-0.016 (1076)
Amount of cement (lb/cy) D101C	526 46	1074					×	0.054 (1065)	0.309 (772)	0.2727 (138)	-	0.011 (1024)	-0.272 (127)	0.1688 (1060)	0.080 (1024)	-0.012 (1073)
Air content (%) D107A	5.26 0.79	1127						×	0.1159 (764)	0.574 (159)	-	-0.275 (1076)	-0.584 (148)	-0.110 (1121)	0.075 (1076)	-0.134 (1126)
Amount of water (lb/cy) D101D	240 19	773							×	-0.181 (128)	-	-0.295 (746)	-0.072 (128)	-0.157 (764)	-0.048 (746)	-0.019 (772)
28-day modulus of rupture (psi) D102A	644 69	199								×	0.333 (26)	-0.026 (199)	-0.311 (188)	-0.073 (163)	0.265 (199)	-0.130 (199)
Present Serviceability Index RIR	3.300 0.620	26									×	-0.781 (26)	-0.160 (26)	-0.164 (26)	-	-0.066 (26)
Age of pavement (years) AGE	16.01 9.78	1129										×	0.565 (188)	-0.027 (1077)	-0.528 (1129)	0.125 (1128)
Cumulative right lane ESAL's TCUMR	5.787 4.53	188											×	0.050 (152)	-0.094 (188)	0.184 (188)
Mean slump (inches) D104A	1.58 0.37	1125												×	0.112 (1077)	-0.057 (1124)
PCC pavement thickness (inches) D41	8.53 0.94	1129													×	-0.019 (1128)
Severe transverse cracking (ft) S37RH	4.264 19.60	1181														×

correlation coefficients are: cement type (D105) at -0.723, percent air content (D107A) at 0.574, and water to cement ratio (WTOC) at -0.499. In each case, the sign of the coefficient is in line with practical engineering expectations. The correlation coefficients, however, are based on a relatively small number of observations due primarily to missing modulus values.

In the process of producing primary and secondary relationships from purely statistical data, it is critical to identify all variables that contribute significantly to the relationship. Failure to include a significant variable can cause a derived relationship to make inaccurate predictions. In this study, the correlation analysis included every available M&C variable. In some cases, however, missing values effectively removed a variable from the data base. It is also possible that a relevant variable was never included in the data base to begin with. Another source of problems can be "spurious" correlations which are those that appear statistically significant but are actually irrelevant. For example, pavement performance might appear highly correlated with slump when in actuality changes in construction practice over time are the cause. Time (i.e., age) is an important variable to observe when looking for these types of phenomena. In the effort of trying to locate these types of problems, pavement age was included in the correlation analyses. In fact, pavement thickness was found to have a somewhat high correlation with age which indicates a possible change in construction practice over time (i.e., pavement thickness has historically increased with time). Identifying significant variables and spurious correlations are both vital to the development of plausible relationships. This is a difficult task and an argument against the use of observational data bases in the development of primary and secondary relationships.

The correlation analysis was extended to include pavement performance variables (i.e., PSI and transverse cracking). The purpose was to study how well primary relationships based on the COPES data base parallel generally accepted primary relationships. By selecting a primary predictor that appears in both the primary and secondary relationship, the "reasonableness" of the data can be measured. Unfortunately, no observations are available to correlate present serviceability index (PSI) with pavement thickness, a known predictor of pavement performance. Only 26 observations of PSI are present in the current version of the COPES data base. One correlation of note relative to primary relationships is between 28-day modulus of rupture and PSI. The Pearson correlation coefficient for this pair of variables is a relatively small (0.333). PSI does, however, show very strong negative correlation with age while showing no correlation with the effect of cumulative ESALs. Once again, low pairwise observation counts make these numbers suspect. The second performance variable, transverse cracking (S37RH), was added and, inexplicably, displayed no correlation with any of the other variables in the study. The effort to show that primary relationships based on COPES data are congruous with generally accepted primary relationships was unsuccessful.

FREQUENCY HISTOGRAMS

Frequency histograms are very helpful in analyzing the distribution of observations for a given variable. Two representative examples from the COPEs data base are presented in figures 74 and 75. The first shows a histogram for water/cement ratio which fits a normal distribution fairly well. In contrast, the second figure shows a frequency distribution for concrete slump in which a mode of 1.5 in (38.1 mm) is represented by 1011 of the 1125 non-zero observations in the data base. These two frequency histograms represent extremes but demonstrate the large variation in the distributions.

The frequency plots allowed for rapid identification of modal values for the variables. Notable examples include: cement content, 53 percent are six-sack mixes; cement type, 97 percent are Type I cement; pavement thickness, 42 percent are 9 in (229 mm); slump, 90 percent are 1.5 in (38.1 mm); air content, 82 percent are 5.5 percent. Potentially relevant variables with unfortunately large numbers of missing values are concrete additives with only 113 available observations, present serviceability index (RIR) with 26, and flexural strength (D102A) with 199 observations. No attempts were made to replace the missing values.

SCATTER PLOTS

Scatter plots were generated to further study the relationships between variables, with emphasis on secondary relationships. The 28-day modulus of rupture, an explicit performance predictor, was chosen as the dependent variable for these analyses. Several M&C factors were selected for use as independent variables. Figure 76 presents one of the most notable plots in which strength as a function of entrained air exhibits a reasonably good linear relationship. This coincides with the correlation analysis performed earlier. However, since 82 percent of the air content measurements are near 5.5 percent, the figure may be misleading.

Several additional scatter plots of interest are presented in figures 77 through 81. In figure 77, water/cement ratio appears to have a sharp effect on 28-day modulus of rupture. The majority of the plots, however, display little if any relationship between the variables. This is consistent with the results of the correlation analyses.

A similar set of plots was created with slump as the independent variable. The large percentage of slump observations at the 1.5 in (38.1 mm) level resulted in mostly meaningless plots. Figure 82 is an example.

Modulus of rupture was plotted against the year opened to traffic to study the correlation between these two variables. As discussed earlier, it is important to be aware of any correlation with time. This often hints at changes in common practice or technology. "Spurious" correlations can also be a problem. As seen in figure 83, though, no relationship exists for the data.

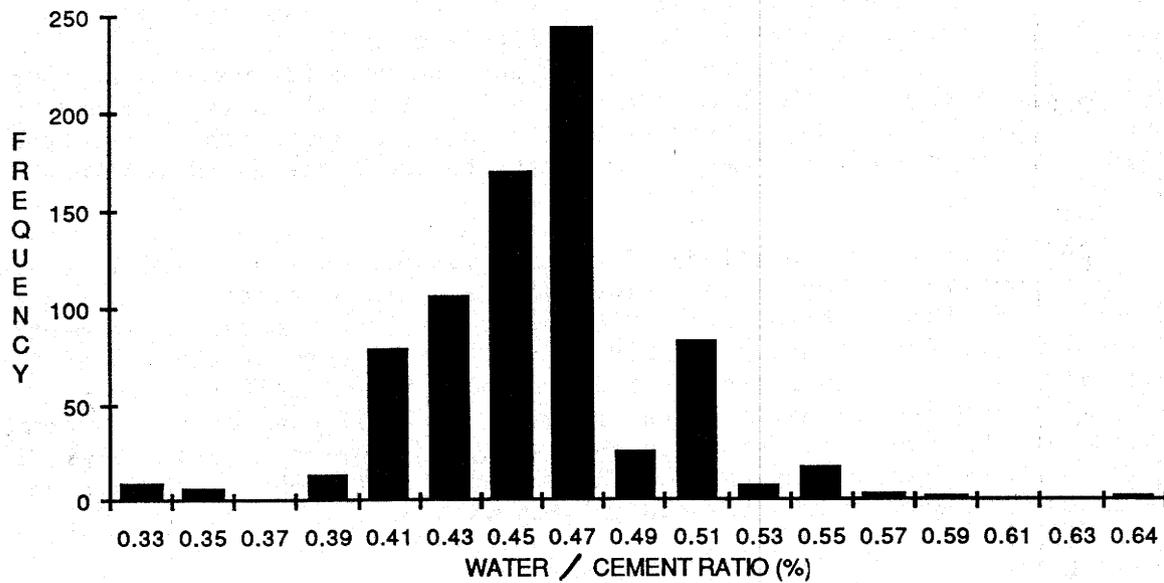


Figure 74. Water/cement ratio.

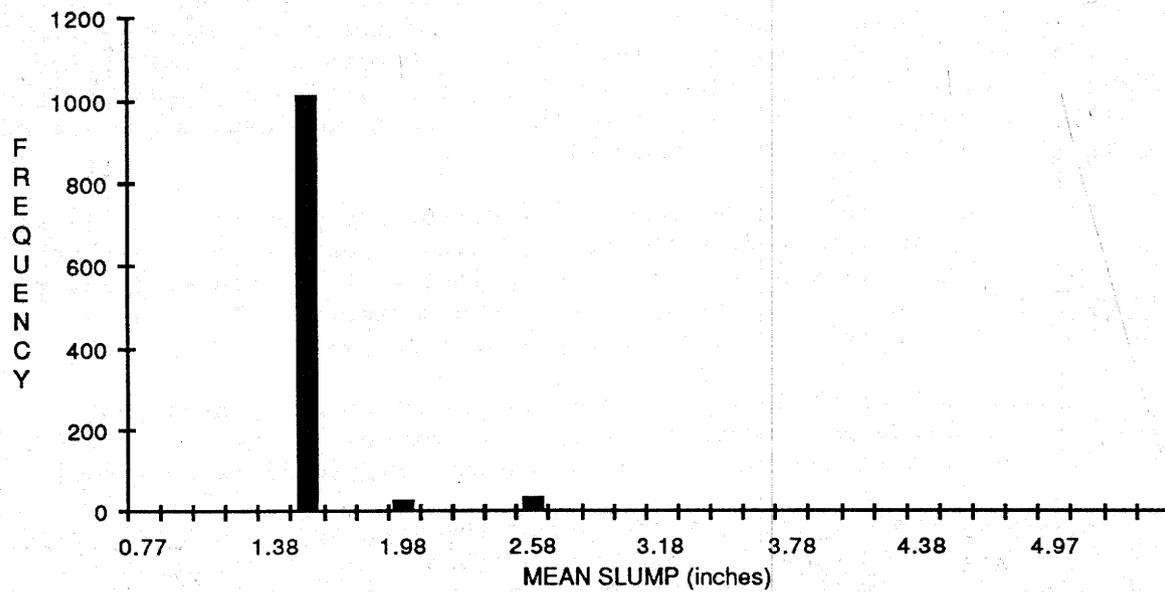
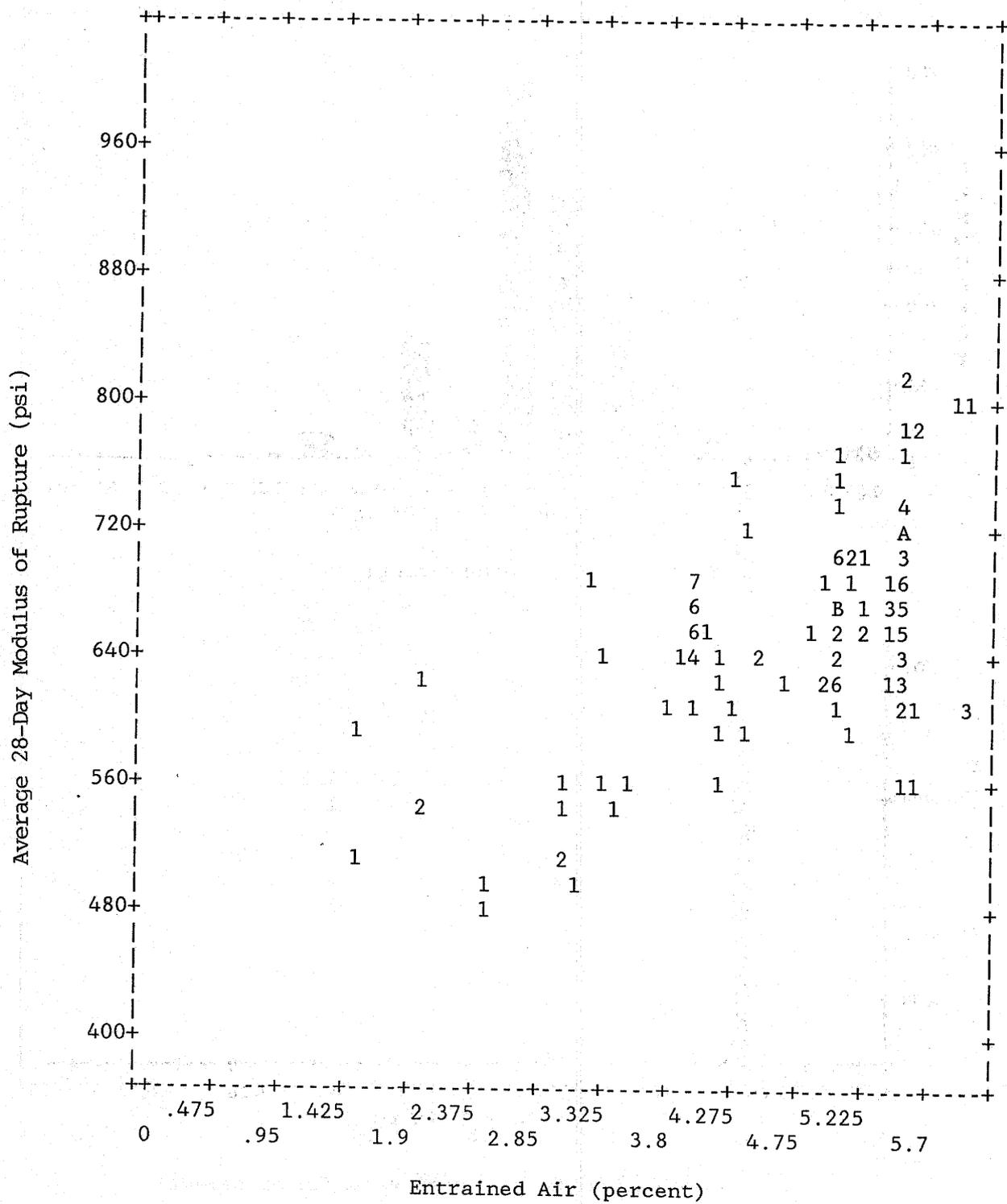
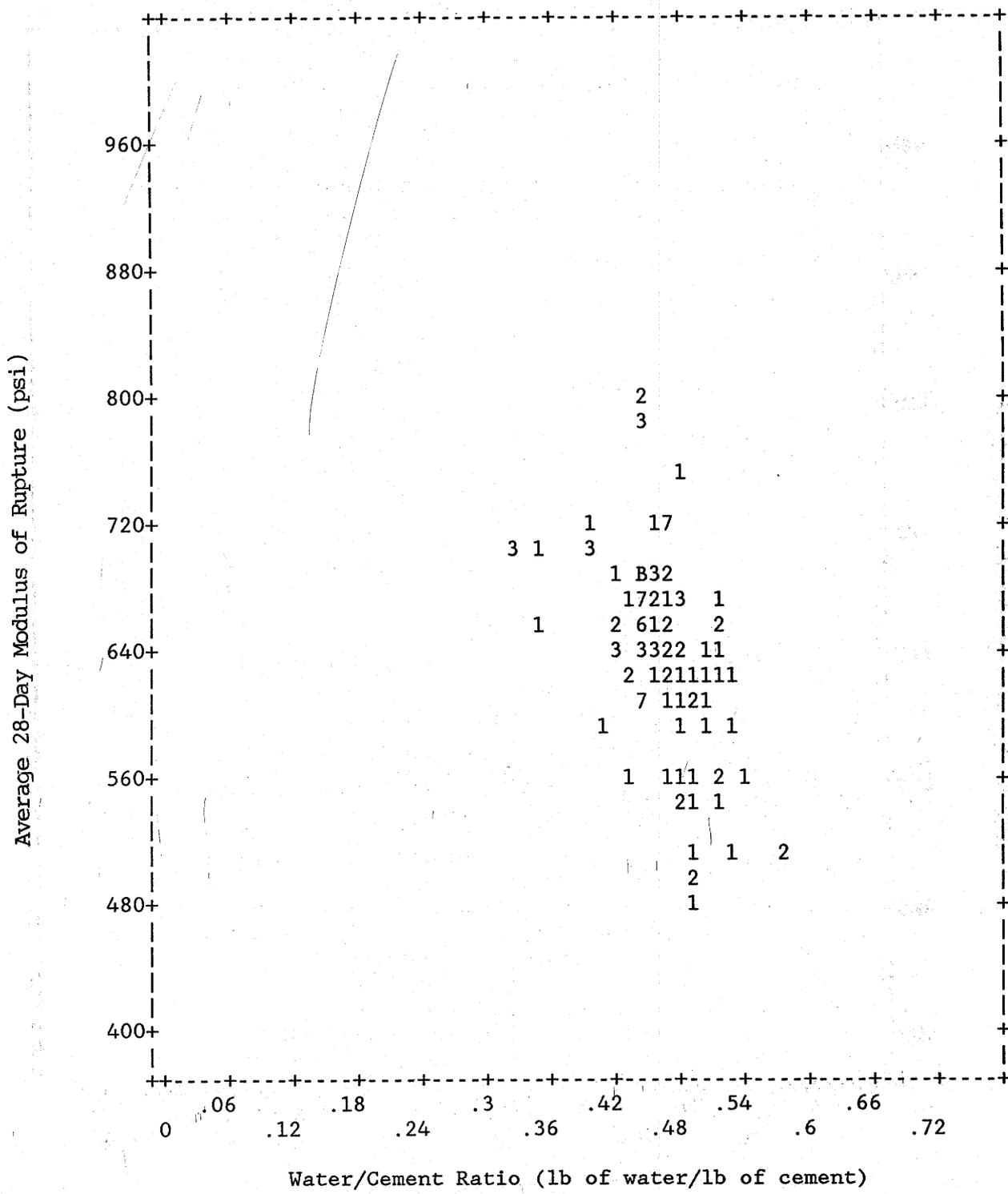


Figure 75. Mean slump.



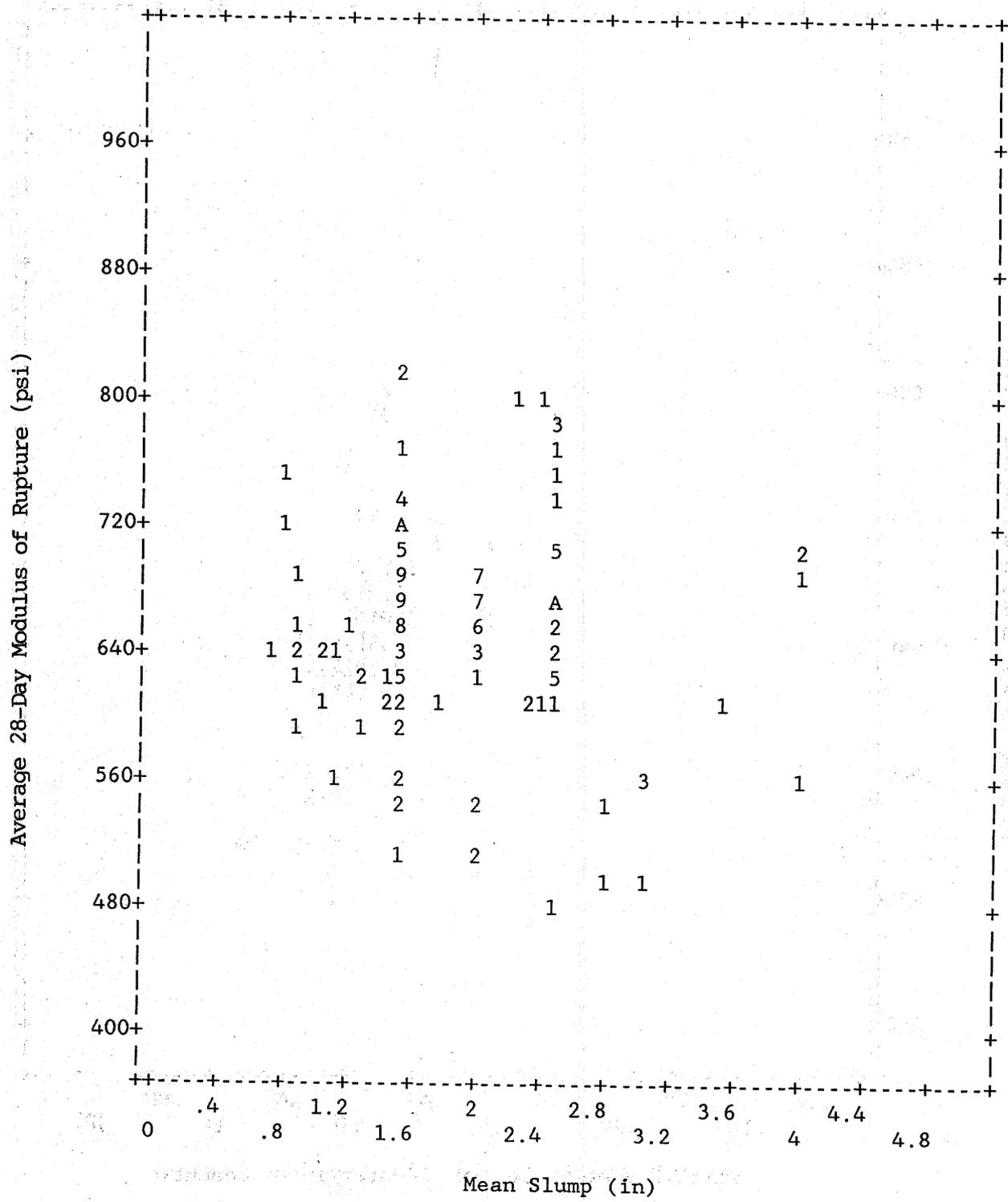
159 cases plotted.

Figure 76. 28-day modulus of rupture versus entrained air.



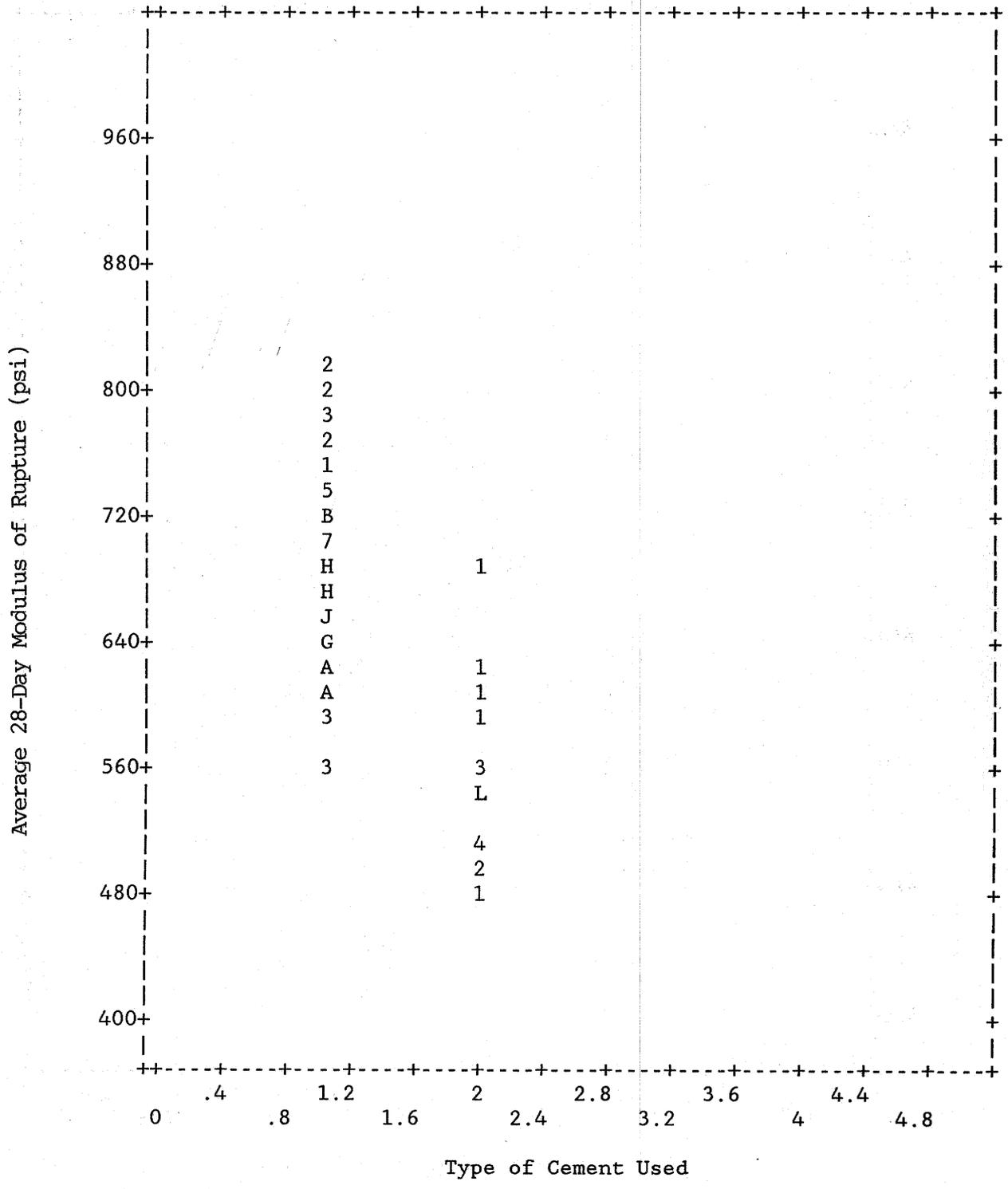
127 cases plotted.

Figure 77. 28-day modulus of rupture versus water/cement ratio.



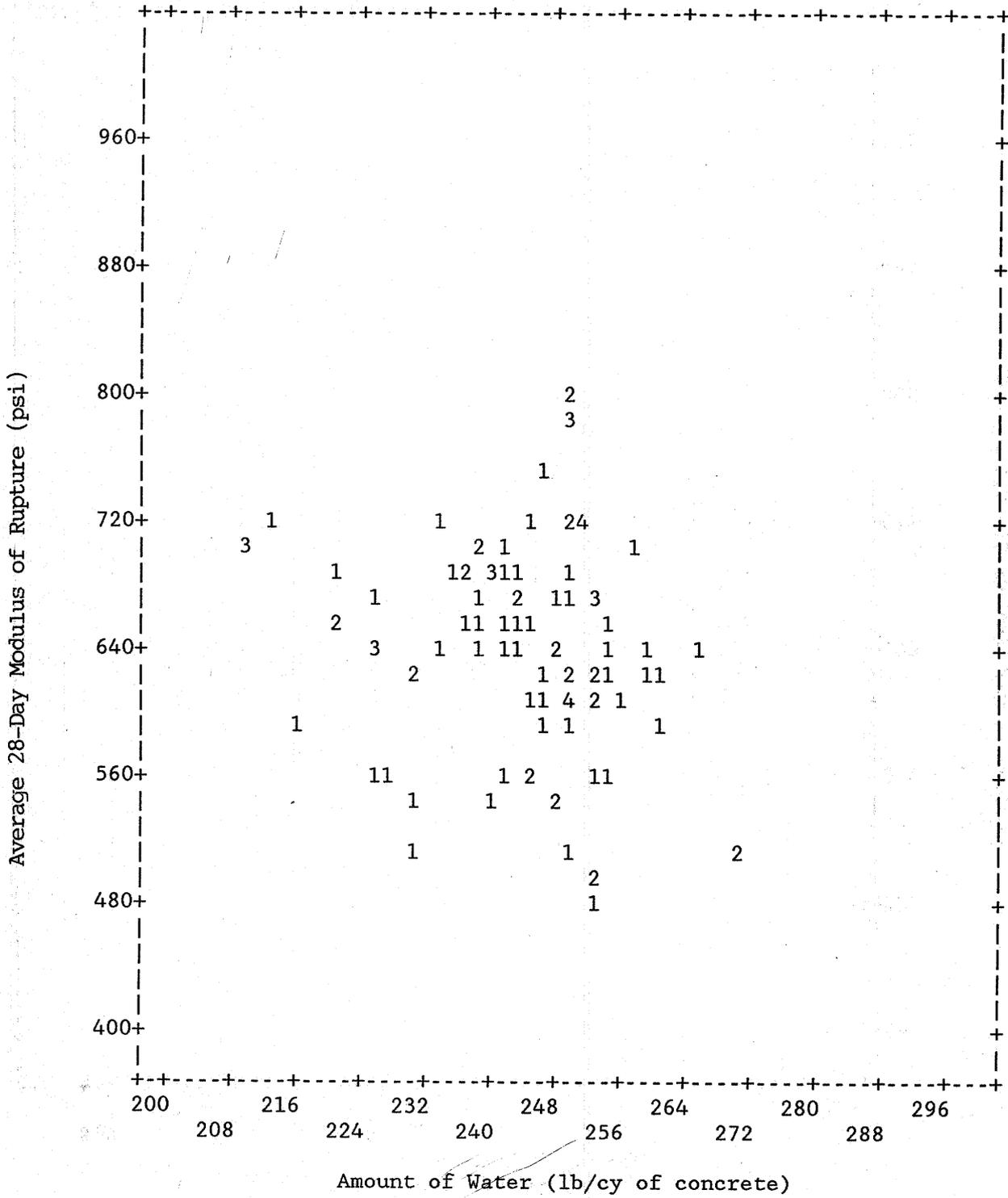
163 cases plotted.

Figure 78. 28-day modulus of rupture versus mean slump.



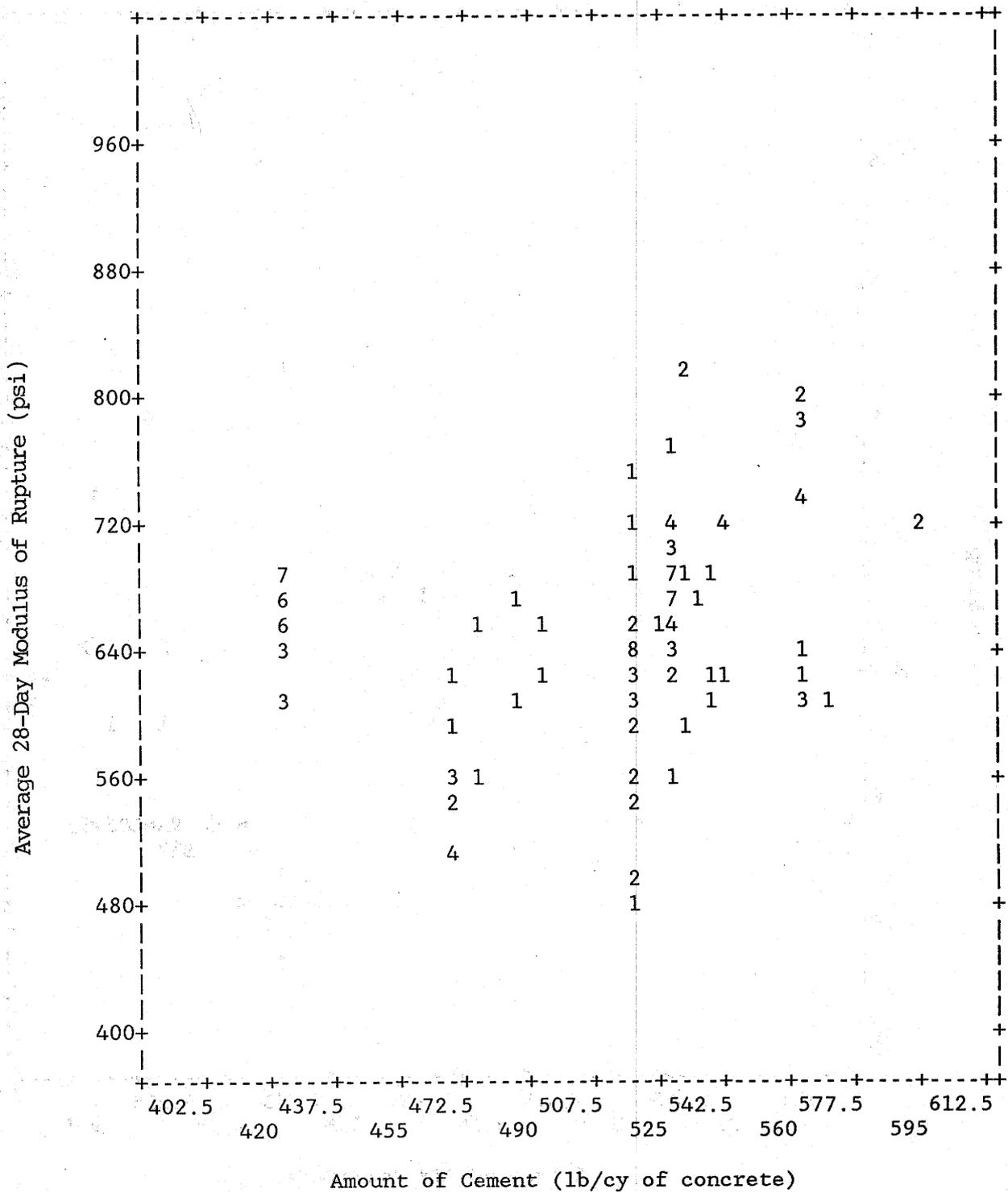
163 cases plotted.

Figure 79. 28-day modulus of rupture versus type of cement used.



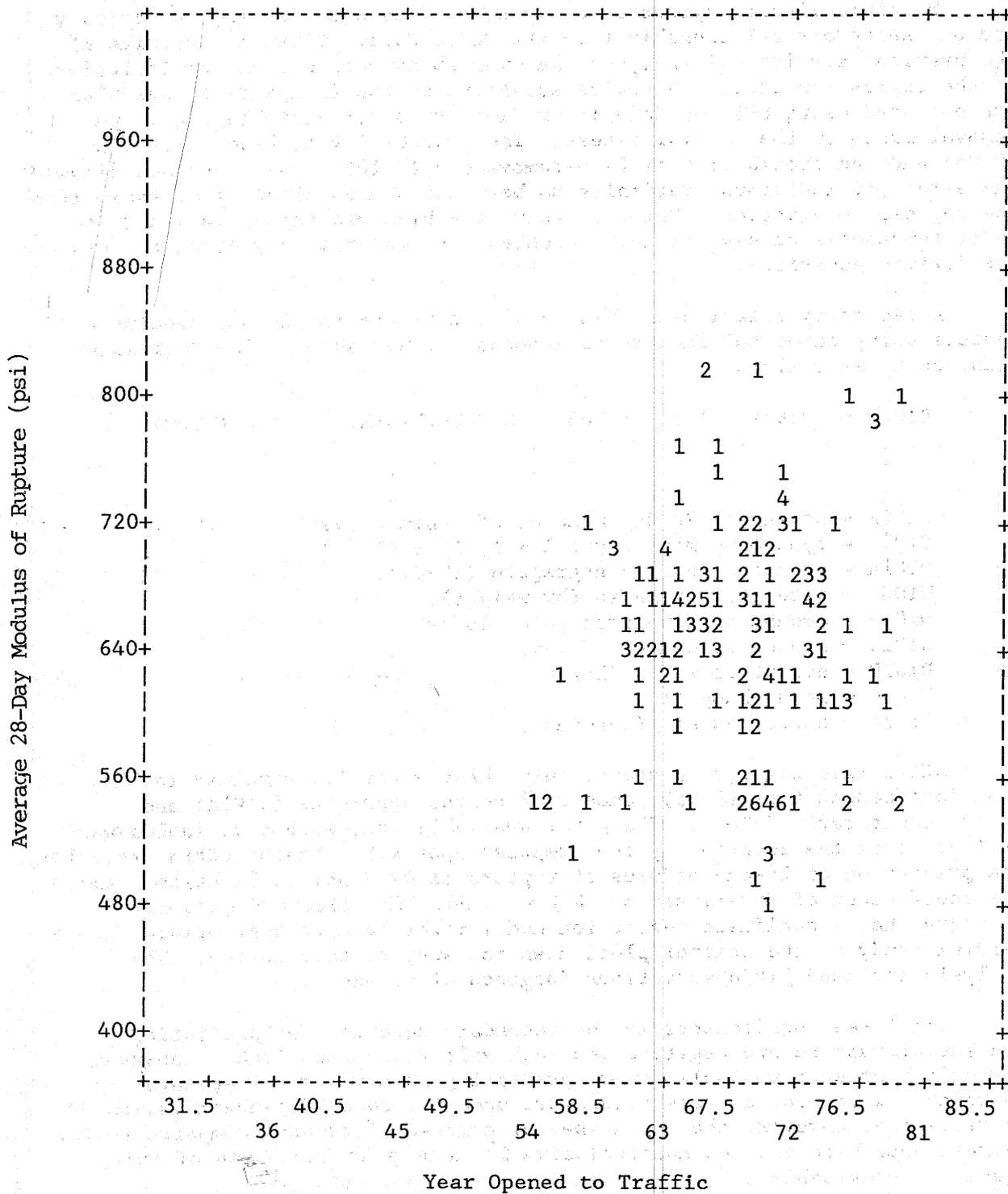
102 cases plotted.

Figure 80. 28-day modulus of rupture versus amount of water.



133 cases plotted.

Figure 81. 28-day modulus of rupture versus amount of cement.



199 cases plotted.

Figure 83. 28-day modulus of rupture versus year opened to traffic.

REGRESSION ANALYSES

Multiple linear regression analyses were used to develop one primary and one secondary relationship from the COPEs data. Given the results of the previous statistical analyses, key variables were chosen for inclusion in the regression study. Stepwise selection of the independent variables was employed using SPSSPC+ default probabilities for variable entry and removal based on the partial F-test. The probability of F-to-enter was (0.05) and the probability of F-to-remove was (0.10). This approach allowed the effect of individual variables to be studied when added to or taken from the regression equation. Unfortunately, the high missing value count and modal tendencies of several key variables cast doubt on any practical use of the derived equations.

A secondary relationship was developed to predict 28-day modulus of rupture using three M&C factors as independent variables. The regression equation is as follows:

$$D102A = 1068.1 - 103.7 (D105) - 0.045 (D101A) - 456.6 (WTOC)$$

where:

- D102A = strength, 28-day modulus of rupture (psi).
- D105 = type of cement (Type I = 1, Type II = 2).
- D101A = amount of coarse aggregate (lb/cy).
- WTOC = water/cement ratio (by weight).
- D101B = amount of fine aggregate (lb/cy).
- D101C = amount of cement (lb/cy).
- D101D = amount of water (lb/cy).
- D104A = mean slump (in).
- D107A = entrained air (percent).

After five regression steps, only three variables remained in the equation; cement type (D105), amount of coarse aggregate (D101A) and water/cement ratio (WTOC). The other available independent variables were excluded from the equation by the stepwise approach. The standard error for the prediction of 28-day modulus of rupture is 42.9 psi (3.02 kg/cm²) and the coefficient of determination (R²) is 0.54. The latter figure may indicate that a nonlinear regression model might be more appropriate though careful study of the scatter plots does not support this notion. The analysis included 118 observations (degrees of freedom).

All three coefficients in the secondary equation for predicting modulus of rupture are negative and each will reduce the 1068.1 constant. The effect of each variable toward predicting 28-day modulus appears reasonable when near average values are used for the independent variables. To reiterate, however, the low number of degrees of freedom compared to the overall data base and the nonconclusive R² value make broad use of this equation inadvisable.

Stepwise regression analysis was also performed to derive a primary performance equation for present serviceability index (R1R). As discussed previously, the purpose of this analysis is to study the "reasonableness" of the variables that are common to both primary and secondary relationships.

The independent variables available for consideration were pavement age (AGE), cumulative 18 kip ESALs (TCUMR), PCC pavement thickness (D41), and modulus of rupture (D102A). As shown below, the stepwise regression process resulted in an equation with only one variable term for pavement age (AGE) omitting modulus (D102A), the variable common to the derived secondary relationship.

$$R1R = 5.28 - 0.126 (AGE)$$

where:

R1R = present serviceability index (PSI) of pavement from roughness and distress measurements (ranges from 0 to 5).

AGE = number of years pavement has been in service.

TCUMR = one-way cumulative 18-kip single-axle loads in the right lane over the life of the pavement.

D102A = 28-day modulus of rupture (psi).

D41 = PCC pavement thickness (in).

This equation is obviously useless since it omits many important factors known to have significant effect on pavement performance. It was hoped that traffic volume, pavement thickness, and PCC flexural strength would enter the equation; unfortunately, there was not enough data. Pavement thickness was omitted because no test section contained both thickness and PSI data. Only 25 observations were included in the regression analysis producing an R^2 of 0.61 and standard error of 0.40. A second regression analysis was performed to derive a primary relationship for severe transverse cracking, another measure of pavement performance. Using the same set of available variables and identical criteria for entry and removal of independent variables, none were added during the stepwise regression. This is consistent with correlation matrix which presents the low Pearson correlation coefficients for transverse cracking (S37RH).

APPENDIX F
LABORATORY TESTING RESULTS

This appendix contains the results of all laboratory testing carried out in this study. The results are contained within three data bases described below.

- Data Base A - Results for all 64 cells, not including 8 replicate batches.
- Data Base B - Results for 8 cells having one original and one replicate batch.
- Data Base C - Results for all 64 cells, including replicate batches.

Design Sequence	Rep A or B	Mix Sequence	Date	CAT 0=CLS 1=SRG	GAM (in) 0.75 1.50	FAM 2.10 or 2.84	AEQ (oz) 0.0 or 6.5	CAQ (lb) 1542 or 1850	CEQ (lb) 426 or 584	WAQ (lb) 236 or 270	FAQ (lb) SSD	WCR (lb/ lb)	ACR (lb/ lb)	SLMP (in)	UNWT (pcf)	YLD (CY)	AIR (X)	TEMP (oF)
1		14 10/05	0	0.75	2.10	0.0	1542	426	236	1772	0.55	7.78	0.00	137.5	28.9	3.1	79.0	
2		17 10/07	0	0.75	2.10	0.0	1542	584	270	1552	0.46	5.30	0.00	141.4	27.9	4.4	80.0	
3	A	30 10/18	0	0.75	2.10	0.0	1850	426	270	1371	0.63	7.56	1.00	140.1	28.0	3.4	81.0	
4		58 11/17	0	0.75	2.10	0.0	1850	584	236	1315	0.40	5.42	0.00	143.7	27.7	3.6	70.0	
5		33 10/20	0	0.75	2.10	6.5	1542	426	270	1507	0.63	7.16	3.50	128.4	29.2	10.5	79.0	
6		18 10/07	0	0.75	2.10	6.5	1542	584	236	1463	0.40	5.15	0.00	138.3	27.7	5.5	79.0	
7		43 10/31	0	0.75	2.10	6.5	1850	426	236	1269	0.55	7.32	4.50	132.4	28.6	10.0	71.0	
8		24 10/11	0	0.75	2.10	6.5	1850	584	270	1064	0.46	4.99	4.75	134.9	27.9	7.3	82.0	
9		40 10/28	0	0.75	2.84	0.0	1542	426	270	1677	0.63	7.56	0.50	142.2	27.5	3.5	80.0	
10		10 09/30	0	0.75	2.84	0.0	1542	584	236	1644	0.40	5.46	0.00	143.9	27.8	3.0	77.0	
11		67 11/21	0	0.75	2.84	0.0	1850	426	236	1425	0.55	7.69	0.00	144.5	27.4	3.1	67.0	
12		65 11/19	0	0.75	2.84	0.0	1850	584	270	1227	0.46	5.27	2.00	144.8	27.1	1.9	76.0	
13		12 10/05	0	0.75	2.84	6.5	1542	426	236	1597	0.55	7.37	0.13	138.7	27.4	6.1	81.0	
14		69 11/21	0	0.75	2.84	6.5	1542	584	270	1367	0.46	4.98	3.50	137.8	27.3	6.8	68.0	
15		53 11/15	0	0.75	2.84	6.5	1850	426	270	1181	0.63	7.12	8.00	128.1	29.0	12.0	79.0	
16	A	5 09/27	0	0.75	2.84	6.5	1850	584	236	1137	0.40	5.11	0.75	144.3	26.4	4.0	87.6	
17		4 09/26	0	1.50	2.10	0.0	1542	426	270	1666	0.63	7.53	0.00	137.8	28.3	5.0	88.0	
18		54 11/16	0	1.50	2.10	0.0	1542	584	236	1614	0.40	5.40	0.00	142.5	27.9	4.7	72.0	
19		31 10/19	0	1.50	2.10	0.0	1850	426	236	1423	0.55	7.68	0.50	141.0	27.9	4.1	79.0	
20		19 10/08	0	1.50	2.10	0.0	1850	584	270	1202	0.46	5.23	5.00	142.0	27.5	2.5	83.0	
21	A	57 11/17	0	1.50	2.10	6.5	1542	426	236	1568	0.55	7.30	0.25	133.7	28.2	6.6	70.0	
22		42 10/31	0	1.50	2.10	6.5	1542	584	270	1352	0.46	4.96	6.00	132.8	28.2	10.0	71.0	
23		21 10/10	0	1.50	2.10	6.5	1850	426	270	1156	0.63	7.06	5.00	134.3	27.6	7.2	85.0	
24		36 10/24	0	1.50	2.10	6.5	1850	584	236	1115	0.40	5.08	3.00	138.7	27.3	5.5	82.0	
25		28 10/14	0	1.50	2.84	0.0	1542	426	236	1747	0.55	7.72	0.50	141.3	28.0	3.8	78.0	
26	A	48 11/03	0	1.50	2.84	0.0	1542	584	270	1530	0.46	5.26	0.75	143.0	27.4	2.8	80.0	
27		39 10/28	0	1.50	2.84	0.0	1850	426	270	1334	0.63	7.47	5.50	144.0	26.9	1.5	80.0	
28		64 11/19	0	1.50	2.84	0.0	1850	584	236	1296	0.40	5.39	0.50	146.3	27.1	2.5	75.0	
29		50 11/15	0	1.50	2.84	6.5	1542	426	270	1481	0.63	7.10	6.25	132.2	28.1	9.0	79.0	
30		11 09/30	0	1.50	2.84	6.5	1542	584	236	1437	0.40	5.10	0.13	142.2	26.7	5.8	77.0	
31		41 10/31	0	1.50	2.84	6.5	1850	426	236	1254	0.55	7.29	5.50	136.2	27.6	7.0	72.0	
32		44 11/02	0	1.50	2.84	6.5	1850	584	270	1034	0.46	4.94	3.25	139.9	26.7	5.3	72.0	
33		70 11/22	1	0.75	2.10	0.0	1542	426	270	1716	0.63	7.65	0.00	141.5	27.9	3.0	68.0	
34		45 11/02	1	0.75	2.10	0.0	1850	584	236	1370	0.40	5.51	0.00	144.9	27.9	8.2	75.0	
35		29 10/14	1	0.75	2.10	0.0	1850	426	236	1501	0.55	7.87	0.25	142.8	28.1	4.0	78.0	
36		51 11/15	1	0.75	2.10	0.0	1850	584	270	1279	0.46	5.36	1.50	144.3	27.6	2.5	79.0	
37		2 09/21	1	0.75	2.10	6.5	1542	426	236	1628	0.55	7.44	0.00	135.8	28.2	11.6	89.0	
38	A	71 12/06	1	0.75	2.10	6.5	1542	584	270	1409	0.46	5.05	2.00	137.6	27.6	7.6	71.0	
39		22 10/10	1	0.75	2.10	6.5	1850	426	270	1237	0.63	7.25	8.75	133.0	28.4	9.4	80.0	
40		56 11/17	1	0.75	2.10	6.5	1850	584	236	1190	0.40	5.21	0.25	143.8	26.8	5.1	70.0	
41	A	49 11/4	1	0.75	2.84	0.0	1542	426	236	1809	0.55	7.87	0.00	140.1	28.7	3.4	85.0	
42		46 11/02	1	0.75	2.84	0.0	1542	584	270	1589	0.46	5.36	0.00	145.4	27.4	3.1	71.0	
43		3 09/23	1	0.75	2.84	0.0	1850	426	270	1409	0.63	7.65	2.25	145.4	27.2	2.3	88.0	
44		7 09/27	1	0.75	2.84	0.0	1850	584	236	1367	0.40	5.51	0.00	148.8	27.1	2.5	84.5	
45		20 10/08	1	0.75	2.84	6.5	1542	426	270	1549	0.63	7.26	7.25	131.1	28.9	11.0	81.0	
46		32 10/20	1	0.75	2.84	6.5	1542	584	236	1501	0.40	5.21	1.50	143.1	27.0	6.0	86.0	
47		26 10/12	1	0.75	2.84	6.5	1850	426	236	1326	0.55	7.46	2.25	137.6	27.9	8.2	80.0	
48		60 11/18	1	0.75	2.84	6.5	1850	584	270	1102	0.46	5.06	4.75	141.4	26.8	5.2	76.0	
49		23 10/11	1	1.50	2.10	0.0	1542	426	236	1803	0.55	7.85	0.00	131.3	30.5	10.0	81.0	
50		13 10/05	1	1.50	2.10	0.0	1542	584	270	1582	0.46	5.35	0.00	144.0	27.6	3.0	86.0	
51		6 09/27	1	1.50	2.10	0.0	1850	426	270	1407	0.63	7.65	1.00	143.0	27.6	4.2	84.5	
52	A	35 10/24	1	1.50	2.10	0.0	1850	584	236	1364	0.40	5.50	0.00	146.4	27.6	3.2	80.0	
53		27 10/13	1	1.50	2.10	6.5	1542	426	270	1537	0.63	7.23	2.25	131.2	28.8	11.0	78.0	
54		9 09/30	1	1.50	2.10	6.5	1542	584	236	1494	0.40	5.20	0.00	141.2	27.3	5.2	75.0	
55		63 11/19	1	1.50	2.10	6.5	1850	426	236	1317	0.55	7.43	1.00	139.1	27.5	7.2	74.0	
56		1 09/21	1	1.50	2.10	6.5	1850	584	270	1099	0.46	5.05	2.00	142.5	26.7	5.0	87.0	
57		8 09/28	1	1.50	2.84	0.0	1542	426	270	1713	0.63	7.64	0.75	143.4	27.6	3.6	84.5	
58		15 10/06	1	1.50	2.84	0.0	1542	584	236	1674	0.40	5.51	0.00	146.4	27.6	3.0	78.0	
59		61 11/18	1	1.50	2.84	0.0	1850	426	236	1495	0.55	7.85	0.25	145.4	27.6	3.6	77.0	
60		66 11/19	1	1.50	2.84	0.0	1850	584	270	1275	0.46	5.35	1.50	148.0	26.9	2.8	75.0	
61		38 10/25	1	1.50	2.84	6.5	1542	426	236	1628	0.55	7.44	5.50	132.3	29.0	11.0	77.0	
62		34 10/20	1	1.50	2.84	6.5	1542	584	270	1409	0.46	5.05	5.75	137.1	27.8	7.5	79.0	
63	A	72 12/06	1	1.50	2.84	6.5	1850	426	270	1228	0.63	7.23	6.75	135.4	27.8	8.5	73.0	
64		47 11/02	1	1.50	2.84	6.5	1850	584	236	1188	0.40	5.20	1.50	145.5	26.5	4.4	70.0	

Table 40. Data base A.

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Design Seq.	Rep Seq.	Mix Seq.	A or B	fr71 fr72fr7AVEfr7DIF (psi)(psi)(psi) (psi)				fpc71fpc72fpc7AVEfpcDIF (psi)(psi) (psi) (psi)				ft71 ft72ft7AVEft7DIF (psi)(psi)(psi) (psi)				fpc283fpc284fpc28AVEfpc28DIF (psi) (psi) (psi) (psi)				Ec283 Ec284 Ec28AVEEc28DIF (MPSI)(MPSI)(MPSI) (MPSI)				PERCENTAGES BY VOLUME					
				7-DAY FLEX. STRENGTH.				7-DAY COMPR. STRENGTH.				7-DAY TENSILE STRENGTH				28-DAY COMPR. STRENGTH				28-DAY MOD. OF ELASTICITY				CAP (%)	CEP (%)	WAP (%)	FAP (%)	AIRP (%)	CUMUL (%)
1		14		420	405	413	15	2230	2240	2235	10	290	300	295	10	2940	2920	2930	20	3.65	3.80	3.73	0.15	33.4	7.5	13.1	37.6	3.1	95
2		17		700	615	658	85	4400	4510	4455	110	435	460	448	25	5080	5140	5110	60	4.60	4.50	4.55	0.10	34.7	10.6	15.5	34.3	4.4	100
3	A	30		495	425	460	70	2830	2990	2910	160	290	310	300	20	3840	4010	3925	170	4.25	4.35	4.30	0.10	41.5	7.7	15.5	30.2	3.4	98
4		58		750	775	763	25	5410	5440	5425	30	450	500	475	50	6570	6740	6655	170	5.05	5.05	5.05	0.00	42.0	10.7	13.7	29.3	3.6	99
5		33		400	425	413	25	1880	2020	1950	140	310	290	300	20	2860	2830	2845	30	3.30	3.40	3.35	0.10	33.2	7.4	14.8	31.8	10.5	98
6		18		665	675	670	10	4480	4790	4635	310	450	445	448	5	5310	4950	5130	360	4.30	4.35	4.33	0.05	35.0	10.7	13.7	32.6	5.5	97
7		43		445	460	453	15	2470	2560	2515	90	275	245	260	30	3250	3330	3290	80	3.80	3.80	3.80	0.00	40.7	7.6	13.2	27.3	10.0	99
8		24		605	580	593	25	3870	3990	3930	120	420	405	413	15	5130	5130	5130	0	4.10	4.00	4.05	0.10	41.7	10.6	15.5	23.5	7.3	99
9		40		540	480	510	60	3040	2920	2980	120	310	290	300	20	4030	4020	4025	10	4.30	4.55	4.43	0.25	35.2	7.9	15.7	37.5	3.5	100
10		10		585	655	620	70	5120	5010	5065	110	435	460	448	25	6110	6060	6085	50	4.80	5.00	4.90	0.20	34.9	10.7	13.6	36.5	3.0	99
11		67		610	645	628	35	4230	3970	4100	260	470	460	465	10	5130	5360	5245	230	4.85	4.85	4.85	0.00	42.4	7.9	13.8	32.1	3.1	99
12		65		725	700	713	25	4810	5100	4955	290	520	535	528	15	6310	6210	6260	100	4.95	4.95	4.95	0.00	42.9	11.0	16.0	27.9	1.9	100
13		12		475	580	528	105	3620	3520	3570	100	380	440	410	60	4570	4370	4470	200	4.45	4.60	4.53	0.15	35.4	7.9	13.8	35.9	6.1	99
14		69		635	635	635	0	4680	4560	4620	120	400	470	435	70	5340	5140	5240	200	4.40	4.25	4.33	0.15	35.5	10.9	15.8	30.9	6.8	100
15		53		345	330	338	15	1990	1950	1970	40	295	235	265	60	2630	2540	2585	90	3.20	2.95	3.08	0.25	40.1	7.5	14.9	25.1	12.0	100
16	A	5		670	680	675	10	5620	5430	5525	190	515	510	513	5	6500	6510	6505	10	5.15	5.10	5.13	0.05	44.0	11.3	14.3	26.5	4.0	100
17		4		445	405	425	40	2520	2350	2435	170	320	310	315	10	3070	3030	3050	40	4.35	4.50	4.43	0.15	34.2	7.7	15.3	36.3	5.0	98
18		54		630	590	610	40	4650	4610	4630	40	505	485	495	20	5910	5640	5775	270	5.10	4.95	5.03	0.15	34.7	10.6	13.6	35.7	4.7	99
19		31		520	515	518	5	3140	3300	3220	160	400	365	383	35	3880	4180	4030	300	4.45	4.45	4.45	0.00	41.7	7.8	13.6	31.4	4.1	99
20		19		590	625	608	35	4400	4290	4345	110	420	430	425	10	5360	5300	5330	60	4.00	4.20	4.10	0.20	42.3	10.8	15.7	26.9	2.5	98
21	A	57		480	515	498	35	2800	2860	2830	60	285	310	298	25	3550	3450	3500	100	3.95	3.65	3.80	0.30	34.4	7.7	13.4	34.3	6.6	96
22		42		445	530	488	85	3390	3460	3425	70	350	415	383	65	4360	4310	4335	50	3.65	3.70	3.68	0.05	34.4	10.5	15.3	29.6	10.0	100
23		21		435	385	410	50	2760	2650	2705	110	335	315	325	20	3240	3430	3335	190	3.80	3.60	3.70	0.20	42.1	7.9	13.7	25.8	7.2	99
24		36		585	590	588	5	4450	4690	4570	240	440	420	430	20	5370	5360	5365	10	4.60	4.50	4.55	0.10	42.6	10.9	13.9	25.2	5.5	98
25		28		510	510	510	0	3470	3320	3395	150	405	390	398	15	3930	4060	3995	130	4.60	4.30	4.45	0.30	34.6	7.7	13.5	38.5	3.8	98
26	A	48	M	660	705	683	45	5090	5240	5165	150	520	510	515	10	6000	6250	6125	250	4.75	5.10	4.93	0.35	35.4	10.8	15.8	34.4	2.8	99
27		39	A	430	490	460	60	2900	2870	2885	30	310	330	320	20	3930	3840	3885	90	4.65	4.20	4.43	0.45	43.2	8.0	16.1	30.5	1.5	99
28		64	T	730	680	705	50	5330	5870	5600	540	530	505	518	25	6900	6690	6795	210	5.05	5.15	5.10	0.10	42.9	11.0	14.0	29.5	2.5	100
29		50	C	360	345	353	15	2060	2240	2150	180	300	245	273	55	2740	2800	2770	60	3.30	3.45	3.38	0.15	34.5	7.7	15.4	32.5	9.0	99
30		11	H	680	630	655	50	5220	5350	5285	130	495	460	478	35	6160	6120	6140	40	4.95	4.80	4.88	0.15	36.3	11.1	14.2	33.2	5.8	101
31		41		430	430	430	0	2560	2595	2578	35	300	350	325	50	3320	3230	3275	90	3.80	4.05	3.93	0.25	42.1	7.9	13.7	28.0	7.0	99
32		44	L	565	565	565	0	4420	4420	4420	0	450	430	440	20	5090	5440	5265	350	4.45	4.45	4.45	0.00	43.5	11.1	16.2	23.9	5.3	100
33		70	I	375	395	385	20	2580	2400	2490	180	325	325	325	0	3460	3390	3425	70	3.95	3.85	3.90	0.10	34.7	7.8	15.5	37.9	3.0	99
34		45	E	570	495	533	75	5480	5840	5660	360	450	485	468	35	6980	6570	6775	410	5.35	5.55	5.45	0.20	41.7	10.6	13.6	30.3	8.2	104
35		29	N	500	505	503	5	3680	3620	3650	60	440	420	430	20	4430	4380	4405	50	4.40	4.35	4.38	0.05	41.4	7.7	13.5	32.9	4.0	99
36		51		635	690	663	55	4170	3900	4035	270	435	375	405	60	4870	4790	4830	80	5.06	4.65	4.86	0.41	42.1	10.8	15.7	28.6	2.5	100
37		2		395	440	418	45	1310	1320	1315	10	225	205	215	20	1940	1810	1875	130	2.60	3.30	2.95	0.70	34.4	7.7	13.4	35.6	11.6	103
38	A	71		580	575	578	5	3720	3640	3680	80	420	390	405	30	4810	4830	4820	20	4.05	4.10	4.08	0.05	35.1	10.8	15.7	31.5	7.6	101
39		22		380	415	398	35	2050	2060	2055	10	270	265	268	5	2700	2790	2745	90	3.50	3.40	3.45	0.10	40.9	7.6	15.2	26.8	9.4	100
40		56		690	700	695	10	4590	4730	4660	140	400	438	419	38	5390	5270	5330	120	5.05	4.90	4.98	0.15	43.4	11.1	14.1	27.4	5.1	101
41	A	49		450	415	433	35	2790	2670	2730	120	295	330	313	35	3510	3440	3475	70	4.20	4.20	4.20	0.00	33.8	7.6	13.2	38.8	3.4	97
42		46		740	755	748	15	4530	4380	4455	150	410	470	440	60	5030	5370	5200	340	4.80	4.80	4.80	0.00	35.4	10.8	15.8	35.7	3.1	101
43		3		470	475	473	5	3520	3250	3385	270	400	410	405	10	4290	4350	4320	60	4.35	3.75	4.05	0.60	42.7	8.0	15.9	31.9	2.3	101
44		7		725	755	740	30	5100	5010	5055	90	460	440	450	20	5900	6120	6010	220	5.85	5.15	5.50	0.70	42.9	11.0	14.0	31.1	2.5	101
45		20		355	340	348	15	2210	2110	2160	100	250	210	230	40	2930	2830	2880	100	3.20	2.85	3.03	0.35	33.5	7.5	15.0	33.0	11.0	100
46		32		640	670	655	30	3750	3820	3785	70	350	350	350	0	4760	4770	4765	10	5.10	4.80	4.95	0.30	35.9	11.0	14.0	34.3	6.0	101
47		26		445	455	450	10	2680	2600	2640	80	355	340	348	15	3410	3370	3390	40	3.65	3.90	3.78	0.25	41.7	7.8	13.6	29.3	8.2	100
48		60		595	625	610	30	3530	3620	3575	90	330	395	363	65	4660	4570	4615	90	4.25	4.15	4.20	0.10	43.4	11.1	16.1	25.4	5.2	101
49		23		200	165	183	35	1160	1220	1190	60	145	150	148	5	1180	1240	1210											

Design Sequence	Rep A or B	Mix Sequence	Date	CAT 0=CLS 1=SRG	CAM	FAM	AEO	CAQ	CEQ	WAQ	FAQ	WCR	ACR	SLMP	UNWT	YLD	AIR	TEMP
					(in) 0.75 or 1.50	2.10 or 2.84	(oz) 0.0 or 6.5	(lb) 1542 or 1850	(lb) 426 or 584	(lb) 236 or 270	(lb) 1371 or 1358	(lb/ lb) 0.63 or 0.63	(lb/ lb) 7.56 or 7.53	(in) 1.00 or 0.75	(pcf) 140.1 or 141.4	(CY) 28.0 or 27.6	(X) 3.4 or 3.1	(oF) 81.0 or 71.0
3 A		30	10/18	0	0.75	2.1	0	1850	426	270	1371	0.63	7.56	1.00	140.1	28.0	3.4	81.0
3 B		55	11/16	0	0.75	2.1	0	1850	426	270	1358	0.63	7.53	0.75	141.4	27.6	3.1	71.0
16 A		5	09/27	0	0.75	2.84	6.5	1850	584	236	1137	0.40	5.11	0.75	144.3	26.4	4.0	87.6
16 B		59	11/17	0	0.75	2.84	6.5	1850	584	236	1139	0.40	5.12	1.75	141.1	27.6	5.4	68.0
21 A		57	11/17	0	1.50	2.1	6.5	1542	426	236	1568	0.55	7.30	0.25	133.7	28.2	6.6	70.0
21 B		25	10/12	0	1.50	2.1	6.5	1542	426	236	1568	0.55	7.30	0.75	133.0	28.4	7.0	81.0
26 A		48	11/03	0	1.50	2.84	0	1542	584	270	1530	0.46	5.26	0.75	143.0	27.4	2.8	80.0
26 B		16	10/07	0	1.50	2.84	0	1542	584	270	1525	0.46	5.25	2.50	142.4	27.5	4.1	76.0
38 A		71	12/06	1	0.75	2.1	6.5	1542	584	270	1409	0.46	5.05	2.00	137.6	27.6	7.6	71.0
38 B		37	10/24	1	0.75	2.1	6.5	1542	584	270	1408	0.46	5.05	2.50	136.9	27.4	7.6	80.0
41 A		49	11/04	1	0.75	2.84	0	1542	426	236	1809	0.55	7.87	0.00	140.1	28.7	3.4	85.0
41 B		52	11/15	1	0.75	2.84	0	1542	426	236	1807	0.55	7.86	0.00	143.2	28.0	3.8	79.0
52 A		35	10/24	1	1.50	2.1	0	1850	584	236	1364	0.40	5.50	0.00	146.4	27.6	3.2	80.0
52 B		62	11/18	1	1.50	2.1	0	1850	584	236	1363	0.40	5.50	0.25	146.2	27.6	4.0	75.0
63 A		72	12/06	1	1.50	2.84	6.5	1850	426	270	1228	0.63	7.23	6.75	135.4	27.8	8.5	73.0
63 B		68	11/21	1	1.50	2.84	6.5	1850	426	270	1228	0.63	7.23	7.75	145.6	27.8	8.6	68.0

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Table 41. Data base B.

Design Rep Seq.	Mlx Seq.	A or B	PERCENTAGES BY VOLUME																									
			fr71 (psi)	fr72fr7AVEfr7DIF (psi)(psi)	fr7AVEfr7DIF (psi)	fr7DIF (psi)	fpc71fpc72fpc7AVEfpcDIF (psi)(psi)(psi)(psi)	fpc72fpc7AVEfpcDIF (psi)(psi)(psi)	fpc7AVEfpcDIF (psi)	ft71 (psi)	ft72ft7AVEft7DIF (psi)(psi)(psi)	ft7AVEft7DIF (psi)	ft7DIF (psi)	fpc283fpc284fpc28AVEfpc28DIF (psi)(psi)(psi)(psi)	fpc284fpc28AVEfpc28DIF (psi)(psi)	fpc28AVEfpc28DIF (psi)	fpc28DIF (psi)	Ec283 (MPSI)	Ec284 (MPSI)	Ec28AVE (MPSI)	Ec28DIF (MPSI)	CAP (%)	CEP (%)	WAP (%)	FAP (%)	AIRP (%)	CUMUL (%)	
			7-DAY FLEX. STRENGTH.				7-DAY COMPR. STRENGTH.				7-DAY TENSILE STRENGTH				28-DAY COMPR. STRENGTH				28-DAY MOD. OF ELASTICITY									
3 A	30		495	425	460	70	2830	2990	2910	160	290	310	300	20	3840	4010	3925	170	4.25	4.35	4.30	0.10	41.5	7.7	15.5	30.2	3.4	98
3 B	55		515	500	508	15	3090	2940	3015	150	300	315	308	15	4120	4300	4210	180	4.45	4.25	4.35	0.20	42.1	7.9	15.7	30.3	3.1	99
16 A	5		670	680	675	10	5620	5430	5525	190	515	510	513	5	6500	6510	6505	10	5.15	5.10	5.13	0.05	44.0	11.3	14.3	26.5	4.0	100
16 B	59		690	655	673	35	5100	5110	5105	10	500	459	480	41	6010	6050	6030	40	4.95	4.55	4.75	0.40	42.1	10.8	13.7	25.4	5.4	97
21 A	57	M	480	515	498	35	2800	2860	2830	60	285	310	298	25	3550	3450	3500	100	3.95	3.65	3.80	0.30	34.4	7.7	13.4	34.3	6.6	96
21 B	25	A	445	490	468	45	2390	2410	2400	20	270	305	288	35	3120	3020	3070	100	3.45	4.30	3.88	0.85	34.1	7.6	13.3	34.0	7.0	96
26 A	48	C	660	705	683	45	3090	5240	5165	150	520	510	515	10	6000	6250	6125	250	4.75	5.10	4.93	0.35	35.4	10.8	15.8	34.4	2.8	99
26 B	16	H	695	685	690	10	4640	4680	4660	40	450	450	450	0	5330	5530	5430	200	4.65	4.85	4.75	0.20	35.2	10.8	15.7	34.2	4.1	100
38 A	71	L	580	575	578	5	3720	3640	3680	80	420	390	405	30	4810	4830	4820	20	4.05	4.10	4.08	0.05	35.1	10.8	15.7	31.5	7.6	101
38 B	37	I	585	560	573	25	3450	3274	3360	176	290	355	323	65	4160	4120	4140	40	4.20	4.05	4.13	0.15	35.4	10.8	15.8	31.7	7.6	101
41 A	49	N	450	415	433	35	2790	2670	2730	120	295	330	313	35	3510	3440	3475	70	4.20	4.20	4.20	0.00	33.8	7.6	13.2	38.8	3.4	97
41 B	52	E	460	430	445	30	2760	3150	2955	390	385	350	368	35	3940	4100	4020	160	4.50	4.40	4.45	0.10	34.6	7.7	13.5	39.8	3.8	99
52 A	35		655	705	680	50	4750	4880	4815	130	450	385	418	65	5020	5260	5140	240	5.00	5.20	5.10	0.20	42.1	10.8	13.7	30.5	3.2	100
52 B	62		650	675	663	25	4350	4390	4370	40	440	400	420	40	5370	5420	5395	50	5.20	4.85	5.03	0.35	42.1	10.8	13.7	30.4	4.0	101
63 A	72		375	390	383	15	2070	2040	2055	30	290	275	283	15	2790	2790	2790	0	3.45	3.45	3.45	0.00	41.8	7.8	15.6	27.2	8.5	101
63 B	68		365	385	375	20	2010	2010	2010	0	305	320	313	15	2960	2910	2935	50	3.75	3.60	3.68	0.15	41.8	7.8	15.6	27.2	8.6	101

Table 41. Data base B (continued).

Design Sequence	Rep	Mix Sequence	Date	CAT	CAM (in)	FAM	AEQ (oz)	CAQ (lb)	CEQ (lb)	WAQ (lb)	FAQ (lb)	WCR (lb/lb)	ACR (lb/lb)	SLMP (in)	UNWT (pcf)	YLD (CY)	AIR (X)	TEMP (oF)
	A			0=CLS	0.75	2.10	0.0	1542	426	236	SSD							
	or B			1=SRG	1.50	2.84	6.5	1850	584	270								
1		14	10/05	0	0.75	2.10	0.0	1542	426	236	1772	0.55	7.78	0.00	137.5	28.9	3.1	79.0
2		17	10/07	0	0.75	2.10	0.0	1542	584	270	1552	0.46	5.30	0.00	141.4	27.9	4.4	80.0
3	A	30	10/18	0	0.75	2.10	0.0	1850	426	270	1371	0.63	7.56	1.00	140.1	28.0	3.4	81.0
3	B	55	11/16	0	0.75	2.10	0.0	1850	426	270	1358	0.63	7.53	0.75	141.4	27.6	3.1	71.0
4		58	11/17	0	0.75	2.10	0.0	1850	584	236	1315	0.40	5.42	0.00	143.7	27.7	3.6	70.0
5		33	10/20	0	0.75	2.10	6.5	1542	426	270	1507	0.63	7.16	3.50	128.4	29.2	10.5	79.0
6		18	10/07	0	0.75	2.10	6.5	1542	584	236	1463	0.40	5.15	0.00	138.3	27.7	5.5	79.0
7		43	10/31	0	0.75	2.10	6.5	1850	426	236	1269	0.55	7.32	4.50	132.4	28.6	10.0	71.0
8		24	10/11	0	0.75	2.10	6.5	1850	584	270	1064	0.46	4.99	4.75	134.9	27.9	7.3	82.0
9		40	10/28	0	0.75	2.84	0.0	1542	426	270	1677	0.63	7.56	0.50	142.2	27.5	3.5	80.0
10		10	09/30	0	0.75	2.84	0.0	1542	584	236	1644	0.40	5.46	0.00	143.9	27.8	3.0	77.0
11		67	11/21	0	0.75	2.84	0.0	1850	426	236	1425	0.55	7.69	0.00	144.5	27.4	3.1	67.0
12		65	11/19	0	0.75	2.84	0.0	1850	584	270	1227	0.46	5.27	2.00	144.8	27.1	1.9	76.0
13		12	10/05	0	0.75	2.84	6.5	1542	426	236	1597	0.55	7.37	0.13	138.7	27.4	6.1	81.0
14		69	11/21	0	0.75	2.84	6.5	1542	584	270	1367	0.46	4.98	3.50	137.8	27.3	6.8	68.0
15		53	11/15	0	0.75	2.84	6.5	1850	426	270	1181	0.63	7.12	8.00	128.1	29.0	12.0	79.0
16	A	5	09/27	0	0.75	2.84	6.5	1850	584	236	1137	0.40	5.11	0.75	144.3	26.4	4.0	87.6
16	B	59	11/17	0	0.75	2.84	6.5	1850	584	236	1139	0.40	5.12	1.75	141.1	27.6	5.4	68.0
17		4	09/26	0	1.50	2.10	0.0	1542	426	270	1666	0.63	7.53	0.00	137.8	28.3	5.0	88.0
18		54	11/16	0	1.50	2.10	0.0	1542	584	236	1614	0.40	5.40	0.00	142.5	27.9	4.7	72.0
19		31	10/19	0	1.50	2.10	0.0	1850	426	236	1423	0.55	7.68	0.50	141.0	27.9	4.1	79.0
20		19	10/08	0	1.50	2.10	0.0	1850	584	270	1202	0.46	5.23	5.00	142.0	27.5	2.5	83.0
21	A	57	11/17	0	1.50	2.10	6.5	1542	426	236	1568	0.55	7.30	0.25	133.7	28.2	6.6	70.0
21	B	25	10/12	0	1.50	2.10	6.5	1542	426	236	1568	0.55	7.30	0.75	133.0	28.4	7.0	81.0
22		42	10/31	0	1.50	2.10	6.5	1542	584	270	1352	0.46	4.96	6.00	132.8	28.2	10.0	71.0
23		21	10/10	0	1.50	2.10	6.5	1850	426	270	1156	0.63	7.06	5.00	134.3	27.6	7.2	85.0
24		36	10/24	0	1.50	2.10	6.5	1850	584	236	1115	0.40	5.08	3.00	138.7	27.3	5.5	82.0
25		28	10/14	0	1.50	2.84	0.0	1542	426	236	1747	0.55	7.72	0.50	141.3	28.0	3.8	78.0
26	A	48	11/03	0	1.50	2.84	0.0	1542	584	270	1530	0.46	5.26	0.75	143.0	27.4	2.8	80.0
26	B	16	10/07	0	1.50	2.84	0.0	1542	584	270	1525	0.46	5.25	2.50	142.4	27.5	4.1	76.0
27		39	10/28	0	1.50	2.84	0.0	1850	426	270	1334	0.63	7.47	5.50	144.0	26.9	1.5	80.0
28		64	11/19	0	1.50	2.84	0.0	1850	584	236	1296	0.40	5.39	0.50	146.3	27.1	2.5	75.0
29		50	11/15	0	1.50	2.84	6.5	1542	426	270	1481	0.63	7.10	6.25	132.2	28.1	9.0	79.0
30		11	09/30	0	1.50	2.84	6.5	1542	584	236	1437	0.40	5.10	0.13	142.2	26.7	5.8	77.0
31		41	10/31	0	1.50	2.84	6.5	1850	426	236	1254	0.55	7.29	5.50	136.2	27.6	7.0	72.0
32		44	11/02	0	1.50	2.84	6.5	1850	584	270	1034	0.46	4.94	3.25	139.9	26.7	5.3	72.0
33		70	11/22	1	0.75	2.10	0.0	1542	426	270	1716	0.63	7.45	0.00	141.5	27.9	3.0	68.0
34		45	11/02	1	0.75	2.10	0.0	1850	584	236	1370	0.40	5.51	0.00	144.9	27.9	8.2	75.0
35		29	10/14	1	0.75	2.10	0.0	1850	426	236	1501	0.55	7.87	0.25	142.8	28.1	4.0	78.0
36		51	11/15	1	0.75	2.10	0.0	1850	584	270	1279	0.46	5.36	1.50	144.3	27.6	2.5	79.0
37		2	09/21	1	0.75	2.10	6.5	1542	426	236	1628	0.55	7.44	0.00	135.8	28.2	11.6	89.0
38	A	71	12/06	1	0.75	2.10	6.5	1542	584	270	1409	0.46	5.05	2.00	137.6	27.6	7.6	71.0
38	B	37	10/24	1	0.75	2.10	6.5	1542	584	270	1408	0.46	5.05	2.50	136.9	27.4	7.6	80.0
39		22	10/10	1	0.75	2.10	6.5	1850	426	270	1237	0.63	7.25	8.75	133.0	28.4	9.4	80.0
40		56	11/17	1	0.75	2.10	6.5	1850	584	236	1190	0.40	5.21	0.25	143.8	26.8	5.1	70.0
41	A	49	11/04	1	0.75	2.84	0.0	1542	426	236	1809	0.55	7.87	0.00	140.1	28.7	3.4	85.0
41	B	52	11/15	1	0.75	2.84	0.0	1542	426	236	1807	0.55	7.86	0.00	143.2	28.0	3.8	79.0
42		46	11/02	1	0.75	2.84	0.0	1542	584	270	1589	0.46	5.36	0.00	145.4	27.4	3.1	71.0
43		3	09/23	1	0.75	2.84	0.0	1850	426	270	1409	0.63	7.65	2.25	145.4	27.2	2.3	88.0
44		7	09/27	1	0.75	2.84	0.0	1850	584	236	1367	0.40	5.51	0.00	148.8	27.1	2.5	84.5
45		20	10/08	1	0.75	2.84	6.5	1542	426	270	1549	0.63	7.26	7.25	131.1	28.9	11.0	81.0
46		32	10/20	1	0.75	2.84	6.5	1542	584	236	1501	0.40	5.21	1.50	143.1	27.0	6.0	86.0
47		26	10/12	1	0.75	2.84	6.5	1850	426	236	1326	0.55	7.46	2.25	137.6	27.9	8.2	80.0
48		60	11/18	1	0.75	2.84	6.5	1850	584	270	1102	0.46	5.06	4.75	141.4	26.8	5.2	76.0
49		23	10/11	1	1.50	2.10	0.0	1542	426	236	1803	0.55	7.85	0.00	131.3	30.5	10.0	81.0
50		13	10/05	1	1.50	2.10	0.0	1542	584	270	1582	0.46	5.35	0.00	144.0	27.6	3.0	86.0
51		6	09/27	1	1.50	2.10	0.0	1850	426	270	1407	0.63	7.65	1.00	143.0	27.6	4.2	84.5
52	A	35	10/24	1	1.50	2.10	0.0	1850	584	236	1364	0.40	5.50	0.00	146.4	27.6	3.2	80.0
52	B	62	11/18	1	1.50	2.10	0.0	1850	584	236	1363	0.40	5.50	0.25	146.2	27.6	4.0	75.0
53		27	10/13	1	1.50	2.10	6.5	1542	426	270	1537	0.63	7.23	2.25	131.2	28.8	11.0	78.0
54		9	09/30	1	1.50	2.10	6.5	1542	584	236	1494	0.40	5.20	0.00	141.2	27.3	5.2	75.0
55		63	11/19	1	1.50	2.10	6.5	1850	426	236	1317	0.55	7.43	1.00	139.1	27.5	7.2	74.0
56		1	09/21	1	1.50	2.10	6.5	1850	584	270	1099	0.46	5.05	2.00	142.5	26.7	5.0	87.0
57		8	09/28	1	1.50	2.84	0.0	1542	426	270	1713	0.63	7.64	0.75	143.4	27.6	3.6	84.5
58		15	10/06	1	1.50	2.84	0.0	1542	584	236	1674	0.40	5.51	0.00	146.4	27.6	3.0	78.0
59		61	11/18	1	1.50	2.84	0.0	1850	426	236	1495	0.55	7.85	0.25	145.4	27.6	3.6	77.0
60		66	11/19	1	1.50	2.84	0.0	1850	584	270	1275	0.46	5.35	1.50	148.0	26.9	2.8	75.0
61		38	10/25	1	1.50	2.84	6.5	1542	426	236	1628	0.55	7.44	5.50	132.3	29.0	11.0	77.0
62		34	10/20	1	1.50	2.84	6.5	1542	584	270	1409	0.55	5.05	5.75	137.1	27.8	7.5	79.0
63	A	72	12/06	1	1.50	2.84	6.5	1850	426	270	1228	0.63	7.23	6.75	135.4	27.8	8.5	73.0
63	B	68	11/21	1	1.50	2.84	6.5	1850	426	270	1228	0.63	7.23	7.75	145.6	27.8	8.6	68.0
64		47	11/02	1	1.50	2.84	6.5	1850	584	236	1188	0.40	5.20	1.50	145.5	26.5	4.4	70.0

Table 42. Data base C.

M A T C H
L I N E

Design Seq.	Rep Seq.	Mix Seq.	fr71 fr72fr7AVEfr7DIF (psi)(psi)(psi) (psi)				fpc71fpc72fpc7AVEfpcDIF (psi)(psi) (psi) (psi)				ft71 ft72ft7AVEft7DIF (psi)(psi)(psi) (psi)				fpc283fpc284fpc28AVEfpc28DIF (psi) (psi) (psi) (psi)				Ec283 Ec284 Ec28AVEEc28DIF (MPSI)(MPSI)(MPSI) (MPSI)				PERCENTAGES BY VOLUME					
			7-DAY FLEX. STRENGTH.				7-DAY COMPR. STRENGTH.				7-DAY TENSILE STRENGTH				28-DAY COMPR. STRENGTH				28-DAY MOD. OF ELASTICITY				CAP (%)	CEP (%)	WAP (%)	FAP (%)	AIRP (%)	CUMUL (%)
1		14	420	405	413	15	2230	2240	2235	10	290	300	295	10	2940	2920	2930	20	3.65	3.80	3.73	0.15	33.4	7.5	13.1	37.6	3.1	95
2		17	700	615	658	85	4400	4510	4455	110	435	460	448	25	5080	5140	5110	60	4.60	4.50	4.55	0.10	34.7	10.6	15.5	34.3	4.4	100
3	A	30	495	425	460	70	2830	2990	2910	160	290	310	300	20	3840	4010	3925	170	4.25	4.35	4.30	0.10	41.5	7.7	15.5	30.2	3.4	98
3	B	55	515	500	508	15	3090	2940	3015	150	300	315	308	15	4120	4300	4210	180	4.45	4.25	4.35	0.20	42.1	7.9	15.7	30.3	3.1	99
4		58	750	775	763	25	5410	5440	5425	30	450	500	475	50	6570	6740	6655	170	5.05	5.05	5.05	0.00	42.0	10.7	13.7	29.3	3.6	99
5		33	400	425	413	25	1880	2020	1950	140	310	290	300	20	2860	2830	2845	30	3.30	3.40	3.35	0.10	33.2	7.4	14.8	31.8	10.5	98
6		18	665	675	670	10	4480	4790	4635	310	450	445	448	5	5310	4950	5130	360	4.30	4.35	4.33	0.05	35.0	10.7	13.7	32.6	5.5	97
7		43	445	460	453	15	2470	2560	2515	90	275	245	260	30	3250	3330	3290	80	3.80	3.80	3.80	0.00	40.7	7.6	13.2	27.3	10.0	99
8		24	605	580	593	25	3870	3990	3930	120	420	405	413	15	5130	5130	5130	0	4.10	4.00	4.05	0.10	41.7	10.6	15.5	23.5	7.3	99
9		40	540	480	510	60	3040	2920	2980	120	310	290	300	20	4030	4020	4025	10	4.30	4.55	4.43	0.25	35.2	7.9	15.7	37.5	3.5	100
10		10	585	655	620	70	5120	5010	5065	110	435	460	448	25	6110	6060	6085	50	4.80	5.00	4.90	0.20	34.9	10.7	13.6	36.5	3.0	99
11		67	610	645	628	35	4230	3970	4100	260	470	460	465	10	5130	5360	5245	230	4.85	4.85	4.85	0.00	42.4	7.9	13.8	32.1	3.1	99
12		65	725	700	713	25	4810	5100	4955	290	520	535	528	15	6310	6210	6260	100	4.95	4.95	4.95	0.00	42.9	11.0	16.0	27.9	1.9	100
13		12	475	580	528	105	3620	3520	3570	100	380	440	410	60	4570	4370	4470	200	4.45	4.60	4.53	0.15	35.4	7.9	13.8	35.9	6.1	99
14		69	635	635	635	0	4680	4560	4620	120	400	470	435	70	5340	5140	5240	200	4.40	4.25	4.33	0.15	35.5	10.9	15.8	30.9	6.8	100
15		53	345	330	338	15	1990	1950	1970	40	295	235	265	60	2630	2540	2585	90	3.20	2.95	3.08	0.25	40.1	7.5	14.9	25.1	12.0	100
16	A	5	670	680	675	10	5620	5430	5525	190	515	510	513	5	6500	6510	6505	10	5.15	5.10	5.13	0.05	44.0	11.3	14.3	26.5	4.0	100
16	B	59	690	655	673	35	5100	5110	5105	10	500	300	400	200	6010	6050	6030	40	4.95	4.55	4.75	0.40	42.1	10.8	13.7	25.4	5.4	97
17		4	445	405	425	40	2520	2350	2435	170	320	310	315	10	3070	3030	3050	40	4.35	4.50	4.43	0.15	34.2	7.7	15.3	36.3	5.0	98
18		54	630	590	610	40	4650	4610	4630	40	505	485	495	20	5910	5640	5775	270	5.10	4.95	5.03	0.15	34.7	10.6	13.6	35.7	4.7	99
19		31	520	515	518	5	3140	3300	3220	160	400	365	383	35	3880	4180	4030	300	4.45	4.45	4.45	0.00	41.7	7.8	13.6	31.4	4.1	99
20		19	590	625	608	35	4400	4290	4345	110	420	430	425	10	5360	5300	5330	60	4.00	4.20	4.10	0.20	42.3	10.8	15.7	26.9	2.5	98
21	A	57	480	515	498	35	2800	2860	2830	60	285	310	298	25	3550	3450	3500	100	3.95	3.65	3.80	0.30	34.4	7.7	13.4	34.3	6.6	96
21	B	25	445	490	468	45	2390	2410	2400	20	270	305	288	35	3120	3020	3070	100	3.45	4.30	3.88	0.85	34.1	7.6	13.3	34.0	7.0	96
22		42	445	530	488	85	3390	3460	3425	70	350	415	383	65	4360	4310	4335	50	3.65	3.70	3.68	0.05	34.4	10.5	15.3	29.6	10.0	100
23		21	435	385	410	50	2760	2650	2705	110	335	315	325	20	3240	3430	3335	190	3.80	3.60	3.70	0.20	42.1	7.9	15.7	25.8	7.2	99
24		36	585	590	588	5	4450	4690	4570	240	440	420	430	20	5370	5360	5365	10	4.60	4.50	4.55	0.10	42.6	10.9	13.9	25.2	5.5	98
25		28	510	510	510	0	3470	3320	3395	150	405	390	398	15	3930	4060	3995	130	4.60	4.30	4.45	0.30	34.6	7.7	13.5	38.5	3.8	98
26	A	48	660	705	683	45	5090	5240	5165	150	520	510	515	10	6000	6250	6125	250	4.75	5.10	4.93	0.35	35.4	10.8	15.8	34.4	2.8	99
26	B	16	695	685	690	10	4640	4680	4660	40	450	450	450	0	5330	5530	5430	200	4.65	4.85	4.75	0.20	35.2	10.8	15.7	34.2	4.1	100
27		39	430	490	460	60	2900	2870	2885	30	310	330	320	20	3930	3840	3885	90	4.65	4.20	4.43	0.45	43.2	8.0	16.1	30.5	1.5	99
28		64	730	680	705	50	5330	5870	5600	540	530	505	518	25	6900	6690	6795	210	5.05	5.15	5.10	0.10	42.9	11.0	14.0	29.5	2.5	100
29		50	360	345	353	15	2060	2240	2150	180	300	245	273	55	2740	2800	2770	60	3.30	3.45	3.38	0.15	34.5	7.7	15.4	32.5	9.0	99
30		11	680	630	655	50	5220	5350	5285	130	495	460	478	35	6160	6120	6140	40	4.95	4.80	4.88	0.15	36.3	11.1	14.2	33.2	5.8	101
31		41	430	430	430	0	2560	2595	2578	35	300	350	325	50	3320	3230	3275	90	3.80	4.05	3.93	0.25	42.1	7.9	13.7	28.0	7.0	99
32		44	565	565	565	0	4420	4420	4420	0	450	430	440	20	5090	5440	5265	350	4.45	4.45	4.45	0.00	43.5	11.1	16.2	23.9	5.3	100
33		70	375	395	385	20	2580	2400	2490	180	325	325	325	0	3460	3390	3425	70	3.95	3.85	3.90	0.10	34.7	7.8	15.5	37.9	3.0	99
34		45	570	495	533	75	5480	5840	5660	360	450	485	468	35	6980	6570	6775	410	5.35	5.55	5.45	0.20	41.7	10.6	13.6	30.3	8.2	104
35		29	500	505	503	5	3680	3620	3650	60	440	420	430	20	4430	4380	4405	50	4.40	4.35	4.38	0.05	41.4	7.7	13.5	32.9	3.0	99
36		51	635	690	663	55	4170	3900	4035	270	435	375	405	60	4870	4790	4830	80	7.10	4.65	5.88	2.45	42.1	10.8	15.7	28.6	2.5	100
37		2	220	440	330	220	1310	1320	1315	10	225	205	215	20	1940	1810	1875	130	2.60	3.30	2.95	0.70	34.4	7.7	13.4	35.6	11.6	103
38	A	71	580	575	578	5	3720	3640	3680	80	420	390	405	30	4810	4830	4820	20	4.05	4.10	4.08	0.05	35.1	10.8	15.7	31.5	7.6	101
38	B	37	585	560	573	25	3450	2710	3080	740	290	355	323	65	4160	4120	4140	40	4.20	4.05	4.13	0.15	35.4	10.8	15.8	31.7	7.6	101
39		22	380	415	398	35	2050	2060	2055	10	270	265	268	5	2700	2790	2745	90	3.50	3.40	3.45	0.10	40.9	7.6	15.2	26.8	9.4	100
40		56	690	700	695	10	4590	4730	4660	140	400	438	419	38	5390	5270	5330	120	5.05	4.90	4.98	0.15	43.4	11.1	14.1	27.4	5.1	101
41	A	49	450	415	433	35	2790	2670	2730	120	295	330	313	35	3510	3440	3475	70	4.20	4.20	4.20	0.00	33.8	7.6	13.2	38.8	3.4	97
41	B	52	460	430	445	30	2760	3150	2955	390	385	350	368	35	3940	4100	4020	160	4.50	4.40	4.45	0.10	34.6	7.7	13.5	39.8	3.8	99
42		46	740	755	748	15	4530	4380	4455	150	410	470	440	60	5030	5370	5200	340	4.80	4.80	4.80	0.00	35.4	10.8	15.8	35.7	3.1	101
43		3	470	475	473	5	3520	3250	3385	270	400	410	405	10	4290	4350	4320	60	4.35	3.75	4.05	0.60	42.7	8.0	15.9	31.9	2.3	101
44		7	725	755	740	30	5100	5010	5055	90	460	440	450	20														

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